



Power Mean of Power Means of Possible Subsets of a Set of Numbers

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ABSTRACT: One property of **power mean** regarding relationship between the **power mean** of the elements of a set of positive real numbers and the **power means** of the respective elements of the respective possible subsets of the set has mathematically been derived in this study. The mathematical derivation of the property has been presented in this article.

KEYWORDS: Set, Positive Real Numbers, Possible Subsets, **Power Mean**, Relationship

I. INTRODUCTION

Concept of average [1, 57] has been found to be the prime player in understanding and developing measures in every branch of science and technology. Several researches had already been done on average [1, 57] since the introduction of its concept. As per the history of development of theory of average is known, Pythagoras was the first mathematician to introduce the concept of measure of average and consequently to develop three measures of average termed as **arithmetic mean** [2, 6, 64], **geometric mean** [2, 6] and **harmonic mean** [2, 6, 64] which together is also popularly known as “Pythagorean Means” [3, 7, 15]. Later on, more measures of average were developed due to necessity of handling different situations some of which are **quadratic mean** or **root mean square**, **square root mean**, **cubic mean**, **cube root mean**, **power mean** etc. [4, 8, 18, 32, 45, 55, 56, 58, 59, 63]. In addition to these, generalized definitions of average namely **generalized f – mean** or **generalized f_A – mean**, **generalized f_G – mean** and **generalized f_H – mean** had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable and successively a generalized method was developed for defining average of a function of a set (or of a list) of values [9, 16, 17, 20]. In another study, four formulations of average were derived from the three Pythagorean means which are **arithmetic- geometric mean**, **arithmetic- harmonic mean**, **geometric - harmonic mean** and **arithmetic-geometric - harmonic** respectively [19, 32].

Usually, each measure of average carries / satisfies / fulfils some properties which may be known or un known. Several studies have already been done on the properties of **arithmetic mean**, **geometric mean**, **harmonic mean**, **quadratic mean**, **cubic mean** & **cube root mean** [2, 3, 6, 39, 40, 42 – 54, 64, 65]. **Arithmetic mean**, **geometric mean** & **harmonic mean** have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7, 15, 21 – 31, 36, 37] and in developing the statistical concept of expectation [5, 33 – 35, 38, 41, 60, 62]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of **arithmetic mean** which states that the **arithmetic mean** of the **arithmetic means** of the respective possible subsets of fixed size of a set of real numbers is the **arithmetic mean** of the original set of numbers and also the **arithmetic mean** of the **arithmetic means** of the respective non-empty possible subsets a set of real numbers is the **arithmetic mean** of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48]. Similar properties of **geometric mean**, **harmonic mean**, **quadratic mean**, **square root mean**, **cubic mean** and **cube root mean** have also been mathematically established in separate studies [49, 50, 51, 52, 53, 54]. The Similar property of **power mean** has mathematically been derived in this study. Derivation of the property has been presented in this article.



II. SET OF REAL NUMBERS: POWER MEAN

Power mean refers to a family of mathematical means, like the arithmetic, geometric, and harmonic means, quadratic mean etc. generalized by a parameter p and it is more specifically termed as power mean with exponent p or Generalized p – mean or simply p – mean [57 , 61].

Definition: If p is a non-zero real number and

$$x_1, x_2, \dots, x_n$$

are n positive real numbers, then the generalized p – mean or simply p – mean or power mean with exponent p of them, denoted by $M_p(x_1, x_2, \dots, x_n)$, is defined by [2, 61]

$$M_p(x_1, x_2, \dots, x_n) = \left\{ \frac{1}{n} (x_1^p + x_2^p + \dots + x_n^p) \right\}^{\frac{1}{p}}$$

Note: The definition of power mean implies that

$$x_1^p + x_2^p + \dots + x_n^p = n \cdot \{ M_p(x_1, x_2, \dots, x_n) \}^p$$

i.e. the sum of p^{th} powers of n positive real numbers is n times of the p^{th} power of the power mean with exponent p of the numbers.

III. POSSIBLE SUBSETS OF FIXED SIZE: POWER MEAN OF POWER MEANS

Let us abbreviate power mean with exponent p by $PM-p$.

Suppose, a set S consists of the N real numbers

$$a_1, a_2, \dots, a_N$$

as elements so that

$$\text{sum of the } p^{\text{th}} \text{ powers of the } N \text{ elements of } S = a_1^p + a_2^p + \dots + a_N^p$$

& the $PM-p$ of the N elements of $S = \left\{ \frac{1}{N} (a_1^p + a_2^p + \dots + a_N^p) \right\}^{\frac{1}{p}} = M$, say

Let us consider the possible subsets of S having n elements in each set.

The number of such possible subsets is $C(N, n)$

$$\text{where } C(N, n) = {}^N C_n = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Among the $C(N, n)$ possible subsets, there are

$C(N - 1, n - 1)$ subsets with a_1 as 1st element,

$C(N - 2, n - 2)$ subsets with a_2 as 1st element and not having a_1 ,

$C(N - 3, n - 3)$ subsets with a_3 as 1st element and not having a_1 & a_2 ,

.....

$C(N - 1, N - 2)$ subsets with a_{N-n+2} as 1st element and not having $a_1, a_2, \dots, a_{N-n+3}$,

$C(N - 1, N - 1)$ subsets with a_{N-n+1} as 1st element and not having $a_1, a_2, \dots, a_{N-n+2}$,

such that



Total number of possible subsets
 $= C(N-1, n-1) + C(N-2, n-2) + C(N-3, n-3) + \dots + C(N-1, N-2) + C(N-1, N-1) = C(N, n)$

and that each a_i appears a total of $C(N-1, n-1)$ times in the set containing all possible $C(N, n)$ subsets.

Suppose,

$$M_{p1}, M_{p2}, M_{p3}, \dots, M_{p C(N, n)-1}, M_{p C(N, n)}$$

are the $PM-p$ s of the respective subsets and

$$S_1, S_2, S_3, \dots, S_{C(N, n)-1}, S_{C(N, n)}$$

are the **sums of p^{th} powers** of the respective n elements in the respective subsets so that

$$S_1 = n M_{p1}^p,$$

$$S_2 = n M_{p2}^p,$$

$$S_3 = n M_{p3}^p,$$

.....

$$S_{C(N, n)-1} = n M_{p C(N, n)-1}^p,$$

$$S_{C(N, n)} = n M_{p C(N, n)}^p.$$

Now,

$$S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)}$$

is the **sum of the p^{th} powers** of all elements in the set containing the elements of all possible $C(N, n)$ subsets of the original set S where each a_i appears a total of $C(N-1, n-1)$ times.

Therefore,

$$S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)} = C(N-1, n-1) (a_1^p + a_2^p + \dots + a_N^p)$$

Accordingly,

$$\begin{aligned} & \text{PM-p of the PM-p s of the respective elements of the respective possible } C(N, n) \text{ subsets} \\ &= \left\{ \frac{1}{C(N, n)} (M_{p1}^p + M_{p2}^p + M_{p3}^p + \dots + M_{p C(N, n)-1}^p + M_{p C(N, n)}^p) \right\}^{\frac{1}{p}} \\ &= \left\{ \frac{1}{n C(N, n)} \frac{1}{n} (S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)}) \right\}^{\frac{1}{p}} \\ &= \left\{ \left\{ \frac{1}{n C(N, n)} \frac{1}{n} C(N-1, n-1) (a_1^p + a_2^p + \dots + a_N^p) \right\}^{\frac{1}{p}} \right\}^{\frac{1}{p}} \\ &= \left\{ \frac{1}{N} (a_1^p + a_2^p + \dots + a_N^p) \right\}^{\frac{1}{p}} \\ &= M = \text{PM-p of the elements of } S \end{aligned}$$

Therefore,

$$\text{PM-p of the PM-p s of the respective possible } C(N, n) \text{ subsets} = \text{PM-p of the elements of } S$$



IV. POSSIBLE NON-EMPTY SUBSETS: POWER ER MEANS

Now, the set S has a total of (2^n - 1) number of non-empty subsets of which

- number of possible subsets having single element in each is C(N, 1),
number of possible subsets having 2 elements in each is C(N, 2),
.....
number of possible subsets having n - 1 elements in each is C(N, n - 1),
number of possible subsets having n elements is in each C(N, n)

such that

Total number of all possible non-empty subsets
= C(N, 1) + C(N, 2) + + C(N, n - 1) + C(N, n) = 2^n - 1

By the results obtained in section III,

PM-p of the PM-p s of the respective elements of the respective possible subsets having 1 element in each =
PM-p of the elements of S = M

Similarly,

PM-p of the PM-p s of the respective elements of the respective possible subsets having 2 elements in each =
M,

PM-p of the PM-p s of the respective elements of the respective possible subsets having n - 1 elements in
each = M,

PM-p of the PM-p s of the respective elements of the respective possible subsets having n elements in each =
M.

Therefore,

PM-p of the PM-p s of the respective elements of the respective non-empty possible subsets of S = M

i.e. PM-p of the PM-p s of the respective elements of the respective non-empty possible subsets of S is PM-
p of the elements of S

V. CONCLUSION

Findings on the property of power mean, obtained in this study, can be summarized as follows:

“The power mean with exponent p of the power means with the same exponent of the respective elements of the
respective possible subsets of fixed size of a set of positive real numbers is the power mean with the same exponent
of the elements of the original set of numbers

and

the power mean with exponent p of the power means with the same exponent of the respective elements of the
respective non-empty possible subsets a set of positive real numbers is the power mean with the same exponent
of the elements of the original set of numbers.”

This property of power mean is a relation between the power mean of the elements of a set of real numbers and
the power means of the respective elements of the respective possible subsets of the set.

The property/result on power mean obtained here is hoped to be useful for analysis of data specially on
estimation based on data available in sample drawn from population.



Moreover, the property/result obtained here is hoped to be a useful in making the literature of average more enriched.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class & 1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (in Vocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.



(Dr. Dhritikesh Chakrabarty with his elder son Sri Mrityunjay Chakrabarty (Right) and younger son Sri Hrimaanjay Chakrabarty (Left) in a busy moment while checking smart mobile phone for purchasing)

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of more than 300 published research items namely research papers, chapter in books / conference



**International Journal of Advanced
Research in Science, Engineering
and Technology**

ISSN: 2350-0328



Vol. 13, Issue 1, January 2026

proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post-doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.