

A Collection of Special Diophantine Ellipse and Pythagorean Equations with Integer Solutions

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ABSTRACT: Learning about the various techniques to solve this higher power Diophantine equation in successfully deriving their solutions help us understand how numbers work and their significance in different areas of mathematics and science. In this paper, First focused to study infinitely many integer solutions of following Diophantine Equations.

$$2x^2 + y^2 = z^2; 3x^2 + y^2 = z^2; 4x^2 + y^2 = z^2; 5x^2 + y^2 = z^2; 6x^2 + y^2 = z^2; \\ 7x^2 + y^2 = z^2; 8x^2 + y^2 = z^2; 9x^2 + y^2 = z^2; 10x^2 + y^2 = z^2; 11x^2 + y^2 = z^2; \\ 12x^2 + y^2 = z^2; 13x^2 + y^2 = z^2; 14x^2 + y^2 = z^2; 15x^2 + y^2 = z^2; 16x^2 + y^2 = z^2; \\ 17x^2 + y^2 = z^2; 18x^2 + y^2 = z^2; 19x^2 + y^2 = z^2; 20x^2 + y^2 = z^2; 21x^2 + y^2 = z^2;$$

Also, $kx^2 + y^2 = z^2$ having ellipse equation form of $k\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Also, focused to study Reciprocal form of above Diophantine Equation $\frac{k}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$. Which is having different sets of integer solutions of $p = yz$, $q = xz$ and $h = xy$. Also, focused to obtained infinitely many Integer solutions of following Special Diophantine Pythagorean Equations.

$$p^2 + q^2 + r^2 + s^2 = t^2 + u^2; \quad p^2 + q^2 + r^2 = s^2; \quad p^2 + q^2 + r^2 + s^2 = t^2; \\ p^4 + q^4 + 2r^2 = s^4; \quad p^2 + q^2 + t^2 = r^2 + s^2 + u^2; \quad p^6 + q^6 + 3r^2 = s^6; \\ x^3 + y^4 = z^5; \quad x^3 + y^3 = z^2; \quad x^2 + y^3 + z^4 = w^5; \quad x^2 + y^3 + z^4 + w^5 = u^2;$$

KEYWORDS: Diophantine Equations, Pythagorean theorem, Reciprocal Pythagorean theorem, Ellipse, Reciprocal ellipse equations.

Mathematics Subject Classifications:11D72,11D61,

I. INTRODUCTION

The fascinating branch of Mathematics is the Theory of Numbers in which the subject of Diophantine equations requiring only the integer solutions is an interesting area to various mathematicians. In other words, the theory of Diophantine equations is an ancient subject that typically involves solving, polynomial equation in two or more variables or a system of polynomial equations with the number of unknowns greater than the number of equations, in integers and occupies a pivotal role in the region of mathematics.

II. METHODS

Now we are focused to study to obtain integer solutions of special Diophantine equation $kx^2 + y^2 = z^2$ and Reciprocal curve equation $\frac{k}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ with using of trial-and-error method. In particularly, focused to obtain integer solutions of above Diophantine equations with k values are varies from 2 to 21.

III.RESULTS:

In this paper, First focused to study integer solutions of following Diophantine Equations.

$$\begin{aligned} 2x^2 + y^2 &= z^2; \quad 3x^2 + y^2 = z^2; \quad 4x^2 + y^2 = z^2; \quad 5x^2 + y^2 = z^2; \quad 6x^2 + y^2 = z^2; \\ 7x^2 + y^2 &= z^2; \quad 8x^2 + y^2 = z^2; \quad 9x^2 + y^2 = z^2; \quad 10x^2 + y^2 = z^2; \quad 11x^2 + y^2 = z^2; \\ 12x^2 + y^2 &= z^2; \quad 13x^2 + y^2 = z^2; \quad 14x^2 + y^2 = z^2; \quad 15x^2 + y^2 = z^2; \quad 16x^2 + y^2 = z^2; \\ 17x^2 + y^2 &= z^2; \quad 18x^2 + y^2 = z^2; \quad 19x^2 + y^2 = z^2; \quad 20x^2 + y^2 = z^2; \quad 21x^2 + y^2 = z^2; \end{aligned}$$

Case 1: Consider Diophantine equation $2x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 3(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 3(2)^n$ are satisfies the integer solution of Diophantine equation $2x^2 + y^2 = z^2$.

$$\text{Since } 2(2^{n+1})^2 + (2^n)^2 = (2^{2n+3}) + (2^{2n}) = (2^{2n})(2^3 + 1) = (3(2)^n)^2.$$

Lemma 1.1: It is having Ellipse equation form of $2\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Which is having simple form of ellipse $2p^2 + q^2 = 1$, whose solution is $p = \frac{2}{3}$, $q = \frac{1}{3}$.

Lemma 1.2: Reciprocal form of above Diophantine Equation $\frac{2}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 3(2^{2n}), \quad q = xz = 3(2^{2n+1}) \text{ and } h = xy = 2^{2n+1}.$$

$$\text{Since } \frac{2}{(3(2^{2n}))^2} + \frac{1}{(3(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2};$$

Case 2: Consider Diophantine equation $3x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^n$, $y = 3^n$ and $z = 2(3)^n$. Here n is positive integer.

Proof: Let $x = 3^n$, $y = 3^n$ and $z = 2(3)^n$ are satisfies the integer solution of Diophantine equation $3x^2 + y^2 = z^2$.

$$\text{Since } 3(3^n)^2 + (3^n)^2 = (3^{2n+1}) + (3^{2n}) = (3^{2n})(3 + 1) = (2(3)^n)^2.$$

Lemma 2.1: It is having Ellipse equation form of $3\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Which is having simple form of ellipse $3p^2 + q^2 = 1$, whose solution is $p = q = \frac{1}{2}$.

Lemma 2: Reciprocal form of above Diophantine Equation $\frac{3}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 2(3^{2n}), \quad q = xz = 2(3^{2n}) \text{ and } h = xy = 3^{2n}.$$

$$\text{Since } \frac{3}{(2(3^{2n}))^2} + \frac{1}{(2(3^{2n}))^2} = \frac{1}{(3^{2n})^2};$$

Case 3: Consider Diophantine equation $4x^2 + y^2 = z^2$ having different sets of integer solutions is $(2x, y, z)$ is a Pythagorean triplet. It is having different sets of Integer solutions for each odd integer x, then $y = x^2 - 1$ and $z = x^2 + 1$.

Proof: Let $y = x^2 - 1$ and $z = x^2 + 1$.

Consider $z^2 - y^2 = (x^2 + 1)^2 - (x^2 - 1)^2 = 4x^2 = (2x)^2$.

Hence if x is an odd, then $(2x, y, z)$ is a Pythagorean triplet.

Case 4: Consider Diophantine equation $5x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 4^{n+1}$, $y = 4^n$ and $z = 3(4)^n$. Here n is positive integer.

Proof: Let $x = 4^{n+1}$, $y = 4^n$ and $z = 3(4)^n$ are satisfies the integer solution of Diophantine equation $5x^2 + y^2 = z^2$.

Since $5(4^{n+1})^2 + (4^n)^2 = (3(4)^n)^2$.

Lemma 4.1: Reciprocal form of above Diophantine Equation $\frac{5}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 3(4^{2n})$, $q = xz = 3(4^{2n+1})$ and $h = xy = 4^{2n+1}$.

Since $\frac{5}{(3(4^{2n}))^2} + \frac{1}{(3(4^{2n+1}))^2} = \frac{1}{(4^{2n+1})^2}$.

Case 5: Consider Diophantine equation $6x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 5(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 5(2)^n$ are satisfies the integer solution of Diophantine equation $6x^2 + y^2 = z^2$.

Since $6(2^{n+1})^2 + (2^n)^2 = (5(2)^n)^2$.

Lemma 5.1: Reciprocal form of above Diophantine Equation $\frac{6}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 5(2^{2n})$, $q = xz = 5(2^{2n+1})$ and $h = xy = 2^{2n+1}$.

Since $\frac{6}{(5(2^{2n}))^2} + \frac{1}{(5(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2}$.

Case 6: Consider Diophantine equation $7x^2 + y^2 = z^2$ having the different sets of integer solutions is $x = 3^n$, $y = 3^{n+1}$ and $z = 4(3)^n$. Here n is positive integer.

Proof: Let $x = 3^n$, $y = 3^{n+1}$ and $z = 4(3)^n$ are satisfies the integer solution of Diophantine equation $7x^2 + y^2 = z^2$.

Since $7(3^n)^2 + (3^{n+1})^2 = (4(3)^n)^2$.

Lemma 6.1: Reciprocal form of above Diophantine Equation $\frac{7}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 4(3^{2n+1})$, $q = xz = 4(3^{2n})$ and $h = xy = 3^{2n+1}$.

Since $\frac{7}{(4(3^{2n+1}))^2} + \frac{1}{(4(3^{2n}))^2} = \frac{1}{(3^{2n+1})^2}$.

Case 7: Consider Diophantine equation $8x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^n$, $y = 2^n$ and $z = 3(2)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 3^{n+2}$ are satisfies the integer solution of Diophantine equation $8x^2 + y^2 = z^2$.

Since $8(3^{n+1})^2 + (3^{n+1})^2 = (3^{n+2})^2$.

Lemma 7.1: Reciprocal form of above Diophantine Equation $\frac{8}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 3^{2n+3}, \quad q = xz = 3^{2n+3} \text{ and } h = xy = 3^{2n+2}.$$

$$\text{Since } \frac{8}{(3^{2n+3})^2} + \frac{1}{(3^{2n+3})^2} = \frac{1}{(3^{2n+2})^2}$$

Case 8: Consider Diophantine equation $9x^2 + y^2 = z^2$ having the two different sets of integer solutions.

If x is odd then $y = \frac{9x^2-1}{2}, z = \frac{9x^2+1}{2}$. If x is even then $y = 9\left(\frac{x}{2}\right)^2 - 1, z = 9\left(\frac{x}{2}\right)^2 + 1$.

Proof: if x is an odd integer. Let $y = \frac{9x^2-1}{2}, z = \frac{9x^2+1}{2}$.

Consider $z^2 - y^2 = \left(\frac{9x^2+1}{2}\right)^2 - \left(\frac{9x^2-1}{2}\right)^2 = 9x^2 = (3x)^2$. Hence $(3x, y, z)$ is a Pythagorean triplet.

if x is an even integer. Let $y = 9\left(\frac{x}{2}\right)^2 - 1, z = 9\left(\frac{x}{2}\right)^2 + 1$.

Consider $z^2 - y^2 = \left(9\left(\frac{x}{2}\right)^2 + 1\right)^2 - \left(9\left(\frac{x}{2}\right)^2 - 1\right)^2 = 9x^2 = (3x)^2$. Hence $(3x, y, z)$ is a Pythagorean triplet.

Case 9: Consider Diophantine equation $10x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^{n+1}, y = 6^n$ and $z = 19(6)^n$. Here n is positive integer.

Proof: Let $x = 6^{n+1}, y = 6^n$ and $z = 19(6)^n$ are satisfies the integer solution of Diophantine equation $10x^2 + y^2 = z^2$.

$$\text{Since } 10(6^{n+1})^2 + (6^n)^2 = (19(6)^n)^2.$$

Lemma 9.1: Reciprocal form of above Diophantine Equation $\frac{10}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 19(6^{2n}), \quad q = xz = 19(6^{2n+1}) \text{ and } h = xy = 6^{2n+1}.$$

$$\text{Since } \frac{10}{(19(6^{2n}))^2} + \frac{1}{(19(6^{2n+1}))^2} = \frac{1}{(6^{2n+1})^2}$$

Case 10: Consider Diophantine equation $11x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}, y = 3^n$ and $z = 10(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}, y = 3^n$ and $z = 10(3)^n$ are satisfies the integer solution of Diophantine equation $11x^2 + y^2 = z^2$.

$$\text{Since } 11(3^{n+1})^2 + (3^n)^2 = (10(3)^n)^2.$$

Lemma 10.1: Reciprocal form of above Diophantine Equation $\frac{11}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 10(3^{2n}), \quad q = xz = 10(3^{2n+1}) \text{ and } h = xy = 3^{2n+1}.$$

$$\text{Since } \frac{11}{(10(3^{2n}))^2} + \frac{1}{(10(3^{2n+1}))^2} = \frac{1}{(3^{2n+1})^2}$$

Case 11: Consider Diophantine equation $12x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}, y = 2^n$ and $z = 7(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}, y = 2^n$ and $z = 7(2)^n$ are satisfies the integer solution of Diophantine equation $12x^2 + y^2 = z^2$.

Since $12(2^{n+1})^2 + (2^n)^2 = (7(2)^n)^2$.

Lemma 11.1: Reciprocal form of above Diophantine Equation $\frac{12}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 7(2^{2n}), \quad q = xz = 7(2^{2n+1}) \text{ and } h = xy = 2^{2n+1}.$$

$$\text{Since } \frac{12}{(7(2^{2n}))^2} + \frac{1}{(7(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2}$$

Case 12: Consider Diophantine equation $13x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^n$, $y = 6^{n+1}$ and $z = 7(6)^n$. Here n is positive integer.

Proof: Let $x = 6^n$, $y = 6^{n+1}$ and $z = 7(6)^n$ are satisfies the integer solution of Diophantine equation $13x^2 + y^2 = z^2$.

$$\text{Since } 13(6^n)^2 + (6^{n+1})^2 = (7(6)^n)^2.$$

Lemma 12.1: Reciprocal form of above Diophantine Equation $\frac{13}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 7(6^{2n+1}), \quad q = xz = 7(6^{2n}) \text{ and } h = xy = 6^{2n+1}.$$

$$\text{Since } \frac{13}{(7(6^{2n+1}))^2} + \frac{1}{(7(6^{2n}))^2} = \frac{1}{(6^{2n+1})^2}$$

Case 13: Consider Diophantine equation $14x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^{n+1}$, $y = 5(6)^n$ and $z = 23(6)^n$. Here n is positive integer.

Proof: Let $x = 6^{n+1}$, $y = 5(6)^n$ and $z = 23(6)^n$ are satisfies the integer solution of Diophantine equation $14x^2 + y^2 = z^2$.

$$\text{Since } 14(6^{n+1})^2 + (5(6)^n)^2 = (23(6)^n)^2.$$

Lemma 13.1: Reciprocal form of above Diophantine Equation $\frac{14}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 115(6^{2n}), \quad q = xz = 23(6^{2n+1}) \text{ and } h = xy = 5(6)^{2n+1}.$$

$$\text{Since } \frac{14}{(115(6^{2n}))^2} + \frac{1}{(23(6^{2n+1}))^2} = \frac{1}{(5(6)^{2n+1})^2}$$

Case 14: Consider Diophantine equation $15x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 12(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 12(3)^n$ are satisfies the integer solution of Diophantine equation $15x^2 + y^2 = z^2$.

$$\text{Since } 15(3^{n+1})^2 + (3^{n+1})^2 = (12(3)^n)^2.$$

Lemma 14.1: Reciprocal form of above Diophantine Equation $\frac{15}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 12(3^{2n+1}), \quad q = xz = 12(3^{2n+1}) \text{ and } h = xy = (3)^{2n+2}.$$

$$\text{Since } \frac{15}{(12(3^{2n+1}))^2} + \frac{1}{(12(3^{2n+1}))^2} = \frac{1}{((3)^{2n+2})^2}$$

Case 15: Consider Diophantine equation $16x^2 + y^2 = z^2$ having different sets of integer solutions. If x is even then $y = 16\left(\frac{x}{2}\right)^2 - 1, z = 16\left(\frac{x}{2}\right)^2 + 1$.

Proof: if x is an even integer. Let $y = 16\left(\frac{x}{2}\right)^2 - 1, z = 16\left(\frac{x}{2}\right)^2 + 1$.

Consider $z^2 - y^2 = \left(16\left(\frac{x}{2}\right)^2 + 1\right)^2 - \left(16\left(\frac{x}{2}\right)^2 - 1\right)^2 = 16x^2 = (4x)^2$. Hence $(4x, y, z)$ is a Pythagorean triplet.

Case 16: Consider Diophantine equation $17x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}, y = 4(3)^n$ and $z = 13(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}, y = 4(3)^n$ and $z = 13(3)^n$ are satisfies the integer solution of Diophantine equation $17x^2 + y^2 = z^2$.

Since $17(3^{n+1})^2 + (4(3)^n)^2 = (13(3)^n)^2$.

Lemma 16.1: Reciprocal form of above Diophantine Equation $\frac{17}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 52(3^{2n}), q = xz = 13(3^{2n+1})$ and $h = xy = 4(3)^{2n+1}$.

Since $\frac{17}{(52(3^{2n}))^2} + \frac{1}{(13(3^{2n+1}))^2} = \frac{1}{(4(3)^{2n+1})^2}$

Case 17: Consider Diophantine equation $18x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}, y = 3(2)^n$ and $z = 9(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}, y = 3(2)^n$ and $z = 9(2)^n$ are satisfies the integer solution of Diophantine equation $18x^2 + y^2 = z^2$.

Since $18(2^{n+1})^2 + (3(2)^n)^2 = (9(2)^n)^2$.

Lemma 17.1: Reciprocal form of above Diophantine Equation $\frac{18}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 27(2^{2n}), q = xz = 9(2^{2n+1})$ and $h = xy = 3(2)^{2n+1}$.

Since $\frac{18}{(27(2^{2n}))^2} + \frac{1}{(9(2^{2n+1}))^2} = \frac{1}{(3(2)^{2n+1})^2}$

Case 18: Consider Diophantine equation $19x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}, y = 5(3)^n$ and $z = 14(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}, y = 5(3)^n$ and $z = 14(3)^n$ are satisfies the integer solution of Diophantine equation $19x^2 + y^2 = z^2$.

Since $19(3^{n+1})^2 + (5(3)^n)^2 = (14(3)^n)^2$.

Lemma 18.1: Reciprocal form of above Diophantine Equation $\frac{19}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 70(3^{2n}), q = xz = 14(3^{2n+1})$ and $h = xy = 5(3)^{2n+1}$.

Since $\frac{19}{(70(3^{2n}))^2} + \frac{1}{(14(3^{2n+1}))^2} = \frac{1}{(5(3)^{2n+1})^2}$

Case 19: Consider Diophantine equation $20x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 9(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 9(2)^n$ are satisfies the integer solution of Diophantine equation $20x^2 + y^2 = z^2$. Since $20(2^{n+1})^2 + ((2)^n)^2 = (9(2)^n)^2$.

Lemma 19.1: Reciprocal form of above Diophantine Equation $\frac{20}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 9(2^{2n}), q = xz = 9(2^{2n+1}) \text{ and } h = xy = (2)^{2n+1}.$$

$$\text{Since } \frac{20}{(9(2^{2n}))^2} + \frac{1}{(9(2^{2n+1}))^2} = \frac{1}{((2)^{2n+1})^2}$$

Case 20: Consider Pythagorean equation $21x^2 + y^2 = z^2$ having different sets of integer solutions (in terms of exponential) is $x = 2^n$, $y = 2^{n+1}$ and

$$z = 5(3)^n. \text{ Here } n \text{ is positive integer.}$$

Proof: Let $x = 2^n$, $y = 2^{n+1}$ and $z = 5(3)^n$ are satisfies the integer solution of Diophantine equation $21x^2 + y^2 = z^2$.

$$\text{Since } 21(2^{n+1})^2 + ((2)^n)^2 = (5(3)^n)^2.$$

Lemma 20.1: Reciprocal form of above Diophantine Equation $\frac{21}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 10(6^n), q = xz = 5(6^n) \text{ and } h = xy = (2)^{2n+1}.$$

$$\text{Since } \frac{21}{(10(6^n))^2} + \frac{1}{(5(6^n))^2} = \frac{1}{((2)^{2n+1})^2}$$

some special collection of Diophantine Equations, whose solutions are obtained from standard Pythagorean theorem.

Case 21: Consider Diophantine equation $x^3 + y^4 = z^5$ having integer solution is $x = 2^8$, $y = 2^6$ and $z = 2^5$.

Case 22: Consider Diophantine equation $x^3 + y^3 = z^2$ having integer solution is $x = 2^8$, $y = 2^9$ and $z = 3(2)^{12}$.

Case 23: Consider Diophantine equation $x^2 + y^3 + z^4 = w^5$ having integer solution is $x = 3^{12}$, $y = 3^8$, $z = 3^6$ and $w = 3^5$.

Case 24: Consider Diophantine equation $x^2 + y^3 + z^4 + w^5 = u^2$ having integer solution is $x = 4^{30}$, $y = 4^{20}$, $z = 4^{15}$, $w = 4^{12}$ and $u = 2^{61}$.

Case 25: Consider the Pythagorean (4;2) tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$p = x^2yh, q = y^2xh, r = yh, s = xh, t = xy, u = xyzh$$

where $x = bc$, $y = ca$, $h = ab$, $z = c^2$ with (a, b, c) is a Pythagorean triplet, which is satisfies $a^2 + b^2 = c^2$.

Proof: We know that if (a, b, c) is a Pythagorean triplet, then is satisfies $a^2 + b^2 = c^2$.

if (a, b, c) is a Pythagorean triplet then (b, c, a) is also a Reciprocal Pythagorean triplet. i.e. if $x = bc$, $y =$

$$ca, h = ab \text{ then } \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{h^2} \dots \dots \dots [1]$$

Also, if (a, b, c) is a Pythagorean triplet then (bc, ac, c^2) is also a Pythagorean triplet.

i.e $x = bc$, $y = ca$, $z = c^2$ then $x^2 + y^2 = z^2$[2]

Adding equations [1], [2], we obtain

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$p = x^2yh, q = y^2xh, r = yh, s = xh, t = xy, u = xyzh$$

where $x = bc$, $y = ca$, $h = ab$, $z = c^2$ with (a, b, c) is a Pythagorean triplet, which satisfies $a^2 + b^2 = c^2$.

E.g.1: Choose One of the Pythagorean triplets (a, b, c) is $(3, 4, 5)$, which follows

$$x = bc = 20, y = ca = 15, h = ab = 12, z = c^2 = 25$$

$$p = x^2yh = 72000, q = y^2xh = 54000, r = yh = 180, s = xh = 240, t = xy = 300,$$

$$u = xyzh = 90000.$$

$$p^2 + q^2 + r^2 + s^2 = 8100090000$$

$$t^2 + u^2 = 8100090000. \text{ Hence } p^2 + q^2 + r^2 + s^2 = t^2 + u^2$$

Also, note that (a, b, c) and (x, y, z) are Pythagorean triplets.

i.e. $a^2 + b^2 = c^2$ and $x^2 + y^2 = z^2$

Also, (x, y, h) is Reciprocal Pythagorean triplet. i.e. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{h^2}$.

Case 25.1: Consider the Pythagorean $(4;2)$ tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$ having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is odd then } q = p + 1, r = \frac{p^2-1}{2}, s = \left(\frac{p+1}{2}\right)^2 - 1, t = \frac{p^2+1}{2}, u = \left(\frac{p+1}{2}\right)^2 + 1.$$

We can verify it easily by replacing some odd integer p .

Suppose $p = 3$ then $q = 4, r = 4, s = 3, t = 5, u = 5$.

$$\text{Hence } p^2 + q^2 + r^2 + s^2 = 3^2 + 4^2 + 4^2 + 3^2 = 50 = t^2 + u^2$$

Case 25.2: Consider the Pythagorean $(4;2)$ tuples equation as follows

$$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$$

different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is even then } q = p + 1, r = \left(\frac{p}{2}\right)^2 - 1, s = \frac{(p+1)^2-1}{2}, t = \left(\frac{p}{2}\right)^2 + 1, u = \frac{(p+1)^2+1}{2}.$$

We can verify it easily by replacing some even integer p .

Suppose $p = 4$ then $q = 5, r = 3, s = 12, t = 5, u = 13$.

$$\text{Hence } p^2 + q^2 + r^2 + s^2 = 4^2 + 5^2 + 3^2 + 12^2 = 194 = t^2 + u^2$$

Case 25.3: Consider the Pythagorean $(4;2)$ tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$ having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is even then } q = p - 1, r = \left(\frac{p}{2}\right)^2 - 1, s = \frac{(p-1)^2-1}{2}, t = \left(\frac{p}{2}\right)^2 + 1, u = \frac{(p-1)^2+1}{2}.$$

We can verify it easily by replacing some even integer p .

Case 25.4: Consider the Pythagorean $(4;2)$ tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is odd then } q = p - 1, r = \frac{p^2-1}{2}, s = \left(\frac{p-1}{2}\right)^2 - 1, t = \frac{p^2+1}{2}, u = \left(\frac{p-1}{2}\right)^2 - 1.$$

We can verify it easily by replacing some odd integer p

Case 26: Consider the Pythagorean (2;2) tuples equation as follows

$$p^2 + q^2 = r^2 + s^2 \text{ Having two types of solutions}$$

Case 26.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p^2-1}{4}\right)^2 + 1, r = \left(\frac{p^2-1}{4}\right)^2 - 1 \text{ and } s = \frac{p^2+1}{2}$$

Proof: From Reference [10],[11],[12], We know that, if p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e.

$$p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2.$$

Also, we know that, if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

If p is odd then $\frac{p^2-1}{2}$ is an even number.

Hence $(\frac{p^2-1}{2}, \left(\frac{p^2-1}{4}\right)^2 - 1, \left(\frac{p^2-1}{4}\right)^2 + 1)$ is a Pythagorean triplet.

It follows that if p is odd then $p^2 + \left(\left(\frac{p^2-1}{4}\right)^2 + 1\right)^2 = \left(\left(\frac{p^2-1}{4}\right)^2 - 1\right)^2 + \left(\frac{p^2+1}{2}\right)^2$.

Hence $p^2 + q^2 = r^2 + s^2$.

Case 26.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}, r = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2} \text{ and } s = \left(\frac{p}{2}\right)^2 + 1.$$

Proof: If p is even then $\left(\frac{p}{2}\right)^2 - 1$ is odd number.

Hence $(\left(\frac{p}{2}\right)^2 - 1, \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}, \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2})$ is a Pythagorean triplet.

if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$p^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}\right)^2 - \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$p^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}\right)^2 = \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}\right)^2 + \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

Hence $p^2 + q^2 = r^2 + s^2$.

If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}, r = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2} \text{ and } s = \left(\frac{p}{2}\right)^2 + 1.$$

We can verify it easily by replacing some even integer p.

Case 27: Consider the Pythagorean (3;1) tuples equation as follows $p^2 + q^2 + r^2 = s^2$

Having two types of solutions

Case 27.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, \quad r = \frac{\left(\frac{p^2+1}{2}\right)^2-1}{2} \quad \text{and} \quad s = \frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}$$

Proof: if p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e. $p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2$.

Also, if p is odd then $\frac{p^2+1}{2}$ is also odd integer. Hence $\left(\frac{p^2+1}{2}, \frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}, \frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right)$ is also a Pythagorean triplet.

$$\text{Hence } \left(\frac{p^2+1}{2}\right)^2 + \left(\frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}\right)^2 = \left(\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right)^2.$$

$$\left(\frac{p^2+1}{2}\right)^2 = \left(\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right)^2 - \left(\frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}\right)^2.$$

If p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e. $p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2$.

$$p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right)^2 - \left(\frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}\right)^2.$$

$$p^2 + \left(\frac{p^2-1}{2}\right)^2 + \left(\frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}\right)^2 = \left(\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right)^2. \text{ Hence } p^2 + q^2 + r^2 = s^2.$$

Case 27.2: If p is an even, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, \quad r = \frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2} \quad \text{and} \quad s = \frac{\left[\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right]^2+1}{2}.$$

Proof: If p is even then $\left(\frac{p}{2}\right)^2 + 1$ is odd. Hence $\left(\left(\frac{p}{2}\right)^2 + 1, \frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2}, \frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right)$ is also a Pythagorean triplet.

Also, we know that if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

$$\text{Hence } p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right)^2 - \left(\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2}\right)^2.$$

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2}\right)^2 = \left(\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right)^2.$$

$$\text{Hence } p^2 + q^2 + r^2 = s^2.$$

Case 27.3: $p^2 + q^2 + r^2 = s^2$ having integer solution in exponential form as follows.

$p = 2^3, q = 2^5, r = 2^6, s = 9 * 2^3$ since $(2^3)^2 + (2^5)^2 + (2^6)^2 = (9 * 2^3)^2$

Case 28: Consider the Pythagorean (4;1) tuples equation as follows

$$p^2 + q^2 + r^2 + s^2 = t^2$$

Having two types of solutions

Case 28.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, r = \frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}, s = \frac{\left[\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right]-1}{2} \text{ and } t = \frac{\left[\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right]+1}{2}$$

Proof: Similar Proof of Case 27.1, we can verify easily $p^2 + q^2 + r^2 + s^2 = t^2$. If p is odd then $p^2 + \left(\frac{p^2-1}{2}\right)^2 +$

$$\left(\frac{\left(\frac{p^2+1}{2}\right)^2-1}{2}\right)^2 + \left(\frac{\left[\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right]-1}{2}\right)^2 = \left(\frac{\left[\frac{\left(\frac{p^2+1}{2}\right)^2+1}{2}\right]+1}{2}\right)^2$$

We can verify it easily by replacing some odd integer p.

Case 28.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, r = \frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2}, s = \frac{\left[\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right]-1}{2} \text{ and } t = \frac{\left[\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right]+1}{2}$$

Proof: Similar Proof of above case.

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2-1}{2}\right)^2 + \left(\frac{\left[\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right]-1}{2}\right)^2 = \left(\frac{\left[\frac{\left(\left(\frac{p}{2}\right)^2+1\right)^2+1}{2}\right]+1}{2}\right)^2$$

We can verify it easily by replacing some even integer p.

Case 29: Consider higher degree Diophantine equation $p^4 + q^4 + 2r^2 = s^4$

Having two types of solutions

Case 29.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, r = \frac{p(p^2-1)}{2} \text{ and } s = \frac{p^2+1}{2}$$

Proof: similar proof of above.

Case 29.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, r = p \left(\left(\frac{p}{2}\right)^2 - 1\right) \text{ and } s = \left(\frac{p}{2}\right)^2 + 1$$

Proof: similar proof of above.

Case 29.3: If (x, y, z) is a Pythagorean triplet then $x^4 + y^4 + 2x^2y^2 = z^4$.

Proof: If (x, y, z) is a Pythagorean triplet. i.e. $x^2 + y^2 = z^2$. Square on Both sides, we obtain

$$(x^2 + y^2)^2 = (z^2)^2 \text{ implies that } x^4 + y^4 + 2x^2y^2 = z^4.$$

Case 30: Consider the Pythagorean (3;3) tuples equation as follows

$$p^2 + q^2 + t^2 = r^2 + s^2 + u^2$$

then different sets of integer solutions is illustrated below

$$p = x^2yh, \quad q = y^2xh, \quad r = yh, \quad s = xh, \quad t = xy, \quad u = xyzh$$

where $x = bc, y = ca, h = ab, z = c^2$ with (a, b, c) is a Pythagorean triplet, which satisfies $a^2 + b^2 = c^2$.

E.g.1: Choose the Pythagorean triplet (a, b, c) is $(3, 4, 5)$, which follows

$$x = bc = 20, \quad y = ca = 15, \quad h = ab = 12, \quad z = c^2 = 25$$

$$p = x^2yh = 72000, \quad q = y^2xh = 54000, \quad r = yh = 180, \quad s = xh = 240, \quad t = xy = 300,$$

$$u = xyzh = 90000$$

$$p^2 + q^2 + t^2 = 8100090000$$

$$r^2 + s^2 + u^2 = 8100090000. \text{ Hence } p^2 + q^2 + t^2 = r^2 + s^2 + u^2.$$

Case 31: Consider higher degree Diophantine equation $p^6 + q^6 + 3r^2 = s^6$

Having two types of solutions

Case 31.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, \quad r = \frac{p(p^4-1)}{4} \text{ and } s = \frac{p^2+1}{2}.$$

Proof: We know that $(z^2)^3 = (z^3)^2$ and If p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e. $p^2 +$

$$\left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2. \text{ Apply cube of both sides, obtain } \left(p^2 + \left(\frac{p^2-1}{2}\right)^2\right)^3 = \left(\frac{p^2+1}{2}\right)^6.$$

Hence, we can verify easily, if p is odd then $p^6 + \left(\frac{p^2-1}{2}\right)^6 + 3\left(\frac{p(p^4-1)}{4}\right)^2 = \left(\frac{p^2+1}{2}\right)^6$. Hence

$$p^6 + q^6 + 3r^2 = s^6 \text{ with } q = \frac{p^2-1}{2}, \quad r = \frac{p(p^4-1)}{4} \text{ and } s = \frac{p^2+1}{2} \text{ whenever } p \text{ is odd.}$$

We can verify it easily by replacing some odd integer p .

Case 31.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, \quad r = p\left(\left(\frac{p}{2}\right)^4 - 1\right) \text{ and } s = \left(\frac{p}{2}\right)^2 + 1$$

Proof: if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

Hence $p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2$. Apply cube on both sides, obtain

$$\left(p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2\right)^3 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^6. \text{ Hence } p^6 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^6 + 3\left(p\left(\left(\frac{p}{2}\right)^4 - 1\right)\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^6.$$

Hence $p^6 + q^6 + 3r^2 = s^6$ with $q = \left(\frac{p}{2}\right)^2 - 1, \quad r = p\left(\left(\frac{p}{2}\right)^4 - 1\right) \text{ and } s = \left(\frac{p}{2}\right)^2 + 1$ with p is even. We can verify it easily by replacing some even integer p .

Conclusion: In this paper, First focused to study infinitely many integer solutions of following Diophantine Equations.

$$2x^2 + y^2 = z^2; 3x^2 + y^2 = z^2; 4x^2 + y^2 = z^2; 5x^2 + y^2 = z^2; 6x^2 + y^2 = z^2;$$

$$7x^2 + y^2 = z^2; 8x^2 + y^2 = z^2; 9x^2 + y^2 = z^2; 10x^2 + y^2 = z^2; 11x^2 + y^2 = z^2;$$

$$12x^2 + y^2 = z^2; 13x^2 + y^2 = z^2; 14x^2 + y^2 = z^2; 15x^2 + y^2 = z^2; 16x^2 + y^2 = z^2;$$

$$17x^2 + y^2 = z^2; 18x^2 + y^2 = z^2; 19x^2 + y^2 = z^2; 20x^2 + y^2 = z^2; 21x^2 + y^2 = z^2;$$

Also, $kx^2 + y^2 = z^2$ having ellipse equation form of $k\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Also, focused to study Reciprocal form of above Diophantine Equation $\frac{k}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$. Which is having different sets of integer solutions of $p = yz$, $q = xz$ and $h = xy$. Also, focused to obtained infinitely many Integer solutions of following Special Diophantine Pythagorean Equations.

$$p^2 + q^2 + r^2 + s^2 = t^2 + u^2; p^2 + q^2 + r^2 = s^2; p^2 + q^2 + r^2 + s^2 = t^2;$$

$$p^4 + q^4 + 2r^2 = s^4; p^2 + q^2 + t^2 = r^2 + s^2 + u^2; p^6 + q^6 + 3r^2 = s^6;$$

$$x^3 + y^4 = z^5; x^3 + y^3 = z^2; x^2 + y^3 + z^4 = w^5; x^2 + y^3 + z^4 + w^5 = u^2;$$

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