



A Collection of Special Diophantine Ellipse and Pythagorean Equations with Integer Solutions

Dr. T. SRINIVAS

Department of FME, Associate Professor, Audi Sankara Deemed to be University, Gudur bypass, Gudur, Spsr Nellore (Dist.), Andhra Pradesh, India.

ABSTRACT: Learning about the various techniques to solve this higher power Diophantine equation in successfully deriving their solutions help us understand how numbers work and their significance in different areas of mathematics and science. In this paper, First focused to study infinitely many integer solutions of following Diophantine Equations.

$2x^2 + y^2 = z^2$; $3x^2 + y^2 = z^2$; $4x^2 + y^2 = z^2$; $5x^2 + y^2 = z^2$; $6x^2 + y^2 = z^2$;
 $7x^2 + y^2 = z^2$; $8x^2 + y^2 = z^2$; $9x^2 + y^2 = z^2$; $10x^2 + y^2 = z^2$; $11x^2 + y^2 = z^2$;
 $12x^2 + y^2 = z^2$; $13x^2 + y^2 = z^2$; $14x^2 + y^2 = z^2$; $15x^2 + y^2 = z^2$; $16x^2 + y^2 = z^2$;
 $17x^2 + y^2 = z^2$; $18x^2 + y^2 = z^2$; $19x^2 + y^2 = z^2$; $20x^2 + y^2 = z^2$; $21x^2 + y^2 = z^2$;

Also, $kx^2 + y^2 = z^2$ having ellipse equation form of $k\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Also, focused to study Reciprocal form of above Diophantine Equation $\frac{k}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$. Which is having different sets of integer solutions of $p = yz$, $q = xz$ and $h = xy$. Also, focused to obtained infinitely many Integer solutions of following Special Diophantine Pythagorean Equations.

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$; $p^2 + q^2 + r^2 = s^2$; $p^2 + q^2 + r^2 + s^2 = t^2$;
 $p^4 + q^4 + 2r^2 = s^4$; $p^2 + q^2 + t^2 = r^2 + s^2 + u^2$; $p^6 + q^6 + 3r^2 = s^6$;
 $x^3 + y^4 = z^5$; $x^3 + y^3 = z^2$; $x^2 + y^3 + z^4 = w^5$; $x^2 + y^3 + z^4 + w^5 = u^2$;

KEYWORDS: Diophantine Equations, Pythagorean theorem, Reciprocal Pythagorean theorem, Ellipse, Reciprocal ellipse equations.

Mathematics Subject Classifications: 11D72, 11D61,

I. INTRODUCTION

The fascinating branch of Mathematics is the Theory of Numbers in which the subject of Diophantine equations requiring only the integer solutions is an interesting area to various mathematicians. In other words, the theory of Diophantine equations is an ancient subject that typically involves solving, polynomial equation in two or more variables or a system of polynomial equations with the number of unknowns greater than the number of equations, in integers and occupies a pivotal role in the region of mathematics.

II. METHODS

Now we are focused to study to obtain integer solutions of special Diophantine equation $kx^2 + y^2 = z^2$ and Reciprocal curve equation $\frac{k}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ with using of trial-and-error method. In particularly, focused to obtain integer solutions of above Diophantine equations with k values are varies from 2 to 21.

III.RESULTS:

In this paper, First focused to study integer solutions of following Diophantine Equations.

$$\begin{aligned}
 2x^2 + y^2 &= z^2; \quad 3x^2 + y^2 = z^2; \quad 4x^2 + y^2 = z^2; \quad 5x^2 + y^2 = z^2; \quad 6x^2 + y^2 = z^2; \\
 7x^2 + y^2 &= z^2; \quad 8x^2 + y^2 = z^2; \quad 9x^2 + y^2 = z^2; \quad 10x^2 + y^2 = z^2; \quad 11x^2 + y^2 = z^2; \\
 12x^2 + y^2 &= z^2; \quad 13x^2 + y^2 = z^2; \quad 14x^2 + y^2 = z^2; \quad 15x^2 + y^2 = z^2; \quad 16x^2 + y^2 = z^2; \\
 17x^2 + y^2 &= z^2; \quad 18x^2 + y^2 = z^2; \quad 19x^2 + y^2 = z^2; \quad 20x^2 + y^2 = z^2; \quad 21x^2 + y^2 = z^2;
 \end{aligned}$$

Case 1: Consider Diophantine equation $2x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 3(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 3(2)^n$ are satisfies the integer solution of Diophantine equation $2x^2 + y^2 = z^2$.

$$\text{Since } 2(2^{n+1})^2 + (2^n)^2 = (2^{2n+3}) + (2^{2n}) = (2^{2n})(2^3 + 1) = (3(2)^n)^2.$$

Lemma 1.1: It is having Ellipse equation form of $2\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Which is having simple form of ellipse $2p^2 + q^2 = 1$, whose solution is $p = \frac{2}{3}$, $q = \frac{1}{3}$.

Lemma 1.2: Reciprocal form of above Diophantine Equation $\frac{2}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 3(2^{2n}), \quad q = xz = 3(2^{2n+1}) \text{ and } h = xy = 2^{2n+1}.$$

$$\text{Since } \frac{2}{(3(2^{2n}))^2} + \frac{1}{(3(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2};$$

Case 2: Consider Diophantine equation $3x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^n$, $y = 3^n$ and $z = 2(3)^n$. Here n is positive integer.

Proof: Let $x = 3^n$, $y = 3^n$ and $z = 2(3)^n$ are satisfies the integer solution of Diophantine equation $3x^2 + y^2 = z^2$.

$$\text{Since } 3(3^n)^2 + (3^n)^2 = (3^{2n+1}) + (3^{2n}) = (3^{2n})(3 + 1) = (2(3)^n)^2.$$

Lemma 2.1: It is having Ellipse equation form of $3\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Which is having simple form of ellipse $3p^2 + q^2 = 1$, whose solution is $p = q = \frac{1}{2}$.

Lemma 2: Reciprocal form of above Diophantine Equation $\frac{3}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 2(3^{2n}), \quad q = xz = 2(3^{2n}) \text{ and } h = xy = 3^{2n}.$$

$$\text{Since } \frac{3}{(2(3^{2n}))^2} + \frac{1}{(2(3^{2n}))^2} = \frac{1}{(3^{2n})^2};$$

Case 3: Consider Diophantine equation $4x^2 + y^2 = z^2$ having different sets of integer solutions is $(2x, y, z)$ is a Pythagorean triplet. It is having different sets of Integer solutions for each odd integer x, then $y = x^2 - 1$ and $z = x^2 + 1$.

Proof: Let $x = x^2 - 1$ and $z = x^2 + 1$.



Consider $z^2 - y^2 = (x^2 + 1)^2 - (x^2 - 1)^2 = 4x^2 = (2x)^2$.

Hence if x is an odd, then (2x, y, z) is a Pythagorean triplet.

Case 4: Consider Diophantine equation $5x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 4^{n+1}$, $y = 4^n$ and $z = 3(4)^n$. Here n is positive integer.

Proof: Let $x = 4^{n+1}$, $y = 4^n$ and $z = 3(4)^n$ are satisfies the integer solution of Diophantine equation $5x^2 + y^2 = z^2$.

Since $5(4^{n+1})^2 + (4^n)^2 = (3(4)^n)^2$.

Lemma 4.1: Reciprocal form of above Diophantine Equation $\frac{5}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 3(4^{2n})$, $q = xz = 3(4^{2n+1})$ and $h = xy = 4^{2n+1}$.

Since $\frac{5}{(3(4^{2n}))^2} + \frac{1}{(3(4^{2n+1}))^2} = \frac{1}{(4^{2n+1})^2}$.

Case 5: Consider Diophantine equation $6x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 5(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 5(2)^n$ are satisfies the integer solution of Diophantine equation $6x^2 + y^2 = z^2$.

Since $6(2^{n+1})^2 + (2^n)^2 = (5(2)^n)^2$.

Lemma 5.1: Reciprocal form of above Diophantine Equation $\frac{6}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 5(2^{2n})$, $q = xz = 5(2^{2n+1})$ and $h = xy = 2^{2n+1}$.

Since $\frac{6}{(5(2^{2n}))^2} + \frac{1}{(5(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2}$

Case 6: Consider Diophantine equation $7x^2 + y^2 = z^2$ having the different sets of integer solutions is $x = 3^n$, $y = 3^{n+1}$ and $z = 4(3)^n$. Here n is positive integer.

Proof: Let $x = 3^n$, $y = 3^{n+1}$ and $z = 4(3)^n$ are satisfies the integer solution of Diophantine equation $7x^2 + y^2 = z^2$.

Since $7(3^n)^2 + (3^{n+1})^2 = (4(3)^n)^2$.

Lemma 6.1: Reciprocal form of above Diophantine Equation $\frac{7}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 4(3^{2n+1})$, $q = xz = 4(3^{2n})$ and $h = xy = 3^{2n+1}$.

Since $\frac{7}{(4(3^{2n+1}))^2} + \frac{1}{(4(3^{2n}))^2} = \frac{1}{(3^{2n+1})^2}$

Case 7: Consider Diophantine equation $8x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^n$, $y = 2^n$ and $z = 3(2)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 3^{n+2}$ are satisfies the integer solution of Diophantine equation $8x^2 + y^2 = z^2$.

Since $8(3^{n+1})^2 + (3^{n+1})^2 = (3^{n+2})^2$.



Lemma 7.1: Reciprocal form of above Diophantine Equation $\frac{8}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 3^{2n+3}$, $q = xz = 3^{2n+3}$ and $h = xy = 3^{2n+2}$.

Since $\frac{8}{(3^{2n+3})^2} + \frac{1}{(3^{2n+3})^2} = \frac{1}{(3^{2n+2})^2}$

Case 8: Consider Diophantine equation $9x^2 + y^2 = z^2$ having the two different sets of integer solutions.

If x is odd then $y = \frac{9x^2-1}{2}$, $z = \frac{9x^2+1}{2}$. If x is even then $y = 9\left(\frac{x}{2}\right)^2 - 1$, $z = 9\left(\frac{x}{2}\right)^2 + 1$.

Proof: if x is an odd integer. Let $y = \frac{9x^2-1}{2}$, $z = \frac{9x^2+1}{2}$.

Consider $z^2 - y^2 = \left(\frac{9x^2+1}{2}\right)^2 - \left(\frac{9x^2-1}{2}\right)^2 = 9x^2 = (3x)^2$. Hence (3x, y, z) is a Pythagorean triplet.

if x is an even integer. Let $y = 9\left(\frac{x}{2}\right)^2 - 1$, $z = 9\left(\frac{x}{2}\right)^2 + 1$.

Consider $z^2 - y^2 = \left(9\left(\frac{x}{2}\right)^2 + 1\right)^2 - \left(9\left(\frac{x}{2}\right)^2 - 1\right)^2 = 9x^2 = (3x)^2$. Hence (3x, y, z) is a Pythagorean triplet.

Case 9: Consider Diophantine equation $10x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^{n+1}$, $y = 6^n$ and $z = 19(6)^n$. Here n is positive integer.

Proof: Let $x = 6^{n+1}$, $y = 6^n$ and $z = 19(6)^n$ are satisfies the integer solution of Diophantine equation $10x^2 + y^2 = z^2$.

Since $10(6^{n+1})^2 + (6^n)^2 = (19(6)^n)^2$.

Lemma 9.1: Reciprocal form of above Diophantine Equation $\frac{10}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 19(6^{2n})$, $q = xz = 19(6^{2n+1})$ and $h = xy = 6^{2n+1}$.

Since $\frac{10}{(19(6^{2n}))^2} + \frac{1}{(19(6^{2n+1}))^2} = \frac{1}{(6^{2n+1})^2}$

Case 10: Consider Diophantine equation $11x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}$, $y = 3^n$ and $z = 10(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 3^n$ and $z = 10(3)^n$ are satisfies the integer solution of Diophantine equation $11x^2 + y^2 = z^2$.

Since $11(3^{n+1})^2 + (3^n)^2 = (10(3)^n)^2$.

Lemma 10.1: Reciprocal form of above Diophantine Equation $\frac{11}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 10(3^{2n})$, $q = xz = 10(3^{2n+1})$ and $h = xy = 3^{2n+1}$.

Since $\frac{11}{(10(3^{2n}))^2} + \frac{1}{(10(3^{2n+1}))^2} = \frac{1}{(3^{2n+1})^2}$

Case 11: Consider Diophantine equation $12x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 7(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 7(2)^n$ are satisfies the integer solution of Diophantine equation $12x^2 + y^2 = z^2$.



Since $12(2^{n+1})^2 + (2^n)^2 = (7(2)^n)^2$.

Lemma 11.1: Reciprocal form of above Diophantine Equation $\frac{12}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 7(2^{2n})$, $q = xz = 7(2^{2n+1})$ and $h = xy = 2^{2n+1}$.

Since $\frac{12}{(7(2^{2n}))^2} + \frac{1}{(7(2^{2n+1}))^2} = \frac{1}{(2^{2n+1})^2}$

Case 12: Consider Diophantine equation $13x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^n$, $y = 6^{n+1}$ and $z = 7(6)^n$. Here n is positive integer.

Proof: Let $x = 6^n$, $y = 6^{n+1}$ and $z = 7(6)^n$ are satisfies the integer solution of Diophantine equation $13x^2 + y^2 = z^2$.

Since $13(6^n)^2 + (6^{n+1})^2 = (7(6)^n)^2$.

Lemma 12.1: Reciprocal form of above Diophantine Equation $\frac{13}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 7(6^{2n+1})$, $q = xz = 7(6^{2n})$ and $h = xy = 6^{2n+1}$.

Since $\frac{13}{(7(6^{2n+1}))^2} + \frac{1}{(7(6^{2n}))^2} = \frac{1}{(6^{2n+1})^2}$

Case 13: Consider Diophantine equation $14x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 6^{n+1}$, $y = 5(6)^n$ and $z = 23(6)^n$. Here n is positive integer.

Proof: Let $x = 6^{n+1}$, $y = 5(6)^n$ and $z = 23(6)^n$ are satisfies the integer solution of Diophantine equation $14x^2 + y^2 = z^2$.

Since $14(6^{n+1})^2 + (5(6)^n)^2 = (23(6)^n)^2$.

Lemma 13.1: Reciprocal form of above Diophantine Equation $\frac{14}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 115(6^{2n})$, $q = xz = 23(6^{2n+1})$ and $h = xy = 5(6)^{2n+1}$.

Since $\frac{14}{(115(6^{2n}))^2} + \frac{1}{(23(6^{2n+1}))^2} = \frac{1}{(5(6)^{2n+1})^2}$

Case 14: Consider Diophantine equation $15x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 12(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 3^{n+1}$ and $z = 12(3)^n$ are satisfies the integer solution of Diophantine equation $15x^2 + y^2 = z^2$.

Since $15(3^{n+1})^2 + (3^{n+1})^2 = (12(3)^n)^2$.

Lemma 14.1: Reciprocal form of above Diophantine Equation $\frac{15}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 12(3^{2n+1})$, $q = xz = 12(3^{2n+1})$ and $h = xy = (3)^{2n+2}$.

Since $\frac{15}{(12(3^{2n+1}))^2} + \frac{1}{(12(3^{2n+1}))^2} = \frac{1}{((3)^{2n+2})^2}$



Case 15: Consider Diophantine equation $16x^2 + y^2 = z^2$ having different sets of integer solutions. If x is even then $y = 16\left(\frac{x}{2}\right)^2 - 1, z = 16\left(\frac{x}{2}\right)^2 + 1$.

Proof: if x is an even integer. Let $y = 16\left(\frac{x}{2}\right)^2 - 1, z = 16\left(\frac{x}{2}\right)^2 + 1$.

Consider $z^2 - y^2 = \left(16\left(\frac{x}{2}\right)^2 + 1\right)^2 - \left(16\left(\frac{x}{2}\right)^2 - 1\right)^2 = 16x^2 = (4x)^2$. Hence (4x, y, z) is a Pythagorean triplet.

Case 16: Consider Diophantine equation $17x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}$, $y = 4(3)^n$ and $z = 13(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 4(3)^n$ and $z = 13(3)^n$ are satisfies the integer solution of Diophantine equation $17x^2 + y^2 = z^2$.

Since $17(3^{n+1})^2 + (4(3)^n)^2 = (13(3)^n)^2$.

Lemma 16.1: Reciprocal form of above Diophantine Equation $\frac{17}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 52(3^{2n}), q = xz = 13(3^{2n+1})$ and $h = xy = 4(3)^{2n+1}$.

Since $\frac{17}{(52(3^{2n}))^2} + \frac{1}{(13(3^{2n+1}))^2} = \frac{1}{(4(3)^{2n+1})^2}$

Case 17: Consider Diophantine equation $18x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 3(2)^n$ and $z = 9(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 3(2)^n$ and $z = 9(2)^n$ are satisfies the integer solution of Diophantine equation $18x^2 + y^2 = z^2$.

Since $18(2^{n+1})^2 + (3(2)^n)^2 = (9(2)^n)^2$.

Lemma 17.1: Reciprocal form of above Diophantine Equation $\frac{18}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 27(2^{2n}), q = xz = 9(2^{2n+1})$ and $h = xy = 3(2)^{2n+1}$.

Since $\frac{18}{(27(2^{2n}))^2} + \frac{1}{(9(2^{2n+1}))^2} = \frac{1}{(3(2)^{2n+1})^2}$

Case 18: Consider Diophantine equation $19x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 3^{n+1}$, $y = 5(3)^n$ and $z = 14(3)^n$. Here n is positive integer.

Proof: Let $x = 3^{n+1}$, $y = 5(3)^n$ and $z = 14(3)^n$ are satisfies the integer solution of Diophantine equation $19x^2 + y^2 = z^2$.

Since $19(3^{n+1})^2 + (5(3)^n)^2 = (14(3)^n)^2$.

Lemma 18.1: Reciprocal form of above Diophantine Equation $\frac{19}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$p = yz = 70(3^{2n}), q = xz = 14(3^{2n+1})$ and $h = xy = 5(3)^{2n+1}$.

Since $\frac{19}{(70(3^{2n}))^2} + \frac{1}{(14(3^{2n+1}))^2} = \frac{1}{(5(3)^{2n+1})^2}$



Case 19: Consider Diophantine equation $20x^2 + y^2 = z^2$ having different sets of integer solutions is $x = 2^{n+1}$, $y = 2^n$ and $z = 9(2)^n$. Here n is positive integer.

Proof: Let $x = 2^{n+1}$, $y = 2^n$ and $z = 9(2)^n$ are satisfies the integer solution of Diophantine equation $20x^2 + y^2 = z^2$. Since $20(2^{n+1})^2 + ((2)^n)^2 = (9(2)^n)^2$.

Lemma 19.1: Reciprocal form of above Diophantine Equation $\frac{20}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 9(2^{2n}), \quad q = xz = 9(2^{2n+1}) \text{ and } h = xy = (2)^{2n+1}.$$

$$\text{Since } \frac{20}{(9(2^{2n}))^2} + \frac{1}{(9(2^{2n+1}))^2} = \frac{1}{((2)^{2n+1})^2}$$

Case 20: Consider Pythagorean equation $21x^2 + y^2 = z^2$ having different sets of integer solutions (in terms of exponential) is $x = 2^n$, $y = 2^{n+1}$ and

$z = 5(3)^n$. Here n is positive integer.

Proof: Let $x = 2^n$, $y = 2^{n+1}$ and $z = 5(3)^n$ are satisfies the integer solution of Diophantine equation $21x^2 + y^2 = z^2$.

$$\text{Since } 21(2^{n+1})^2 + ((2)^n)^2 = (5(3)^n)^2.$$

Lemma 20.1: Reciprocal form of above Diophantine Equation $\frac{21}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$.

Which is having different sets of integer solutions

$$p = yz = 10(6^n), \quad q = xz = 5(6^n) \text{ and } h = xy = (2)^{2n+1}.$$

$$\text{Since } \frac{21}{(10(6^n))^2} + \frac{1}{(5(6^n))^2} = \frac{1}{((2)^{2n+1})^2}$$

some special collection of Diophantine Equations, whose solutions are obtained from standard Pythagorean theorem.

Case 21: Consider Diophantine equation $x^3 + y^4 = z^5$ having integer solution is

$$x = 2^8, \quad y = 2^6 \text{ and } z = 2^5.$$

Case 22: Consider Diophantine equation $x^3 + y^3 = z^2$ having integer solution is

$$x = 2^8, \quad y = 2^9 \text{ and } z = 3(2)^{12}.$$

Case 23: Consider Diophantine equation $x^2 + y^3 + z^4 = w^5$ having integer solution is

$$x = 3^{12}, \quad y = 3^8, \quad z = 3^6 \text{ and } w = 3^5.$$

Case 24: Consider Diophantine equation $x^2 + y^3 + z^4 + w^5 = u^2$ having integer solution is $x = 4^{30}$, $y = 4^{20}$, $z = 4^{15}$, $w = 4^{12}$ and $u = 2^{61}$.

Case 25: Consider the Pythagorean (4;2) tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$p = x^2yh, \quad q = y^2xh, \quad r = yh, \quad s = xh, \quad t = xy, \quad u = xyzh$$

where $x = bc$, $y = ca$, $h = ab$, $z = c^2$ with (a, b, c) is a Pythagorean triplet, which is satisfies $a^2 + b^2 = c^2$.

Proof: We know that if (a, b, c) is a Pythagorean triplet, then is satisfies $a^2 + b^2 = c^2$.

if (a, b, c) is a Pythagorean triplet then (b c, a c, a b) is also a Reciprocal Pythagorean triplet. i.e. if $x = bc$, $y = ca$, $h = ab$ then $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{h^2}$[1]



Also, if (a, b, c) is a Pythagorean triplet then (bc, ac, c^2) is also a Pythagorean triplet.

i.e $x = bc$, $y = ca$, $z = c^2$ then $x^2 + y^2 = z^2$[2]

Adding equations [1] , [2], we obtain

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$p = x^2yh, q = y^2xh, r = yh, s = xh, t = xy, u = xyzh$$

where $x = bc$, $y = ca$, $h = ab$, $z = c^2$ with (a, b, c) is a Pythagorean triplet, which is satisfies $a^2 + b^2 = c^2$.

E.g.1: Choose One of the Pythagorean triplets (a, b, c) is $(3, 4, 5)$, which follows

$$x = bc = 20, y = ca = 15, h = ab = 12, z = c^2 = 25$$

$$p = x^2yh = 72000, q = y^2xh = 54000, r = yh = 180, s = xh = 240, t = xy = 300,$$

$$u = xyzh = 90000.$$

$$p^2 + q^2 + r^2 + s^2 = 8100090000$$

$$t^2 + u^2 = 8100090000. \text{ Hence } p^2 + q^2 + r^2 + s^2 = t^2 + u^2$$

Also, note that (a, b, c) and (x, y, z) are Pythagorean triplets.

i.e. $a^2 + b^2 = c^2$ and $x^2 + y^2 = z^2$

Also, (x, y, h) is Reciprocal Pythagorean triplet. i.e. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{h^2}$.

Case 25.1: Consider the Pythagorean (4;2) tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$ having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is odd then } q = p + 1, r = \frac{p^2-1}{2}, s = \left(\frac{p+1}{2}\right)^2 - 1, t = \frac{p^2+1}{2}, u = \left(\frac{p+1}{2}\right)^2 + 1.$$

We can verify it easily by replacing some odd integer p .

Suppose $p = 3$ then $q = 4, r = 4, s = 3, t = 5, u = 5$.

$$\text{Hence } p^2 + q^2 + r^2 + s^2 = 3^2 + 4^2 + 4^2 + 3^2 = 50 = t^2 + u^2$$

Case 25.2: Consider the Pythagorean (4;2) tuples equation as follows

$$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$$

different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is even then } q = p + 1, r = \left(\frac{p}{2}\right)^2 - 1, s = \frac{(p+1)^2-1}{2}, t = \left(\frac{p}{2}\right)^2 + 1, u = \frac{(p+1)^2+1}{2}.$$

We can verify it easily by replacing some even integer p .

Suppose $p = 4$ then $q = 5, r = 3, s = 12, t = 5, u = 13$.

$$\text{Hence } p^2 + q^2 + r^2 + s^2 = 4^2 + 5^2 + 3^2 + 12^2 = 194 = t^2 + u^2$$

Case 25.3: Consider the Pythagorean (4;2) tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$ having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is even then } q = p - 1, r = \left(\frac{p}{2}\right)^2 - 1, s = \frac{(p-1)^2-1}{2}, t = \left(\frac{p}{2}\right)^2 + 1, u = \frac{(p-1)^2+1}{2}.$$

We can verify it easily by replacing some even integer p .

Case 25.4: Consider the Pythagorean (4;2) tuples equation as follows

$p^2 + q^2 + r^2 + s^2 = t^2 + u^2$, having different sets of integer solutions is illustrated below:

$$\text{if } p \text{ is odd then } q = p - 1, r = \frac{p^2-1}{2}, s = \left(\frac{p-1}{2}\right)^2 - 1, t = \frac{p^2+1}{2}, u = \left(\frac{p-1}{2}\right)^2 - 1.$$



We can verify it easily by replacing some odd integer p

Case 26: Consider the Pythagorean (2;2) tuples equation as follows

$$p^2 + q^2 = r^2 + s^2 \text{ Having two types of solutions}$$

Case 26.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p^2-1}{4}\right)^2 + 1, \quad r = \left(\frac{p^2-1}{4}\right)^2 - 1 \text{ and } s = \frac{p^2+1}{2}$$

Proof: From Reference [10],[11],[12], We know that, if p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e.

$$p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2.$$

Also, we know that, if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

If p is odd then $\frac{p^2-1}{2}$ is an even number.

Hence $(\frac{p^2-1}{2}, \left(\frac{p^2-1}{4}\right)^2 - 1, \left(\frac{p^2-1}{4}\right)^2 + 1)$ is a Pythagorean triplet.

$$\text{It follows that if p is odd then } p^2 + \left(\left(\frac{p^2-1}{4}\right)^2 + 1\right)^2 = \left(\left(\frac{p^2-1}{4}\right)^2 - 1\right)^2 + \left(\frac{p^2+1}{2}\right)^2.$$

$$\text{Hence } p^2 + q^2 = r^2 + s^2.$$

Case 26.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}, \quad r = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2} \text{ and } s = \left(\frac{p}{2}\right)^2 + 1.$$

Proof: If p is even then $\left(\frac{p}{2}\right)^2 - 1$ is odd number.

Hence $(\left(\frac{p}{2}\right)^2 - 1, \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}, \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2})$ is a Pythagorean triplet.

if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$p^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}\right)^2 - \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$p^2 + \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}\right)^2 = \left(\frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2}\right)^2 + \left(\left(\frac{p}{2}\right)^2 + 1\right)^2.$$

$$\text{Hence } p^2 + q^2 = r^2 + s^2.$$

If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + 1}{2}, \quad r = \frac{\left(\left(\frac{p}{2}\right)^2 - 1\right)^2 - 1}{2} \text{ and } s = \left(\frac{p}{2}\right)^2 + 1.$$

We can verify it easily by replacing some even integer p.

Case 27: Consider the Pythagorean (3;1) tuples equation as follows $p^2 + q^2 + r^2 = s^2$

$p = 2^3, q = 2^5, r = 2^6, s = 9 * 2^3$ since $(2^3)^2 + (2^5)^2 + (2^6)^2 = (9 * 2^3)^2$

Case 28: Consider the Pythagorean (4;1) tuples equation as follows

$$p^2 + q^2 + r^2 + s^2 = t^2$$

Having two types of solutions

Case 28.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, \quad r = \frac{\left(\frac{(p^2+1)^2-1}{2}\right)^2-1}{2}, \quad s = \frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2-1}{2} \quad \text{and} \quad t = \frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2+1}{2}$$

Proof: Similar Proof of Case 27.1, we can verify easily $p^2 + q^2 + r^2 + s^2 = t^2$. If p is odd then $p^2 + \left(\frac{p^2-1}{2}\right)^2 +$

$$\left(\frac{\left(\frac{(p^2+1)^2-1}{2}\right)^2-1}{2}\right)^2 + \left(\frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2-1}{2}\right)^2 = \left(\frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2+1}{2}\right)^2.$$

We can verify it easily by replacing some odd integer p.

Case 28.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, \quad r = \frac{\left(\frac{(p^2+1)^2-1}{2}\right)^2-1}{2}, \quad s = \frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2-1}{2} \quad \text{and} \quad t = \frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2+1}{2}$$

Proof: Similar Proof of above case.

$$p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 + \left(\frac{\left(\frac{(p^2+1)^2-1}{2}\right)^2-1}{2}\right)^2 + \left(\frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2-1}{2}\right)^2 = \left(\frac{\left(\frac{(p^2+1)^2+1}{2}\right)^2+1}{2}\right)^2.$$

We can verify it easily by replacing some even integer p.

Case 29: Consider higher degree Diophantine equation $p^4 + q^4 + 2r^2 = s^4$

Having two types of solutions

Case 29.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, \quad r = \frac{p(p^2-1)}{2} \quad \text{and} \quad s = \frac{p^2+1}{2}.$$

Proof: similar proof of above.

Case 29.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, \quad r = p \left(\left(\frac{p}{2}\right)^2 - 1\right) \quad \text{and} \quad s = \left(\frac{p}{2}\right)^2 + 1$$

Proof: similar proof of above.

Case 29.3: If (x, y, z) is a Pythagorean triplet then $x^4 + y^4 + 2x^2y^2 = z^4$.



Proof: If (x, y, z) is a Pythagorean triplet. i.e. $x^2 + y^2 = z^2$. Square on Both sides, we obtain

$(x^2 + y^2)^2 = (z^2)^2$ implies that $x^4 + y^4 + 2x^2y^2 = z^4$.

Case 30: Consider the Pythagorean (3;3) tuples equation as follows

$$p^2 + q^2 + t^2 = r^2 + s^2 + u^2$$

then different sets of integer solutions is illustrated below

$$p = x^2yh, q = y^2xh, r = yh, s = xh, t = xy, u = xyzh$$

where $x = bc$, $y = ca$, $h = ab$, $z = c^2$ with (a, b, c) is a Pythagorean triplet, which is satisfies $a^2 + b^2 = c^2$.

E.g.1: Choose the Pythagorean triplet (a, b, c) is $(3, 4, 5)$, which follows

$$x = bc = 20, y = ca = 15, h = ab = 12, z = c^2 = 25$$

$$p = x^2yh = 72000, q = y^2xh = 54000, r = yh = 180, s = xh = 240, t = xy = 300,$$

$$u = xyzh = 90000$$

$$p^2 + q^2 + t^2 = 8100090000$$

$$r^2 + s^2 + u^2 = 8100090000. \text{ Hence } p^2 + q^2 + t^2 = r^2 + s^2 + u^2.$$

Case 31: Consider higher degree Diophantine equation $p^6 + q^6 + 3r^2 = s^6$

Having two types of solutions

Case 31.1: If p is an odd, then different sets of integer solutions is illustrated below

$$q = \frac{p^2-1}{2}, r = \frac{p(p^4-1)}{4} \text{ and } s = \frac{p^2+1}{2}.$$

Proof: We know that $(z^2)^3 = (z^3)^2$ and If p is odd then $(p, \frac{p^2-1}{2}, \frac{p^2+1}{2})$ is a Pythagorean triplet. i.e. $p^2 + \left(\frac{p^2-1}{2}\right)^2 = \left(\frac{p^2+1}{2}\right)^2$. Apply cube of both sides, obtain $\left(p^2 + \left(\frac{p^2-1}{2}\right)^2\right)^3 = \left(\frac{p^2+1}{2}\right)^6$.

Hence, we can verify easily, if p is odd then $p^6 + \left(\frac{p^2-1}{2}\right)^6 + 3\left(\frac{p(p^4-1)}{4}\right)^2 = \left(\frac{p^2+1}{2}\right)^6$. Hence

$$p^6 + q^6 + 3r^2 = s^6 \text{ with } q = \frac{p^2-1}{2}, r = \frac{p(p^4-1)}{4} \text{ and } s = \frac{p^2+1}{2} \text{ whenever } p \text{ is odd.}$$

We can verify it easily by replacing some odd integer p .

Case 31.2: If p is an even integer, then different sets of integer solutions is illustrated below

$$q = \left(\frac{p}{2}\right)^2 - 1, r = p\left(\left(\frac{p}{2}\right)^4 - 1\right) \text{ and } s = \left(\frac{p}{2}\right)^2 + 1$$

Proof: if p is even then $(p, \left(\frac{p}{2}\right)^2 - 1, \left(\frac{p}{2}\right)^2 + 1)$ is a Pythagorean triplet.

Hence $p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^2$. Apply cube on both sides, obtain

$$\left(p^2 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^2\right)^3 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^6. \text{ Hence } p^6 + \left(\left(\frac{p}{2}\right)^2 - 1\right)^6 + 3\left(p\left(\left(\frac{p}{2}\right)^4 - 1\right)\right)^2 = \left(\left(\frac{p}{2}\right)^2 + 1\right)^6.$$

Hence $p^6 + q^6 + 3r^2 = s^6$ with $q = \left(\frac{p}{2}\right)^2 - 1$, $r = p\left(\left(\frac{p}{2}\right)^4 - 1\right)$ and $s = \left(\frac{p}{2}\right)^2 + 1$ with p is even. We can verify it easily by replacing some even integer p .

Conclusion: In this paper, First focused to study infinitely many integer solutions of following Diophantine Equations.



$$\begin{aligned}
 2x^2 + y^2 &= z^2; \quad 3x^2 + y^2 = z^2; \quad 4x^2 + y^2 = z^2; \quad 5x^2 + y^2 = z^2; \quad 6x^2 + y^2 = z^2; \\
 7x^2 + y^2 &= z^2; \quad 8x^2 + y^2 = z^2; \quad 9x^2 + y^2 = z^2; \quad 10x^2 + y^2 = z^2; \quad 11x^2 + y^2 = z^2; \\
 12x^2 + y^2 &= z^2; \quad 13x^2 + y^2 = z^2; \quad 14x^2 + y^2 = z^2; \quad 15x^2 + y^2 = z^2; \quad 16x^2 + y^2 = z^2; \\
 17x^2 + y^2 &= z^2; \quad 18x^2 + y^2 = z^2; \quad 19x^2 + y^2 = z^2; \quad 20x^2 + y^2 = z^2; \quad 21x^2 + y^2 = z^2;
 \end{aligned}$$

Also, $kx^2 + y^2 = z^2$ having ellipse equation form of $k\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 = 1$; Also, focused to study Reciprocal form of above Diophantine Equation $\frac{k}{p^2} + \frac{1}{q^2} = \frac{1}{h^2}$. Which is having different sets of integer solutions of $p = yz$, $q = xz$ and $h = xy$. Also, focused to obtained infinitely many Integer solutions of following Special Diophantine Pythagorean Equations.

$$\begin{aligned}
 p^2 + q^2 + r^2 + s^2 &= t^2 + u^2; \quad p^2 + q^2 + r^2 = s^2; \quad p^2 + q^2 + r^2 + s^2 = t^2; \\
 p^4 + q^4 + 2r^2 &= s^4; \quad p^2 + q^2 + t^2 = r^2 + s^2 + u^2; \quad p^6 + q^6 + 3r^2 = s^6; \\
 x^3 + y^4 &= z^5; \quad x^3 + y^3 = z^2; \quad x^2 + y^3 + z^4 = w^5; \quad x^2 + y^3 + z^4 + w^5 = u^2;
 \end{aligned}$$

REFERENCES

- [1]. Thiruchinapalli, S., Ashok Kumar, C. (2024). Construction Of Pythagorean And Reciprocal Pythagorean N-Tuples. In: Accelerating Discoveries In Data Science And Artificial Intelligence II. ICDSAI 2023. Springer Proceedings In Mathematics & Statistics, Vol 438. Springer, Cham. [Https://Doi.Org/10.1007/978-3-031-51163-9_4](https://doi.org/10.1007/978-3-031-51163-9_4)
- [2]. Srinivas Thiruchinapalli, Sridevi Katterapalle (2024). A New Approach To Determine Constant Coefficients In Higher Order Linear Recurrence Relations And Repeated Steps Of Their Residues With Mth Integer Modulo Of Some Fibonacci Type Numbers. In AIP Conf. Proc. 2986, 030177. [Https://Doi.Org/10.1063/5.0192504](https://doi.org/10.1063/5.0192504)
- [3]. Srinivas Thiruchinapalli, Sridevi Katterapalle (2022). A New Approach To Define A New Integer Sequences Of Fibonacci Type Numbers With Using Of Third Order Linear Recurrence Relations. In AIP Conf. Proc. 2385, 130005 [Https://Doi.Org/10.1063/5.0070691](https://doi.org/10.1063/5.0070691)
- [4]. Dr.THIRUCHINAPALLI SRINIVAS (2024). Additive And Multiplicative Operations On Set Of Polygonal Numbers, QEIOS. [Https://Doi.Org/10.32388/MyOOLE](https://doi.org/10.32388/MyOOLE)
- [5]. Dr.T. Srinivas¹, Dr. C. Ashok Kumar² And K. Neeraja³ (2024). An Enumerative And Analytical Study Of The Sustainable Energy In An Economic Perspectives, International Journal Of Business & Management Research 12 (2), Volume 6. [Doi.Org/10.37391/IJBMR.120204](https://doi.org/10.37391/IJBMR.120204)
- [6]. Sridevi, K., & Srinivas, T. (2023). Transcendental Representation Of Diophantine Equation And Some Of Its Inherent Properties. *Materials Today: Proceedings*, 80, Part , Pages1822-1825. [Https://Doi.Org/10.1016/j.matpr.2021.05.619](https://doi.org/10.1016/j.matpr.2021.05.619)
- [7]. Sridevi, K., & Srinivas, T. (2023). Existence Of Inner Addition And Inner Multiplication On Set Of Triangular Numbers And Some Inherent Properties Of Triangular Numbers. *Materials Today: Proceedings*, 80, Part 3, Pages 1760-1764. [Https://Doi.Org/10.1016/j.matpr.2021.05.502](https://doi.org/10.1016/j.matpr.2021.05.502)
- [8]. Sridevi, K., & Srinivas, T. (2023). Cryptographic Coding To Define Binary Operation On Set Of Pythagorean Triples. *Materials Today: Proceedings*, 80, Part 3, Pages 2027-2031. [Https://Doi.Org/10.1016/j.matpr.2021.06.102](https://doi.org/10.1016/j.matpr.2021.06.102)
- [9]. Dr T SRINIVAS (2025). "A Study On Integer Design Of Exponential Solutions Of Diophantine Equation (X^4+Y^4)^B (P^2+Q^2+R^2+S^2)=C(T^2+U^2)(Z^2-W^2) A^B With A>0,B>1,P Is Odd And Y<X<W<Z. INTERNATIONAL JOURNAL OF ADVANCED SCIENTIFIC AND TECHNICAL RESEARCH (IJASTR), Vol. 15, No. 6, 2025, Pp. 269-274. DOI: 10.26808/RS.2025.20bc63
- [10]. Dr T SRINIVAS (2025). "A Study On Integer Design Of Exponential Solutions Of Diophantine Equation (X^4-Y^4)^B (P^2+Q^2+R^2+S^2)=C(T^2+U^2)(Z^2-W^2) A^B With A>0,B>1,P Is Odd And Y<X<W<Z". *INTERNATIONAL JOURNAL OF ADVANCED SCIENTIFIC AND TECHNICAL RESEARCH (IJASTR)*, Vol. 15, No. 6, 2025, Pp. 263-268. DOI: 10.26808/RS.2025.8a797.
- [11] Srinivas, T., & Sridevi, K. (2021, November). A New Approach To Define Algebraic Structure And Some Homomorphism Functions On Set Of Pythagorean Triples And Set Of Reciprocal Pythagorean Triples " In JSR (Journal Of Scientific Research) Volume 65, Issue 9, November 2021, Pages 86-92.
- [12] Sridevi, K., & Srinivas, T. (2020). A New Approach To Define Two Types Of Binary Operations On Set Of Pythagorean Triples To Form As At Most Commutative Cyclic Semi Group. *Journal Of Critical Reviews*, 7(19), 9871-9878
- [13] Srinivas, T. (2023). Some Inherent Properties Of Pythagorean Triples. *Research Highlights In Mathematics And Computer Science* Vol. 7, 156-169.
- [14]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations $\alpha(X^4+Y^4)^2=(C^2+D^2)(Z^2+W^2)^2P^\beta$ With $\alpha>0$, $\beta=1,2,3,4,5,6,7$ and $x< y < w < z$. International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.
- [15]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations $\alpha(X^4+Y^4)^2=(C^2+D^2)(Z^2-W^2)^2P^\beta$ With $\alpha>0$, $\beta=1,2,3,4,5,6,7$ and $x< y < w < z$. International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.
- [16]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations $\alpha(X^4+Y^4)^2=(C^2+D^2)(Z^2+W^2)^2P^\beta$ With $\alpha>0$, $\beta=1,2,3,4,5,6,7$ and $x< y < w < z$. International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.
- [17]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation $\alpha(X^4+Y^4)^2(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^2P^\beta$ With $\alpha>0$, $\beta=1,2,3,4,5,6,7$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-2882.



[18]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation $\alpha(X^4+Y^4)^2(2U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)P^\beta$ With $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[19]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation $\alpha(X^4+Y^4)^2(3U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)P^\beta$ With $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[20]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation $\alpha(X^4+Y^4)^2(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ With $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[21]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation $\alpha(X^4+Y^4)^2(\gamma U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ With $\alpha, \beta>0, \gamma=2,3$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[22]. Srinivas,T.,G.Sujatha(2025). Solving For Stoichiometric Coefficients Of Chemical Diophantine Equation $\alpha(X^4+Y^4)^2(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ With $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$, 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[23]. Srinivas,T.,G.Sujatha(2025), SOLVING FOR STOICHIOMETRIC OEFFICIENTS OF CHEMICAL DIOPHANTINE EQUATION $\alpha(X^4+Y^4)^2(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ WITH $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$. International Research Journal of Modernization in Engineering Technology and Science Volume:07/Issue:11/November-2025.

[24]. Srinivas,T.,G.Sujatha(2025), Integer Design Of Solutions Of One Of The Complex Stoichiometric Reaction System $\alpha(X^4+Y^4)^2(21U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ WITH $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x< y < w < z$. YMER || ISSN : 0044-0477,Volume 24,issue 11.

[25]. Srinivas,T (2025). A STUDY OF k-GONAL NUMBERS, Palestine Journal of Mathematics, Vol 14 (Special Issue IV), (2025), 1–17.

[26]. Srinivas,T.K.Umadevi(2025), INTEGER DESIGN OF SOLUTIONS OF ONE OF THE DIOPHANTINE EQUATIONS $\alpha(X^4 + Y^4)(P^2 + Q^2 + R^2 + S^2) = (T^2 + U^2)(C^2 - D^2)(Z^2 - W^2)P^\beta$ WITH X<Y<W<Z and P is ODD, $\alpha > 0, \beta > 0$. YMER || ISSN : 0044-0477,Volume 24,issue 11.

[27]. Srinivas,T.,K.Umadevi(2025), INTEGER DESIGN OF SOLUTIONS OF ONE OF THE DIOPHANTINE EQUATIONS $\alpha(X^4 + Y^4)(P^2 + Q^2 + R^2 + S^2) = (T^2 + U^2)(C^2 - D^2)(Z^2 + W^2)P^\beta$ WITH X<Y<W<Z and P is ODD, $\alpha > 0, \beta > 0$. International Research Journal of Modernization in Engineering Technology and Science Volume:07/Issue:11/November-2025.

[28]. Srinivas, T., Umadevi, K., Anitha, V., Babu, L.M. (2025). Role of Corporate Social Responsibility of Banks in Emerging Markets in India. In: Mishra, B.K., Rocha, Á., Mallik, S. (eds) International Conference on Technology Advances for Green Solutions and Sustainable Development. ICT4GS 2024. Information Systems Engineering and Management, vol 56. Springer, Cham. https://doi.org/10.1007/978-3-031-94997-5_8

[29]. Srinivas, T., Ashok Kumar, C., Appa Rao, N., Neeraja, K. (2025). Supply Chain Management for Coconut Farmers to Formulate New Marketing Strategies. In: Mishra, B.K., Rocha, Á., Mallik, S. (eds) International Conference on Technology Advances for Green Solutions and Sustainable Development. ICT4GS 2024. Information Systems Engineering and Management, vol 56. Springer, Cham. https://doi.org/10.1007/978-3-031-94997-5_18