

Inductive Parameterizations of Multi-Unknown Diophantine Equation Yielding Pythagorean Triples

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ABSTRACT:

Integer solutions to high-degree Diophantine equations with over twelve unknowns, parameterized via mathematical induction, enable the generation of special elliptic curves vital for cryptographic applications. The paper examines two lemmas providing explicit integer designs for these solutions, linking them to Pythagorean triplets under specific parity conditions on parameters.

This paper focused on a study to find integer design of solutions Diophantine Equation

$$\alpha(X^4 - Y^4)(\gamma U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

with Mathematical induction method for $\beta = 1, 2, 3, 4, \dots$ and so on.

In this paper, I was focused given Diophantine equation with more than 12 unknowns with two cases

Lemma 1: At $\gamma = 2$, the Diophantine equation

$$\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions,

for $\beta > 2$ is parameterized by integers k and n , with variables defined as:

$$Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

for $\beta = 1$ is $Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$

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Lemma 2: At $\gamma = 3$, the Diophantine equation

$$\alpha(X^4 - Y^4)(3U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

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In all cases, solve for α , whenever $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet.

It implies that $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on $k^4 - 1$.

$(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on whether $k^4 - 1$ is belongs to One of the first element of followings subsets $S_1, S_2, S_3, S_4, S_5, S_6$ of Set of Pythagorean triplets $\{(x, y, z): x^2 + y^2 = z^2\}$.

$$S_1 = \left\{ (k^4 - 1, D, C): \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\}$$

$$S_2 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{(2p - 1)^2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\}$$

$$S_3 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is geometric power of 2} \right\}$$

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If $k^4 - 1$ is even, then $S_5 = (k^4 - 1, \left(\frac{k^4 - 1}{2}\right)^2 - 1, \left(\frac{k^4 - 1}{2}\right)^2 + 1)$ is a Pythagorean triplet.

If $k^4 - 1$ is odd, then $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$ is a Pythagorean triplet.

KEYWORDS: Diophantine Equation, exponential, Pythagorean triplet, Integer design.

I. INTRODUCTION

Diophantine equations—polynomial equations with integer solutions—are a central topic in number theory. Among their many variants, **exponential Diophantine equations** involve terms where variables appear as exponents. Finding integer solutions to such equations is notably complex and has implications in mathematics, cryptography, and several scientific fields. Historical Context and Theoretical Background

Classical Diophantine Equations: Traditionally, research started with linear and polynomial forms, such as the well-known cases of Pythagorean triples.

Exponential Generalization: The study of exponential forms expanded from these roots, posing questions that often lack general solution methods and in some cases are proven to be undecidable.

II. RESULTS & DISCUSSIONS:

In this paper, focused to find the general exponential integer solution of the general exponential integer solution of $(X^4 - Y^4)(\gamma U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta$ at $\gamma = 2$ and $\gamma = 3$

Lemma 1: at $\gamma = 2$, the diophantine equation $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta$

With $\alpha > 0, \beta > 0$ is derived from fixed value of $\beta = 1, \beta = 2$ and $\beta > 2$.

Proposition 1: A Study on exponential integer solution of above Diophantine Equation at

$\beta = 1$ is $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P$

Explanation: Let $Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n$

Consider $\alpha(X^4 - Y^4)(2U^2 + V^2) = \alpha k^{4n}(k^4 - 1)(3(2)^n)^2$.

Again consider $T^2(C^2 - D^2)(Z^2 - W^2)P = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2$

It follows that $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P$ implies that

$\alpha k^{4n}(k^4 - 1)(3(2)^n)^2 = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2$ implies $\alpha(k^4 - 1) = (C^2 - D^2)(k^6 - k^4)$.

Solve for α , whenever $(k^4 - 1, D, C)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet, now I chosen one of the technique of

$$S_1 = \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{x^2 - 1} \text{ if } x \text{ is an odd prime number or its power} \right\}$$

$$S_2 = \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1} \text{ if } x \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\}$$

$$S_3 = \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1} \text{ if } x \text{ is geometric power of 2} \right\}$$

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It implies that $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on whether $k^4 - 1$ is belongs to One of the first element of $S_1, S_2, S_3, S_4, S_5, S_6$

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If $k^4 - 1$ is even, then $S_5 = (k^4 - 1, \left(\frac{k^4 - 1}{2}\right)^2 - 1, \left(\frac{k^4 - 1}{2}\right)^2 + 1)$ is a Pythagorean triplet.

If $k^4 - 1$ is odd, then $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$ is a Pythagorean triplet.

Proportion 2: A Study on exponential integer solution of above Diophantine Equation at

$$\beta = 2 \text{ is } \alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^2$$

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$$\text{Consider } \alpha(X^4 - Y^4)(2U^2 + V^2) = \alpha k^{4n}(k^4 - 1)(3(2)^n)^2$$

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It follows that $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^2$ implies that

$$\alpha k^{4n}(k^4 - 1)(3(2)^n)^2 = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2 \text{ implies } \alpha(k^4 - 1) = (C^2 - D^2)(k^6 - k^4).$$

Solve for α , whenever $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet.

It implies that $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on $k^4 - 1$.

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III. CONCLUSION

This equation generalizes classical Diophantine problems, blending sums of fourth powers with multiplicative factorizations. While challenging, targeted parametrization and modular analysis can yield solutions. Future work may classify solutions for specific α, β or link to broader number-theoretic frameworks. The parametric framework provides infinite families of solutions by exploiting algebraic identities and modular arithmetic. Future work could explore non-parametric solutions or generalizations to higher exponents.

In this paper, I focused to find integer design of solutions as two lemmas.

Lemma 1: At $\gamma = 2$, the Diophantine equation

$$\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions **for $\beta > 2$** is parameterized by integers k and n , with variables defined as:

$$Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

for $\beta = 1$ is $Y = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$

for $\beta = 2$ is $X = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$

Lemma 2: At $\gamma = 3$, the Diophantine equation

$$\alpha(X^4 - Y^4)(3U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions **for $\beta > 2$** is parameterized by integers k and n , with variables defined as:

$$Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 3^n, V = 3^n, T = 2(3)^n,$$

for $\beta = 1$ is $Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 3^n, V = 3^n, T = 2(3)^n,$

for $\beta = 2$ is $Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 3^n, V = 3^n, T = 2(3)^n,$

Solve for α , whenever $(1 + k^4, D, C)$ becomes a Pythagorean Triplet.

It implies that $(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on $k^4 - 1$.

$(k^4 - 1, D, C)$ becomes a Pythagorean Triplet depending on *whether* $k^4 - 1$ is belongs to One of the first element of $S_1, S_2, S_3, S_4, S_5, S_6$

$$S_1 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\}$$

$$S_2 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{(2p - 1)^2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\}$$

$$S_3 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is geometric power of } 2 \right\}$$

$$S_4 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{2p^2}\right)^2 - 1}, \text{ otherwise } (k^4 - 1 \text{ is even composite or its power}) \right\}$$

If $k^4 - 1$ is even, then $S_5 = (k^4 - 1, \left(\frac{k^4 - 1}{2}\right)^2 - 1, \left(\frac{k^4 - 1}{2}\right)^2 + 1)$ is a Pythagorean triplet.

If $k^4 - 1$ is odd, then $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$ is a Pythagorean triplet

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