

# Inductive Parameterizations of Multi-Unknown Diophantine Equation Yielding Pythagorean Triples

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**ABSTRACT:**

Integer solutions to high-degree Diophantine equations with over twelve unknowns, parameterized via mathematical induction, enable the generation of special elliptic curves vital for cryptographic applications. The paper examines two lemmas providing explicit integer designs for these solutions, linking them to Pythagorean triplets under specific parity conditions on parameters.

This paper focused on a study to find integer design of solutions Diophantine Equation

$$\alpha(X^4 - Y^4)(YU^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

with Mathematical induction method for  $\beta = 1, 2, 3, 4, \dots$  and so on.

In this paper, I was focused given Diophantine equation with more than 12 unknowns with two cases

**Lemma 1: At  $\gamma = 2$ , the Diophantine equation**

$$\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions,

for  $\beta > 2$  is parameterized by integers k and n, with variables defined as:

$$Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

$$\text{for } \beta = 1 \text{ is } Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

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**Lemma 2: At  $\gamma = 3$ , the Diophantine equation**

$$\alpha(X^4 - Y^4)(3U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions,

for  $\beta > 2$  is parameterized by integers k and n, with variables defined as:

$$Y = k^n, X = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, U = 3^n, V = 3^n, T = 2(3)^n,$$

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In all cases, solve for  $\alpha$ , whenever  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet.

It implies that  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on  $k^4 - 1$ .

$(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on whether  $k^4 - 1$  is belongs to One of the first element of followings subsets  $S_1, S_2, S_3, S_4, S_5, S_6$  of Set of Pythagorean triplets  $\{(x, y, z) : x^2 + y^2 = z^2\}$ .

$$S_1 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\}$$

$$S_2 = \left\{ \left( k^4 - 1, D, C \right) : \frac{D}{C} = 1 + \frac{2}{\left( \frac{k^4 - 1}{(2p-1)^2} \right)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\}$$

$$S_3 = \left\{ \left( k^4 - 1, D, C \right) : \frac{D}{C} = 1 + \frac{2}{\left( \frac{k^4 - 1}{2} \right)^2 - 1} \text{ if } k^4 - 1 \text{ is geometric power of 2} \right\}$$

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If  $k^4 - 1$  is even, then  $S_5 = (k^4 - 1, \left( \frac{k^4 - 1}{2} \right)^2 - 1, \left( \frac{k^4 - 1}{2} \right)^2 + 1)$  is a Pythagorean triplet.

If  $k^4 - 1$  is odd, then  $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$  is a Pythagorean triplet.

**KEYWORDS:** Diophantine Equation, exponential, Pythagorean triplet, Integer design.

## I. INTRODUCTION

Diophantine equations—polynomial equations with integer solutions—are a central topic in number theory. Among their many variants, **exponential Diophantine equations** involve terms where variables appear as exponents. Finding integer solutions to such equations is notably complex and has implications in mathematics, cryptography, and several scientific fields. Historical Context and Theoretical Background

**Classical Diophantine Equations:** Traditionally, research started with linear and polynomial forms, such as the well-known cases of Pythagorean triples.

**Exponential Generalization:** The study of exponential forms expanded from these roots, posing questions that often lack general solution methods and in some cases are proven to be undecidable.

## II. RESULTS & DISCUSSIONS:

In this paper, focused to find the general exponential integer solution of the general exponential integer solution of  $(X^4 - Y^4)(\gamma U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta$  at  $\gamma = 2$  and  $\gamma = 3$

**Lemma 1:** at  $\gamma = 2$ , the diophantine equaiton  $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta$

With  $\alpha > 0, \beta > 0$  is derived from fixed value of  $\beta = 1, \beta = 2$  and  $\beta > 2$ .

**Proportion 1:** A Study on exponential integer solution of above Diophantine Equation at

$\beta = 1$  is  $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P$

**Explanation:** Let  $Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n$

Consider  $\alpha(X^4 - Y^4)(2U^2 + V^2) = \alpha k^{4n}(k^4 - 1)(3(2)^n)^2$ .

Again consider  $T^2(C^2 - D^2)(Z^2 - W^2)P = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2$

It follows that  $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P$  implies that

$\alpha k^{4n}(k^4 - 1)(3(2)^n)^2 = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2$  implies  $\alpha(k^4 - 1) = (C^2 - D^2)(k^6 - k^4)$ .

Solve for  $\alpha$ , whenever  $(k^4 - 1, D, C)$  is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet, now I chosen one of the technique of

$$\begin{aligned}
 S_1 &= \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{x^2 - 1} \text{ if } x \text{ is an odd prime number or its power} \right\} \\
 S_2 &= \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1} \text{ if } x \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\} \\
 S_3 &= \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1} \text{ if } x \text{ is geometric power of 2} \right\} \\
 S_4 &= \left\{ (x, y, z) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}, \text{ otherwise (x is even composite or its power)} \right\}
 \end{aligned}$$

It implies that  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on whether  $k^4 - 1$  is belongs to One of the first element of  $S_1, S_2, S_3, S_4, S_5, S_6$

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 S_1 &= \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\} \\
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If  $k^4 - 1$  is even, then  $S_5 = (k^4 - 1, \left(\frac{k^4 - 1}{2}\right)^2 - 1, \left(\frac{k^4 - 1}{2}\right)^2 + 1)$  is a Pythagorean triplet.

If  $k^4 - 1$  is odd, then  $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$  is a Pythagorean triplet.

**Proportion 2:** A Study on exponential integer solution of above Diophantine Equation at

$$\beta = 2 \text{ is } \alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^2$$

**Explanation:** Let  $Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n$

Consider  $\alpha(X^4 - Y^4)(2U^2 + V^2) = \alpha k^{4n}(k^4 - 1)(3(2)^n)^2$

Again consider  $T^2(Z^2 - W^2)P^2 = k^{4n}(k^6 - k^4)(3(2)^n)^2$ .

It follows that  $\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^2$  implies that

$$\alpha k^{4n}(k^4 - 1)(3(2)^n)^2 = (C^2 - D^2)k^{4n}(k^6 - k^4)(3(2)^n)^2 \text{ implies } \alpha(k^4 - 1) = (C^2 - D^2)(k^6 - k^4).$$

Solve for  $\alpha$ , whenever  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet.

It implies that  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on  $k^4 - 1$ .

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$$S_1 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\}$$

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$$(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ with } \alpha > 0, \beta > 0 \text{ and } Y < X < z < w$$

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**Lemma 2:**  $\gamma = 3$ , the Diophantine equation  $\alpha(X^4 - Y^4)(3U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta$  With  $\alpha > 0, \beta > 0$  is derived from fixed value of  $\beta = 1, \beta = 2$  and  $\beta > 2$ .

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If  $k^4 - 1$  is odd, then  $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$  is a Pythagorean triplet

### III. CONCLUSION

This equation generalizes classical Diophantine problems, blending sums of fourth powers with multiplicative factorizations. While challenging, targeted parametrization and modular analysis can yield solutions. Future work may classify solutions for specific  $\alpha, \beta$  or link to broader number-theoretic frameworks. The parametric framework provides infinite families of solutions by exploiting algebraic identities and modular arithmetic. Future work could explore non-parametric solutions or generalizations to higher exponents.

In this paper, I focused to find integer design of solutions as two lemmas.

#### Lemma 1: At $\gamma = 2$ , the Diophantine equation

$$\alpha(X^4 - Y^4)(2U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions for  $\beta > 2$  is parameterized by integers  $k$  and  $n$ , with variables defined as:

$$Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

$$\text{for } \beta = 1 \text{ is } Y = k^n, Y = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^{2n}, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

$$\text{for } \beta = 2 \text{ is } X = k^n, Y = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^n, U = 2^{n+1}, V = 2^n, T = 3(2)^n,$$

#### Lemma 2: At $\gamma = 3$ , the Diophantine equation

$$\alpha(X^4 - Y^4)(3U^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)P^\beta \text{ With } \alpha > 0, \beta > 0 \text{ and } Y < X < W < Z$$

Having integer design of solutions for  $\beta > 2$  is parameterized by integers  $k$  and  $n$ , with variables defined as:

$Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^n, U = 3^n, V = 3^n, T = 2(3)^n$ ,

for  $\beta = 1$  is  $Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^{2n}, U = 3^n, V = 3^n, T = 2(3)^n$ ,

for  $\beta = 2$  is  $Y = k^n, X = k^{n+1}, Z = k^{n+3}, W = k^{n+2}, P = k^n, U = 3^n, V = 3^n, T = 2(3)^n$ ,

Solve for  $\alpha$ , whenever  $(1 + k^4, D, C)$  becomes a Pythagorean Triplet.

It implies that  $(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on  $k^4 - 1$ .

$(k^4 - 1, D, C)$  becomes a Pythagorean Triplet depending on whether  $k^4 - 1$  is belongs to One of the first element of  $S_1, S_2, S_3, S_4, S_5, S_6$

$$S_1 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{(k^4 - 1)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd prime number or its power} \right\}$$

$$S_2 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{(2p - 1)^2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is an odd composite or its Power, for some } p = 1, 2, 3, \dots \right\}$$

$$S_3 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{2}\right)^2 - 1} \text{ if } k^4 - 1 \text{ is geometric power of 2} \right\}$$

$$S_4 = \left\{ (k^4 - 1, D, C) : \frac{D}{C} = 1 + \frac{2}{\left(\frac{k^4 - 1}{2p^2}\right)^2 - 1}, \text{otherwise } (k^4 - 1 \text{ is even composite or its power}) \right\}$$

If  $k^4 - 1$  is even, then  $S_5 = (k^4 - 1, \left(\frac{k^4 - 1}{2}\right)^2 - 1, \left(\frac{k^4 - 1}{2}\right)^2 + 1)$  is a Pythagorean triplet.

If  $k^4 - 1$  is odd, then  $S_6 = (k^4 - 1, \frac{(k^4 - 1)^2 - 1}{2}, \frac{(k^4 - 1)^2 + 1}{2})$  is a Pythagorean triplet

## REFERENCES

- [1]. Thiruchinapalli, S., Ashok Kumar, C. (2024). Construction Of Pythagorean And Reciprocal Pythagorean N-Tuples. In: Accelerating Discoveries In Data Science And Artificial Intelligence II. ICDSAI 2023. Springer Proceedings In Mathematics & Statistics, Vol 438. Springer, Cham. [https://doi.org/10.1007/978-3-031-51163-9\\_4](https://doi.org/10.1007/978-3-031-51163-9_4)
- [2]. Srinivas Thiruchinapalli, Sridevi Katterapalle (2024). A New Approach To Determine Constant Coefficients In Higher Order Linear Recurrence Relations And Repeated Steps Of Their Residues With M<sup>th</sup> Integer Modulo Of Some Fibonacci Type Numbers. In AIP Conf. Proc. 2986, 030177. <https://doi.org/10.1063/5.0192504>
- [3]. Srinivas Thiruchinapalli, Sridevi Katterapalle (2022). A New Approach To Define A New Integer Sequences Of Fibonacci Type Numbers With Using Of Third Order Linear Recurrence Relations. In AIP Conf. Proc. 2385, 130005 <https://doi.org/10.1063/5.0070691>
- [4]. Dr. THIRUCHINAPALLI SRINIVAS (2024). [Additive And Multiplicative Operations On Set Of Polygonal Numbers, QEIOS](#). <https://doi.org/10.32388/MY0OLE>
- [5]. Dr.T. Srinivas<sup>1</sup>, Dr. C. Ashok Kumar<sup>2</sup> And K. Neeraja<sup>3</sup> (2024). [An Enumerative And Analytical Study Of The Sustainable Energy In An Economic Perspectives](#), International Journal Of Business & Management Research 12 (2), Volume 6. [doi.org/10.37391/IJBMR.120204](https://doi.org/10.37391/IJBMR.120204)
- [6]. Sridevi, K., & Srinivas, T. (2023). Transcendental Representation Of Diophantine Equation And Some Of Its Inherent Properties. Materials Today: Proceedings, 80, Part , Pages1822-1825. <https://doi.org/10.1016/j.matpr.2021.05.619>
- [7]. Sridevi, K., & Srinivas, T. (2023). Existence Of Inner Addition And Inner Multiplication On Set Of Triangular Numbers And Some Inherent Properties Of Triangular Numbers. Materials Today: Proceedings, 80, Part 3, Pages 1760-1764. <https://doi.org/10.1016/j.matpr.2021.05.502>
- [8]. Sridevi, K., & Srinivas, T. (2023). Cryptographic Coding To Define Binary Operation On Set Of Pythagorean Triples. Materials Today: Proceedings, 80, Part 3, Pages 2027-2031. <https://doi.org/10.1016/j.matpr.2021.06.102>
- [9]. Dr T SRINIVAS (2025). "A Study On Integer Design Of Exponential Solutions Of Diophantine Equation (X<sup>4</sup>+Y<sup>4</sup>)<sup>B</sup> (P<sup>2</sup>+Q<sup>2</sup>+R<sup>2</sup>+S<sup>2</sup>)=C(T<sup>2</sup>+U<sup>2</sup>)(Z<sup>2</sup>-W<sup>2</sup>) A<sup>B</sup> With A>0,B>1,P Is Odd And Y<X<W<Z. INTERNATIONAL JOURNAL OF ADVANCED SCIENTIFIC AND TECHNICAL RESEARCH (IJASTR), Vol. 15, No. 6, 2025, Pp. 269-274. DOI: 10.26808/RS.2025.20bc63



[10]. Dr T Srinivas (2025). "A Study On Integer Design Of Exponential Solutions Of Diophantine Equation  $(X^4+Y^4)^{\alpha}B(P^2+Q^2+R^2+S^2)=C(T^2+U^2)(Z^2-W^2)$  With  $A>0, B>1, P$  Is Odd And  $Y<X<W<Z$ ". INTERNATIONAL JOURNAL OF ADVANCED SCIENTIFIC AND TECHNICAL RESEARCH (IJASTR), Vol. 15, No. 6, 2025, Pp. 263-268. DOI: 10.26808/RS.2025.8a7b97.

[11] Srinivas, T., & Sridevi, K. (2021, November). A New Approach To Define Algebraic Structure And Some Homomorphism Functions On Set Of Pythagorean Triples And Set Of Reciprocal Pythagorean Triples " In JSR (Journal Of Scientific Research) Volume 65, Issue 9, November 2021, Pages 86-92.

[12] Sridevi, K., & Srinivas, T. (2020). A New Approach To Define Two Types Of Binary Operations On Set Of Pythagorean Triples To Form As At Most Commutative Cyclic Semi Group. Journal Of Critical Reviews, 7(19), 9871-9878

[13] Srinivas, T. (2023). Some Inherent Properties Of Pythagorean Triples. Research Highlights In Mathematics And Computer Science Vol. 7, 156-169.

[14]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[15]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[16]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[17]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-2882.

[18]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[19]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(3U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[20]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28.

[21]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(YU^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha, \beta>0, \gamma=2,3$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[22]. Srinivas,T.,G.Sujatha(2025). Solving For Stoichiometric Coefficients Of Chemical Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[23]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[24]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[25]. Srinivas,T.(2025). A Study on Integer Design of Exponential Solutions of Diophantine Equations  $\alpha(X^4+Y^4)^{\alpha}=(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Journal of Advanced Research in Science, Engineering and Technology Vol. 12, Issue 10, October 2025.

[26]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-2882. [30].

[27]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[28]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(3U^2+V^2)=T^2(C^2+D^2)(Z^2+W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[29]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(2U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28.

[30]. Srinivas,T.(2025). A Study On Integer Design Of Solutions Of Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(YU^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha, \beta>0, \gamma=2,3$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[31]. Srinivas,T.,G.Sujatha(2025). Solving For Stoichiometric Coefficients Of Chemical Diophantine Equation  $\alpha(X^4+Y^4)^{\alpha}(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  With  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ , 2025 IJCRT | Volume 13, Issue 11 November 2025 | ISSN: 2320-28

[32]. Srinivas,T.,G.Sujatha(2025), SOLVING FOR STOICHIOMETRIC OEFFICIENTS OF CHEMICAL DIOPHANTINE EQUATION  $\alpha(X^4+Y^4)^{\alpha}(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  WITH  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . International Research Journal of Modernization in Engineering Technology and Science Volume:07/Issue:11/November-2025.

[33]. Srinivas,T.,G.Sujatha(2025), Integer Design Of Solutions Of One Of The Complex Stoichiometric Reaction System  $\alpha(X^4+Y^4)^{\alpha}(21U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)^{\beta}P^{\beta}$  WITH  $\alpha>0, \beta=1,2,3,4,5,6,7$  and  $x<y<w<z$ . YMER || ISSN : 0044-0477

[34]. Srinivas,T (2025). A STUDY OF k-GONAL NUMBERS, Palestine Journal of Mathematics, Vol 14 (Special Issue IV), (2025) , 1-17.

[35]. Srinivas,T.K.Umadevi(2025), INTEGER DESIGN OF SOLUTIONS OF ONE OF THE DIOPHANTINE EQUATIONS  $\alpha(X^4+Y^4)(P^2+Q^2+R^2+S^2)=(T^2+U^2)(C^2-D^2)(Z^2-W^2)^{\beta}P^{\beta}$  WITH  $X<Y<W<Z$  and  $P$  is ODD,  $\alpha > 0, \beta > 0$ . YMER || ISSN : 0044-0477,Volume 24,issue 11.

[36]. Srinivas,T.,K.Umadevi(2025), INTEGER DESIGN OF SOLUTIONS OF ONE OF THE DIOPHANTINE EQUATIONS  $\alpha(X^4+Y^4)(P^2+Q^2+R^2+S^2)=(T^2+U^2)(C^2-D^2)(Z^2-W^2)^{\beta}P^{\beta}$  WITH  $X<Y<W<Z$  and  $P$  is ODD,  $\alpha > 0, \beta > 0$ . International Research Journal of Modernization in Engineering Technology and Science Volume:07/Issue:11/November-2025.

[37]. Srinivas, T., Umadevi, K., Anitha, V., Babu, L.M. (2025). Role of Corporate Social Responsibility of Banks in Emerging Markets in India. In: Mishra, B.K., Rocha, A., Malik, S. (eds) International Conference on Technology Advances for Green Solutions and Sustainable



Development. ICT4GS 2024. Information Systems Engineering and Management, vol 56. Springer, Cham. [https://doi.org/10.1007/978-3-031-94997-5\\_8](https://doi.org/10.1007/978-3-031-94997-5_8)

[38]. Srinivas, T., Ashok Kumar, C., Appa Rao, N., Neeraja, K. (2025). Supply Chain Management for Coconut Farmers to Formulate New Marketing Strategies. In: Mishra, B.K., Rocha, Á., Mallik, S. (eds) International Conference on Technology Advances for Green Solutions and Sustainable Development. ICT4GS 2024. Information Systems Engineering and Management, vol 56. Springer, Cham. [https://doi.org/10.1007/978-3-031-94997-5\\_18](https://doi.org/10.1007/978-3-031-94997-5_18)

[39]. Srinivas, T. (2026), A STUDY ON INTEGER DESIGN OF SOLUTIONS OF DIOPHANTINE EQUATION  $\alpha(p^3 + q^3 + r^3)(X^4 + Y^4)(yU^2 + V^2) = T^2(C^2 - D^2)(Z^2 - W^2)S^\beta$  WITH  $\alpha > 0, \beta > 0, \gamma = 2, 3$  and  $X < Y < W < Z$  , INTERNATIONAL JOURNAL OF COMPUTER APPLICATION (IJCA), Vol. 16, no. 1, 2026, pp. 11-21. DOI: 10.26808/RS.2026.311c51

[40]. Srinivas, T. (2026), INTEGER DESIGN OF SOLUTIONS OF ONE OF THE DIOPHANTINE EQUATIONS  $\alpha(X^4 + Y^4)(P^2 + Q^2 + R^2 + S^2) = (T^2 + U^2)(a^3 + b^3 + c^3)(C^2 - D^2)(Z^2 - W^2)P^\beta$  WITH  $X < Y < W < Z$  and  $P$  is ODD,  $\alpha > 0, \beta > 0$  , International Journal of Advanced Research in Science, Engineering and Technology Vol. 13, Issue 1, January 2026

[41]. Srinivas, T., Ashok Kumar, (2021). Using goal programming for transportation planning decisions problem in petroleum companies. Materials Today Proceedings. <https://doi.org/10.1016/j.matpr.2020.11.322>

[42]. Srinivas, T. (2026), Transcendental Pythagorean Cryptography: Symmetric Key Generation via Diophantine Triples and Algebraic Structures, International Journal of Advanced Research in Science, Engineering and Technology Vol. 13, Issue 1, January 2026.