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Weighted Arithmetic Mean of Weighted Arithmetic Means of Subsets of a Set of Numbers

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ABSTRACT: One property of *weighted arithmetic mean* on the relationship between the *weighted arithmetic mean* of the *weighted arithmetic mean*s of the respective elements of the respective possible subsets of a set of real numbers and the weighted arithmetic mean of the elements of the original set has mathematically been proved since no research publication on mathematical proof of this property has been found available.

KEYWORDS: Real Numbers, Set, Possible Susets, Weighted Arithmetic Mean, Mathemattical Property

I. INTRODUCTION

The term *average* is an entity that describes a set of many entities [1 , 49]. The great mathematician Pythagoras is the pioneer of developing measures of *average*, who derived three measures of average namely *arithmetic mean* [2 , 6 , 54], *geometric mean* [2 , 6 , 55] and *harmonic mean* [2 , 6 , 56] popularly known as "*Pythagorean Means*" [3 , 7 , 15]. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are *quadratic mean* or *root mean square*, *square root mean* , *cubic mean*, *cube root mean*, *generalized p mean* (also known as *power mean*) & *generalized p^{th} root mean* etc. which are *simple* as well as *weighted* [8 , 32 , 57]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 − 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are *arithmetic-geometric mean*, *arithmetic-harmonic mean*, *geometric-harmonic mean* and *arithmetic-geometric-harmonic* respectively [19 , 32]. Moreover, several studies have already been done on properties of *arithmetic mean*, *geometric mean* & *harmonic mean* [2 , 3 , 6 , 39 , 40 , 42 – 44 , 46 , 55 , 56] which have been found to be widely used in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 − 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 − 35 , 38 , 41 , 52 , 53. However, more properties of these measures of average are yet to be identified due to their importance in in analysis of numerical data as well as in the development of the associated theory. One interesting mathematical property of *arithmetic mean* which states that the arithmetic mean of the arithmetic means of the respective elements of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective elements of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, was proved algebraically in a recent study since no research publication on the proof of this property has been found available [47]. One similar property has here been proved algebraically in the case of *weighted arithmetic mean*.

II. WEIGHTED ARITHMETIC MEAN OF A SET OF ELEMENTS

Let us first mention the definition of *weighted arithmetic mean* of a set of real numbers.

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Definition

Let us consider a set of n real numbers namely

$$x_1 , x_2 , \ \dots\dots\dots \ , x_n$$

which correspond the weights

$$w_1 , w_2 , \ \dots\dots\dots \ , w_n$$

respectively.

Then the *weighted arithmetic mean* of them, denoted by $A(w: x_1 , x_2 , \ \dots\dots \ , x_n)$, is defined by

$$A(w: x_1 , x_2 , \ \dots\dots \ , x_n) = \frac{1}{w_1 + w_2 + \ \dots\dots\dots \ + w_n}(w_1x_1 + w_2x_2 + \dots\dots + w_nx_n)$$

The definition of *weighted arithmetic mean* implies that

$$w_1x_1 + w_2x_2 + \dots\dots\dots + w_nx_n = (w_1 + w_2 + \ \dots\dots\dots \ + w_n)\, A(w: x_1 , x_2 , \ \dots\dots\dots \ , x_n)$$

i.e. the *weighted sum of the n real numbers is* $(w_1 + w_2 + \ \dots\dots\dots \ + w_n)$ times of the *weighted arithmetic mean* of the numbers.

III. WEIGHTED ARITHMETIC MEAN OF WEIGHTED ARITHMETIC MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Now suppose, a set P consists of the N real numbers

$$a_1 , a_2 , \ \dots\dots\dots \ , a_N$$

which correspond the weights

$$w_1 , w_2 , \ \dots\dots\dots \ , w_N$$

respectively so that the *Weighted Sum* S_w and the *Weighted Arithmetic Mean* A_w of the N elements are respectively

$$S_w = w_1a_1 + w_2a_2 + \dots\dots\dots + w_Na_N$$

& $$A_w = \frac{1}{w_1 + w_2 + \ \dots\dots\dots \ + w_N}(w_1a_1 + w_2a_2 + \dots\dots\dots + w_Na_N)$$

Let us abbreviate *Weighted Sum* and *Weighted Arithmetic Mean* by *WS* and *WAM* respectively.

Let us consider the possible subsets of P having n elements in each set.

The number of such possible subsets is $C(N , n)$ where

$$C(N , n) = \frac{N!}{n!(N-n)!}$$

Among the $C(N , n)$ possible subsets, there are

$C(N - 1 , n - 1)$ subsets with a_1 as 1^{st} element,

$C(N - 2 , n - 2)$ subsets with a_2 as 1^{st} element and not having a_1 ,

$C(N - 3 , n - 3)$ subsets with a_3 as 1^{st} element and not having a_1 & a_2 ,

……………………………………………………………………………………

$C(N - 1 , N - 2)$ subsets with a_{N-n+2} as 1^{st} element and not having $a_1 , a_2 , \ \dots\dots\dots \ a_{N-n+3}$,

$C(N - 1 , N - 1)$ subsets with a_{N-n+1} as 1^{st} element and not having $a_1 , a_2 , \ \dots\dots\dots \ a_{N-n+2}$,

so that the total number of possible subsets is $C(N , n)$ and that each a_i appears a total of $C(N - 1 , n - 1)$ times in the set containing all possible $C(N , n)$ subsets [47].

Let

$$P_1 , P_2 , P_3 , \ \dots\dots\dots \ , P_{C(N,n)-1} , P_{C(N,n)}$$

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be the (N, n) possible subsets of P
Suppose,

$$A_{w1}, A_{w2}, A_{w3}, \ \ldots\ldots\ldots, A_{w\,C(N,\ n)-1}, A_{w\,C(N,\ n)}$$

are the individual WAM s,

$$S_{w1}, S_{w2}, S_{w3}, \ \ldots\ldots\ldots, S_{w\,C(N,\ n)-1}, S_{w\,C(N,\ n)}$$

are the individual WSs.
and

$$w(1), w(2), w(3), \ldots\ldots\ldots\ldots, w(c(N,\ n)-1), w(c(N, n))$$

are the individual *sum*s of the *weights* of the elements in the corresponding subsets

$$P_1, P_2, P_3, \ldots\ldots\ldots, P_{C(N,n)-1}, P_{C(N,n)}$$

respectively.
Then

$$S_{w1} = w(1)\, A_{w1},$$
$$S_{w2} = w(2)\, A_{w2},$$
$$S_{w3} = w(3)\, A_{w3},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$S_{w\,C(N,\ n)-1} = w(C(N,\ n)-1)\, A_{w\,C(N,\ n)-1},$$
$$S_{w\,C(N,\ n)} = w(C(N,\ n))\, A_{w\,C(N,\ n)}$$

Now,

$$S_{w1} + S_{w2} + S_{w3} + \ \ldots\ldots\ldots + S_{w\,C(N,\ n)-1} + S_{w\,C(N,\ n)}$$

is the WS of all elements in the combined set of all possible $C(N, n)$ subsets of the original set P where each a_i appears a total of $C(N-1, n-1)$ times.
Therefore,

$$S_{w1} + S_{w2} + S_{w3} + \ \ldots\ldots\ldots + S_{w\,C(N,\ n)-1} + S_{w\,C(N,\ n)}$$
$$= C(N-1, n-1)\,(w_1a_1 + w_2a_2 + \ldots\ldots\ldots + w_Na_N)$$

Similarly,

$$w(1) + w(2) + w(3) + \ \ldots\ldots\ldots + w(c(N,\ n)-1) + w(c(N, n))$$
$$= C(N-1, n-1)\,(w_1 + w_2 + \ldots\ldots\ldots + w_N)$$

Accordingly,

WAM of the individual $C(N, n)$ WAMs

$$= \frac{1}{C(N-1, n-1)\,(w_1 + w_2 + \ldots\ldots\ldots + w_N)}\{w(1)\, A_{w1} + w(2)A_{w2} + w(3)\, A_{w3} + \ \ldots\ldots\ldots$$
$$+ w(c(N,\ n)-1)A_{w\,C(N,\ n)-1} + w(c(N, n))A_{w\,C(N,\ n)}\}$$

$$= \frac{1}{C(N-1, n-1)\,(w_1 + w_2 + \ldots\ldots\ldots + w_N)}\{S_{w1} + S_{w2} + S_{w3} + \ \ldots\ldots\ldots + S_{w\,C(N,\ n)-1}$$
$$+ S_{w\,C(N,\ n)}\}$$

$$= \frac{1}{C(N-1, n-1)\,(w_1 + w_2 + \ldots\ldots\ldots + w_N)}\ C(N-1, n-1)\,(w_1a_1 + w_2a_2 + \ldots\ldots + w_Na_N)$$

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$$= \frac{1}{(w_1 + w_2 + \dots\dots\dots + w_N)} (w_1 a_1 + w_2 a_2 + \dots\dots\dots + w_N a_N)$$

= *WAM* of the elements of ***P***

This result can be summarized as the following theorem:

***Theorem* (3.1):** The *weighted arithmetic mean* of the *weighted arithmetic mean*s of the respective elements of the respective possible subsets, of fixed size, of a set of real numbers is the weighted arithmetic mean of the elements of the original set of numbers.

IV. WEIGHTED ARITHMETIC MEAN OF WEIGHTED ARITHMETIC MEANS OF All NON-EMPTY POSSIBLE SUBSETS

Now, the set ***P*** has a total of 2^n non-empty subsets of which

number of possible subsets having single element in each is ${}^N C_1$,
number of possible subsets having 2 elements in each is ${}^N C_2$,
………………………………………………………
number of possible subsets having $n-1$ elements in each is ${}^N C_{n-1}$,
number of possible subsets having n elements is in each ${}^N C_n$

such that

Total number of all possible non-empty subsets

$$= C(N, 1) + C(N, 2) + C(N, 3) + \dots\dots\dots + C(N, n-1) + C(N, n) = 2^n - 1$$

By the results obtained in section III,

WAM of the *WAM*s of the respective elements of the respective possible subsets having 1 element
= *WAM* of the elements of ***P***,
WAM of the *WAM*s of the respective elements of the respective possible subsets having 2 elements
= *WAM* of the elements of ***P***,
…………………………………………………………..…………………….………
WAM of the *WAM*s of the respective elements of the respective possible subsets having $n-1$ elements
= *WAM* of the elements of ***P***,
WAM of the *WAM*s of the respective elements of the respective possible subsets having n elements
= *WAM* of the elements of ***P***.

Therefore,
WAM of the *WAM*s of the respective elements of the respective possible non-empty subsets of ***P***
= *WAM* of the elements of ***P***
This result can be summarized as the following theorem:

***Theorem* (4.1):** The *weighted arithmetic mean*s of the respective elements of the respective non-empty possible subsets a set of real numbers is the *weighted arithmetic mean* of the elements of the original set of numbers.

V. NUMERICAL EXAMPLE

Suppose, ***S*** is a set of five real numbers given by

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$$S = \{2 , 4 , 6 , 8 , 10\}$$

with the set W of the corresponding weights given by

$$W = \{1 , 3 , 2 , 3 , 1\}$$

so that

$$WAM \text{ of the elements of } S = 6.0$$

Now, ${}^5C_1 = 5$ possible subsets of S having single element are

$$\{2\} , \{4\} , \{6\} , \{8\} , \{10\}$$

with the corresponding subsets of the corresponding weights

$$\{1\} , \{3\} , \{2\} , \{3\} , \{1\}$$

The 5 corresponding WAMs of the element if the respective subsets are

$$2 , 4 , 6 , 8 , 10$$

and the WAM of these 5 WAMs is 6.0 which is the WAM of the elements of S.

Similarly, ${}^5C_2 = 10$ possible subsets of S having 2 elements are

$$\{2 , 4\} , \{2 , 6\} , \{2 , 8\} , \{2 , 10\} , \{4 , 6\} , \{4 , 8\} , \{4 , 10\} , \{6 , 8\} , \{6 , 10\} , \{8 , 10\}$$

with the corresponding subsets of the corresponding weights

$$\{1 , 3\} , \{1 , 2\} , \{1 , 3\} , \{1 , 1\} , \{3 , 2\} , \{3 , 3\} , \{3 , 1\} , \{2 , 3\} , \{2 , 1\} , \{3 , 1\}$$

The 10 corresponding WAMs of the element if the respective subsets are

3.5 , 4.666666666666666666666666666667 , 6.5 , 6.0 , 4.8 , 6.0 , 5.5 , 7.2 ,
7.333333333333333333333333333333 , 8.5

and the WAM of these 10 WAMs is 6.0 which is the WAM of the elements of S.

Again, ${}^5C_3 = 10$ possible subsets of S having 3 elements are

$$\{2 , 4 , 6\} , \{2 , 4 , 8\} , \{2 , 4 , 10\} , \{2 , 6 , 8\} , \{2 , 6 , 10\} , \{2 , 8 , 10\} , \{4 , 6 , 8\} ,$$
$$\{4 , 6 , 10\} , \{4 , 8 , 10\} , \{6 , 8 , 10\}$$

with the corresponding subsets of the corresponding weights

$$\{1 , 3 , 2\} , \{1 , 3 , 3\} , \{1 , 3 , 1\} , \{1 , 2 , 3\} , \{1 , 2 , 1\} , \{1 , 3 , 1\} , \{3 , 2 , 3\} ,$$
$$\{3 , 2 , 1\} , \{3 , 3 , 1\} , \{2 , 3 , 1\}$$

The 10 corresponding WAMs of the element if the respective subsets are

4.333333333333333333333333333333 , 5.428571428571428571428571428571 4 , 4.8 ,
6.333333333333333333333333333333 , 6.0 , 7.2 , 6.0 ,
5.666666666666666666666666666667 , 6.571428571428571428571428571428 6 ,
7.666666666666666666666666666667

and the WAM of these 10 WAMs is 6.0 which is the WAM of the elements of S.

Moreover, ${}^5C_4 = 5$ possible subsets of S having 4 elements are

$$\{2 , 4 , 6 , 8\} , \{2 , 4 , 6 , 10\} , \{2 , 4 , 8 , 10\} , \{2 , 6 , 8 , 10\} , \{4 , 6 , 8 , 10\}$$

with the corresponding subsets of the corresponding weights

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$$\{1, 3, 2, 3\}, \{1, 3, 2, 1\}, \{1, 3, 3, 1\}, \{1, 2, 3, 1\}, \{3, 2, 3, 1\}$$

The 5 corresponding WAMs of the element if the respective subsets are

$$5.5555555555555555555555555556,\ 5.1428571428571428571428571,\ 6.0,\ 6.8571428571428571428571428571429,\ 6.444444444444444444444444444444$$

and the WAM of these 5 WAMs is 6.0 which is the WAM of the elements of S.

Moreover, ${}^5C_5 = 1$ possible subset of S having 5 elements is

$$\{2, 4, 6, 8, 10\}$$

with the corresponding subset of the corresponding weights

$$\{1, 3, 2, 3, 1\}$$

WAM of the elements in this subset is also 6.0 which is the WAM of the elements of S.

Finally, the WAM of all the 31 WAMs of the corresponding elements of the respective 31 subsets is is also equal to 6.0 which is the WAM of the elements of S.

VI. CONCLUSION

Findings of the study can be summarized as follows:

The *weighted arithmetic mean* of the *weighted arithmetic mean*s of the respective elements of the respective possible subsets, of fixed size, of a set of real numbers is the weighted arithmetic mean of the elements of the original set of numbers and the *weighted arithmetic mean* of the *weighted arithmetic mean*s of the respective elements of the respective non-empty possible subsets a set of real numbers is the *weighted arithmetic mean* of the elements of the original set of numbers."

The property/result of *weighted arithmetic mean* obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

In this connection, it is to be mentioned that the other measures of *weighted average* are also required to be computed/estimated in similar type of situation. Therefore, there is necessity of study on whether the other measures of *weighted average* possess similar properties like this property of *weighted arithmetic mean*.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

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(Dr. Dhritikesh Chakrabarty (at the middle) with students of National Institute of Pharmaceutical Education and Research (NIPER) Guwahati in a holly meet just after the completion of end semester classes in 2014)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of more than 300 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post-doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.

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