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A Study on Integer Design of Solutions of Quadratic Fibonacci Diophantine Equation

$$\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=$$

$$\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^{\beta}$$

With $\alpha>0,\beta>0\ and\ X<Y<W<Z$

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ABSTRACT:

This paper focused on a study to find integer design of solutions Quadratic Fibonacci Diophantine Equation $\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^{\beta}$ with $\alpha>0,\beta>0\ and\ X<Y<W<Z$ with Mathematical induction method for $\beta=1,2,3,4,\dots\ and\ so\ on.$ This paper focuses on a study of integer designs of solutions for a **quadratic Fibonacci-type Diophantine equation**, analyzed via the **mathematical-induction method**. Diophantine equations of higher degrees, such as the **quartic form**, play a meaningful role in generating **special families of elliptic curves** that are crucial for **elliptic-curve cryptography (ECC)** and secure communications.

Diophantine equations of higher degrees, like the quartic form $\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^{\beta}$, play a meaningful role in generating special elliptic curves that are crucial for cryptography and secure communications. Having integer design of solutions **for $\beta>2$** is parameterized by integers k and n, with variables defined as: $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n$,

$$\alpha=(1+k^4)(k^6+k^4)k^{(\beta-2)n}n^2,\left\{\begin{array}{l} C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n,\quad if\ 1+k^4\ is\ even\\ C=\left(\frac{(1+k^4)^2+1}{2}\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n,\quad if\ 1+k^4\ is\ odd\end{array}\right\}.$$

for $\beta=1$ is $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^{2n}, \alpha=(1+k^4)(k^6-k^4)n^2$,

$$\left\{\begin{array}{l} C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n,\quad if\ 1+k^4\ is\ even\\ C=\left(\frac{(1+k^4)^2+1}{2}\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n,\quad if\ 1+k^4\ is\ odd\end{array}\right\}.$$

for $\beta=2$ is $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n, \alpha=(1+k^4)(k^6+k^4)n^2.$

$$\left\{\begin{array}{l} C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n,\quad if\ 1+k^4\ is\ even\\ C=\left(\frac{(1+k^4)^2+1}{2}\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n,\quad if\ 1+k^4\ is\ odd\end{array}\right\}.$$ Let $\emptyset:\mathbf{Z}^2\to\mathbf{Z}^3(\boldsymbol{P})$ with

$$\emptyset(F_n,F_{n+1})=\left(\left(2F_{n+1}(F_n+F_{n+1})\right), F_n(2F_{n+1}+F_n), F_{n+1}^2+(F_n+F_{n+1})^2\right).$$

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From Reference [5], the sequence of Fibonacci numbers is $(\{1,1,2,3,5,8,13,21\ldots\ldots\})$ following Recurrence Relation $F_n = F_{n-1} + F_{n-2}$ **for** $n \geq 2$, **with** $F_0 = 1$, $F_1 = 1$ **is also, satisfies Pythagorean rule as follows**

$$\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + \left(F_n(2F_{n+1} + F_n)\right)^2 = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2$$

KEYWORDS: Diophantine Equation, exponential, Pythagorean triplet, Integer design.

I.INTRODUCTION

Diophantine equations—polynomial equations with integer solutions—are a central topic in number theory. Among their many variants, **exponential Diophantine equations** involve terms where variables appear as exponents. Finding integer solutions to such equations is notably complex and has implications in mathematics, cryptography, and several scientific fields. Historical Context and Theoretical Background

Classical Diophantine Equations: Traditionally, research started with linear and polynomial forms, such as the well-known cases of Pythagorean triples .

Exponential Generalization: The study of exponential forms expanded from these roots, posing questions that often lack general solution methods and in some cases are proven to be undecidable. In this paper, focused to find the general exponential integer solution of

$$\alpha(X^4 + Y^4)\left(\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + F_n(2F_{n+1} + F_n)^2\right) = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2 (C^2 - D^2)(Z^2 + W^2)P^{\beta}$$

With $\alpha > 0, \beta > 0$ **is derived from fixed value of** $\beta = 1, \beta = 2$ ***and*** $\beta > 2$.

II.RESULTS & DISCUSSIONS

Lemma 1: Introduce to Generate Pythagorean Triples with using of sequence of Fibonacci numbers as follows. Let $\emptyset : \mathbf{Z}^2 \to \mathbf{Z}^3(P)$ with

$$\emptyset(F_n, F_{n+1}) = (\left(2F_{n+1}(F_n + F_{n+1})\right), F_n(2F_{n+1} + F_n), F_{n+1}^2 + (F_n + F_{n+1})^2).$$

From Reference 5, the sequence of Fibonacci numbers is $(\{1,1,2,3,5,8,13,21\ldots\ldots\})$ following Recurrence Relation $F_n = F_{n-1} + F_{n-2}$ **for** $n \geq 2$, **with** $F_0 = 1$, $F_1 = 1$

$$\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + \left(F_n(2F_{n+1} + F_n)\right)^2 = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2$$

Proportion 1: A Study on exponential integer solution of above Diophantine Equation at $\beta = 1$ **is**

$$\alpha(X^4 + Y^4)\left(\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + F_n(2F_{n+1} + F_n)^2\right) = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2 (C^2 - D^2)(Z^2 + W^2)P$$

with $\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + \left(F_n(2F_{n+1} + F_n)\right)^2 = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2$ **is**

$$\alpha(X^4 + Y^4) = (C^2 - D^2)(Z^2 + W^2)P$$

Explanation: Let x = $k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}$.

Consider $\alpha(X^4 + Y^4) = \alpha k^{4n}(1 + k^4)$ Again consider $(Z^2 + W^2)P = k^{4n}(k^6 + k^4)$.

It follows that $\alpha(X^4 + Y^4) = (C^2 - D^2)(Z^2 + W^2)P$ implies that

$\alpha k^{4n}(1 + k^4) = (C^2 - D^2)k^{4n}(k^6 - k^4)$ implies $\alpha(1 + k^4) = (C^2 - D^2)(k^6 + k^4)$.

Solve for α, whenever $(1 + k^4, D, C)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet, now I chosen one of the technique of

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1 = \left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{x^2-1} \text{ if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2 = \left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1} \text{ if } x \text{ is an odd composite or its Power, for some } p = 1,2,3\ldots\right\}$$

$$S_3 = \left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1} \quad if\ x\ is \text{ geometric power of } 2\right\}$$

$$S_4 = \left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}, \text{otherwise (x is even composite or its power)}\right\}.$$

now I chosen one of the technique of

if r is an even number, then (r, $\left(\frac{r}{2}\right)^2 - 1$, $\left(\frac{r}{2}\right)^2 + 1$) is a Pythagorean triplet.

If r is an odd number, then (r, $\frac{r^2-1}{2}$, $\frac{r^2+1}{2}$) is a Pythagorean triplet.

It implies that $(1 + k^4, D, C)$ becomes a Pythagorean Triplet depending on *whether* $1 + k^4$ is odd or even.

Case 1: **If $1 + k^4$ is even**, then ($1 + k^4$, $\left(\frac{1+k^4}{2}\right)^2 - 1$, $\left(\frac{1+k^4}{2}\right)^2 + 1$) is a Pythagorean triplet.

It follows that $\alpha(1 + k^4) = (C^2 - D^2)(k^6 - k^4)$ and solve for α,

whenever $(1 + k^4, D, C)$ becomes a Pythagorean Triplet with $C = \left(\left(\frac{1+k^4}{2}\right)^2 + 1\right)n$,

$D = \left(\left(\frac{1+k^4}{2}\right)^2 - 1\right)n$ *and* $C^2 - D^2 = (1 + k^4)^2 n^2$ and hence $\alpha = (1 + k^4)(k^6 - k^4)n^2$.

Hence, **we obtain**

$$\boldsymbol{\alpha(X^4 + Y^4)\left(\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + F_n(2F_{n+1} + F_n)^2\right) = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2 (C^2 - D^2)(Z^2 + W^2)P}$$

having integer design of solution is $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, C = \left(\left(\frac{1+k^4}{2}\right)^2 + 1\right)n,$

$D = \left(\left(\frac{1+k^4}{2}\right)^2 - 1\right)n, \alpha = (1 + k^4)(k^6 + k^4)n^2$.

Verification:

Consider **LHS**

$\alpha(X^4 + Y^4) = (1 + k^4)(k^6 - k^4)n^2(k^{4n} + k^{4n+4}) = k^{4n}(k^6 + k^4)(1 + k^4)^2 n^2$

Consider RHS

$(C^2 - D^2)(Z^2 + W^2)P = (1 + k^4)^2 n^2 (k^{2n+6} - k^{2n+4})k^{2n} = k^{4n}(k^6 + k^4)(1 + k^4)^2 n^2$.

Hence LHS = *RHS*.

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Case 2: If $1+k^4$ is odd, then $\left(1+k^4, \frac{(1+k^4)^2-1}{2}, \frac{(1+k^4)^2+1}{2}\right)$ is a Pythagorean triplet. It follows that $\alpha(1+k^4) = (C^2-D^2)(k^6-k^4)$ and solves for α, whenever $(1+k^4, D, C)$ becomes a Pythagorean Triplet with $C = \left(\frac{(1+k^4)^2+1}{2}\right)n$, $D = \left(\frac{(1+k^4)^2-1}{2}\right)n$. Hence $C^2-D^2 = (1+k^4)^2n^2$ and hence $\alpha = (1+k^4)(k^6-k^4)n^2$.

Hence**, we obtain** $\boldsymbol{\alpha(X^4+Y^4) = (C^2-D^2)(Z^2+W^2)P}$**.**

Having an integer design of solution is $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}$,

$$C = \left(\frac{(1+k^4)^2+1}{2}\right)n,\ D = \left(\frac{(1+k^4)^2-1}{2}\right)n,\ \alpha = (1+k^4)(k^6+k^4).$$

Verification:

Consider **LHS**

$$\alpha(X^4+Y^4) = (1+k^4)(k^6-k^4)n^2(k^{4n}+k^{4n+4}) = k^{4n}(k^6+k^4)(1+k^4)^2n^2$$

Consider RHS

$$(C^2-D^2)(Z^2+W^2)P = (1+k^4)^2n^2(k^{2n+6}-k^{2n+4})k^{2n} = k^{4n}(k^6+k^4)(1+k^4)^2n^2.$$

Hence LHS $=$ ***RHS*****.**

E.g. 1: Suppose k = 2 then $1+k^4 = 17$, is odd; Having an integer design of solution is

$x = 2^n, y = 2^{n+1}, z = 2^{n+3}, w = 2^{n+2}, p = 2^{2n}$,

$$C = \left(\frac{(1+2^4)^2+1}{2}\right)n = 145n,\ D = \left(\frac{(1+2^4)^2-1}{2}\right) = 144n$$

$$C^2-D^2 = (1+2^4)^2n^2,\ \alpha = (1+2^4)(2^6-2^4)n^2.$$

Suppose n = 1; then $x = 2, y = 4, z = 16, w = 8, p = 4$,

$$C = \left(\frac{(1+2^4)^2+1}{2}\right) = 145,\ D = \left(\frac{(1+2^4)^2-1}{2}\right) = 144$$

$$C^2-D^2 = (1+2^4)^2 = 289,\ \alpha = (1+2^4)(2^6+2^4) = 1360.$$

Consider **LHS=** $\boldsymbol{\alpha(X^4+Y^4)}$ **=1360**$\mathbf{(2^4+4^4) = 369920}$**.**

RHS $= \mathbf{(C^2-D^2)(Z^2+W^2)P = 289 * (16^2+8^2) * 4 = 369920.}$

E.g. 2: Suppose k = 3 then $1+k^4 = 82$, is even; Having an integer design of solution is

$x = 3^n, y = 3^{n+1}, z = 3^{n+3}, w = 3^{n+2}, p = 3^{2n}$,

$$C = \left(\left(\frac{1+3^4}{2}\right)^2+1\right)n = 1681n,\ D = \left(\left(\frac{1+3^4}{2}\right)^2-1\right)n = 1600n$$

$$C^2-D^2 = (1+3^4)^2n^2 = 6724n,\ \alpha = (1+3^4)(3^6+3^4)n^2 = 66420n^2.$$

Suppose n = 1; then $x = 3, y = 9, z = 81, w = 27, p = 9, C^2-D^2 = 6724,\ \alpha = 66420$

Consider **LHS=** $\boldsymbol{\alpha(X^4+Y^4)}$ **=66420**$\mathbf{(3^4+9^4) = 441161640}$

RHS $= \mathbf{(C^2-D^2)(Z^2+W^2)P = 6724 * (81^2+27^2) * 9 = 441161640.}$

Proportion 2: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 2}$ **is**

$$\boldsymbol{\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right) = \left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^2}$$

Explanation: Let $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n$

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Consider $\alpha(X^4+Y^4)=\alpha k^{4n}(1+k^4)$

Again consider $(Z^2+W^2)\boldsymbol{P^2}=k^{4n}(k^6+k^4)$.

It follows that

$$\alpha(X^4+Y^4)\left(\left(\mathbf{2F_{n+1}(F_n+F_{n+1})}\right)^2+\mathbf{F_n(2F_{n+1}+F_n)^2}\right)=\left(\mathbf{F_{n+1}^2}+(\mathbf{F_n}+\mathbf{F_{n+1}})^2\right)^2(C^2-D^2)(Z^2+W^2)\boldsymbol{P^2}$$

implies that

$\alpha k^{4n}(1+k^4)=(C^2-D^2)k^{4n}(k^6+k^4)$ implies $\alpha(1+k^4)=(C^2-D^2)(k^6+k^4)$. Solve for α, whenever $(1+k^4,D,C)$ becomes a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{x^2-1}\quad \text{if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2-1}\quad \text{if } x \text{ is an odd composite or its Power, for some } p=1,2,3\ldots\right\}$$

$$S_3=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2}\right)^2-1}\quad if\ x \text{ is geometric power of 2}\right\}$$

$$S_4=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2p^2}\right)^2-1}, \text{otherwise (x is even composite or its power)}\right\}.$$

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if r is an even number, then (r, $\left(\frac{r}{2}\right)^2-1$, $\left(\frac{r}{2}\right)^2+1$) is a Pythagorean triplet.

If r is an odd number, then (r, $\frac{r^2-1}{2}$, $\frac{r^2+1}{2}$) is a Pythagorean triplet.

It implies that $(1+k^4,D,C)$ becomes a Pythagorean Triplet depending on $whether$ $1+k^4$ is odd or even.

Case 1: If $1+k^4$ is even, then ($1+k^4$, $\left(\frac{1+k^4}{2}\right)^2-1$, $\left(\frac{1+k^4}{2}\right)^2+1$) is a Pythagorean triplet. It follows that $\alpha(1+k^4)=(C^2-D^2)(k^6+k^4)$ and solve for α,

whenever $(1+k^4,D,C)$ becomes a Pythagorean Triplet with $C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n$,

$D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n$, $C^2-D^2=(1+k^4)^2n^2$ and hence $\alpha=(1+k^4)(k^6+k^4)n^2$.

Hence, **we obtain** $\boldsymbol{\alpha(X^4+Y^4)=(C^2-D^2)(Z^2-W^2)P^2}$ **having integer design of solution is** $\mathbf{x}=\boldsymbol{k^n}, \boldsymbol{y}=\boldsymbol{k^{n+1}}, \boldsymbol{z}=\boldsymbol{k^{n+3}}, \boldsymbol{w}=\boldsymbol{k^{n+2}}, \boldsymbol{p}=\boldsymbol{k^n}, \boldsymbol{C}=\boldsymbol{\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n}$,

$\boldsymbol{D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \alpha=(1+k^4)(k^6+k^4)n^2}$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4}) = (\mathbf{1}+\boldsymbol{k^4})(\boldsymbol{k^6}+\boldsymbol{k^4})\boldsymbol{n^2}(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}}) = \boldsymbol{k^{4n}}(k^6+k^4)(\mathbf{1}+\boldsymbol{k^4})^2\boldsymbol{n^2}$

Consider RHS

$(C^2-D^2)(Z^2+W^2)\boldsymbol{P^2} = (1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{2n} = \boldsymbol{k^{4n}}(k^6+k^4)(\mathbf{1}+\boldsymbol{k^4})^2n^2.$

Hence LHS $= \boldsymbol{RHS}$.

Case 2: If $1+k^4$ is odd, then $\left(1+k^4, \frac{(1+k^4)^2-1}{2}, \frac{(1+k^4)^2+1}{2}\right)$ is a Pythagorean triplet. It follows that $\alpha(1+k^4) = (C^2-D^2)(k^6+k^4)$ and solves for α,

whenever $(1+k^4, D, C)$ becomes a Pythagorean Triplet with $C = \left(\frac{(1+k^4)^2+1}{2}\right)n$,

$D = \left(\frac{(1+k^4)^2-1}{2}\right)n$. Hence $C^2-D^2 = (1+k^4)^2n^2$ and hence $\alpha = (1+k^4)(k^6+k^4)n^2$.

Hence**, we obtain $\boldsymbol{\alpha}(\boldsymbol{X^4}+\boldsymbol{Y^4}) = (\boldsymbol{C^2}-\boldsymbol{D^2})(\boldsymbol{Z^2}+\boldsymbol{W^2})\boldsymbol{P^2}$, having an integer design of solution is x $= \boldsymbol{k^n}, \boldsymbol{y} = \boldsymbol{k^{n+1}}, \boldsymbol{z} = \boldsymbol{k^{n+3}}, \boldsymbol{w} = \boldsymbol{k^{n+2}}, \boldsymbol{p} = \boldsymbol{k^n}$,**

$\boldsymbol{C} = \left(\frac{(1+k^4)^2+1}{2}\right)\boldsymbol{n}, \boldsymbol{D} = \left(\frac{(1+k^4)^2-1}{2}\right)\boldsymbol{n},\ \boldsymbol{\alpha} = (\mathbf{1}+\boldsymbol{k^4})(\boldsymbol{k^6}+\boldsymbol{k^4})\boldsymbol{n^2}.$

Verification:

Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4}) = (\mathbf{1}+\boldsymbol{k^4})(\boldsymbol{k^6}+\boldsymbol{k^4})\boldsymbol{n^2}(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}}) = \boldsymbol{k^{4n}}(k^6+k^4)(\mathbf{1}+\boldsymbol{k^4})^2\boldsymbol{n^2}$

Consider RHS

$(C^2-D^2)(Z^2+W^2)\boldsymbol{P^2} = (1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{2n} = \boldsymbol{k^{4n}}(k^6+k^4)(\mathbf{1}+\boldsymbol{k^4})^2n^2.$

Hence LHS $= \boldsymbol{RHS}$.

E.g. 3: Suppose k = 2 then $1+k^4 = 17$, is odd; Having an integer design of solution is

x $= 2^n, y = 2^{n+1}, z = 2^{n+3}, w = 2^{n+2}, p = 2^n$,

$C = \left(\frac{(1+2^4)^2+1}{2}\right)n = 145n, D = \left(\frac{(1+2^4)^2-1}{2}\right) = 144n$

$C^2-D^2 = (1+2^4)^2n^2,\ \alpha = (1+2^4)(2^6-2^4)n^2.$

Suppose n = 1; then x $= 2, y = 4, z = 16, w = 8, p = 2, C = \left(\frac{(1+2^4)^2+1}{2}\right) = 145,$

$D = \left(\frac{(1+2^4)^2-1}{2}\right) = 144.\ C^2-D^2 = (1+2^4)^2 = 289,\ \alpha = (1+2^4)(2^6+2^4) = 1360.$

Consider **LHS= $\boldsymbol{\alpha}(\boldsymbol{X^4}+\boldsymbol{Y^4})$ =1360($\mathbf{2^4}+\mathbf{4^4}$) =.369920**

RHS = $(\boldsymbol{C^2}-\boldsymbol{D^2})(\boldsymbol{Z^2}+\boldsymbol{W^2})\boldsymbol{P^2} = \mathbf{289} * (\mathbf{16^2}+\mathbf{8^2}) * \mathbf{4} = \mathbf{369920}.$

E.g. 4: Suppose k = 3 then $1+k^4 = 82$, is even; Having an integer design of solution is

x $= 3^n, y = 3^{n+1}, z = 3^{n+3}, w = 3^{n+2}, p = 3^{2n}, C = \left(\left(\frac{1+3^4}{2}\right)^2+1\right)n = 1681n,$

$D = \left(\left(\frac{1+3^4}{2}\right)^2-1\right)n = 1600n,\ C^2-D^2 = (1+3^4)^2n^2 = 6724n,$

$\alpha = (1+3^4)(3^6+3^4)n^2 = 66420n^2.$

Suppose n = 1; then x $= 3, y = 9, z = 81, w = 27, p = 3, C = \left(\left(\frac{1+3^4}{2}\right)^2+1\right) = 1681,$

$D = \left(\left(\frac{1+3^4}{2}\right)^2 - 1\right) = 1600,\ C^2 - D^2 = 6724,\ \alpha = 66420$

Consider **LHS**= $\boldsymbol{\alpha(X^4 + Y^4)}$ =**66420**$\mathbf{(3^4 + 9^4) = 441161640}$

RHS = $\mathbf{(C^2 - D^2)(Z^2 + W^2)P^2 = 6724 * (81^2 + 27^2) * 9 = 441161640}$

Proportion 3: A Study on exponential integer solution of above Diophantine Equation at $\boldsymbol{\beta = 3}$ is $\boldsymbol{\alpha(X^4 + Y^4)\left(\left(2F_{n+1}(F_n + F_{n+1})\right)^2 + F_n(2F_{n+1} + F_n)^2\right) = \left(F_{n+1}^2 + (F_n + F_{n+1})^2\right)^2 (C^2 - D^2)(Z^2 - W^2)P^3}$**.**

Explanation: Let x = $k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n$

Consider $\alpha(X^4 + Y^4) = \alpha k^{4n}(1 + k^4)$

Again consider $(Z^2 - W^2)\boldsymbol{P^3} = k^{5n}(k^6 + k^4)$.

It follows that $\alpha(X^4 + Y^4) = (C^2 - D^2)(Z^2 + W^2)\boldsymbol{P^3}$ implies that

$\alpha k^{4n}(1 + k^4) = (C^2 - D^2)k^{5n}(k^6 - k^4)$ implies $\alpha(1 + k^4) = (C^2 - D^2)(k^6 + k^4)k^n$**.** Solve for α, whenever $(1 + k^4, D, C)$ becomes a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1 = \left\{(x, y, z): \frac{z}{y} = 1 + \frac{2}{x^2 - 1} \text{ if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2 = \left\{(x, y, z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1} \text{ if } x \text{ is an odd composite or its Power, for some } p = 1,2,3\ldots\right\}$$

$$S_3 = \left\{(x, y, z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1} \quad if\ x\ is \text{ geometric power of 2}\right\}$$

$$S_4 = \left\{(x, y, z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}, \text{otherwise (x is even composite or its power)}\right\}.$$

now I chosen one of the technique of

if r is an even number, then (r, $\left(\frac{r}{2}\right)^2 - 1, \left(\frac{r}{2}\right)^2 + 1$) is a Pythagorean triplet.

If r is an odd number, then (r, $\frac{r^2-1}{2}, \frac{r^2+1}{2}$) is a Pythagorean triplet.

It implies that $(1 + k^4, D, C)$ becomes a Pythagorean Triplet depending on $whether$ $1 + k^4$ is odd or even.

Case 1: If $1 + k^4$ is even, then ($1 + k^4, \left(\frac{1+k^4}{2}\right)^2 - 1, \left(\frac{1+k^4}{2}\right)^2 + 1$) is a Pythagorean triplet. It follows that $\alpha(1 + k^4) = (C^2 - D^2)(k^6 + k^4)k^n$ and solve for α, whenever $(1 + k^4, D, C)$ becomes a Pythagorean Triplet with $C = \left(\left(\frac{1+k^4}{2}\right)^2 + 1\right)n$, $D = \left(\left(\frac{1+k^4}{2}\right)^2 - 1\right)n$

$C^2 - D^2 = (1 + k^4)^2 n^2$ and hence $\alpha = (1 + k^4)(k^6 + k^4)k^n n^2$.

Hence, **we obtain**

$$\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^3$$

having integer design of solution is $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n, C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n,$

$D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \alpha=(1+k^4)(k^6+k^4)k^n n^2.$

Verification:

Consider **LHS**

$\alpha(X^4+Y^4)=(1+k^4)(k^6-k^4)k^n n^2(k^{4n}+k^{4n+4})=k^{5n}(k^6+k^4)(1+k^4)^2n^2$

Consider RHS

$(C^2-D^2)(Z^2-W^2)P^3=(1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{3n}=k^{5n}(k^6+k^4)(1+k^4)^2n^2.$

Hence LHS $=$ ***RHS*.**

Case 2: If $1+k^4$ is odd, then ($1+k^4, \frac{(1+k^4)^2-1}{2}, \frac{(1+k^4)^2+1}{2}$) is a Pythagorean triplet. It follows that $\alpha(1+k^4)=(C^2-D^2)(k^6+k^4)k^n$ and solve for α, whenever $(1+k^4, D, C)$ becomes a Pythagorean Triplet with $C=\left(\frac{(1+k^4)^2+1}{2}\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n$. Hence

$C^2-D^2=(1+k^4)^2n^2$ and hence $\alpha=(1+k^4)(k^6+k^4)k^n n^2$.

Hence**, we obtain**

$$\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^3,$$

having an integer design of solution is $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n,$

$C=\left(\frac{(1+k^4)^2+1}{2}\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n,\ \alpha=(1+k^4)(k^6+k^4)k^n n^2.$

Verification:

Consider **LHS**

$\alpha(X^4+Y^4)=(1+k^4)(k^6-k^4)k^n n^2(k^{4n}+k^{4n+4})=k^{5n}(k^6+k^4)(1+k^4)^2n^2$

Consider RHS

$(C^2-D^2)(Z^2+W^2)P^3=(1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{3n}=k^{5n}(k^6+k^4)(1+k^4)^2n^2.$

Hence LHS $=$ ***RHS*.**

Proportion 4: A Study on Diophantine Equation

$$\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^4$$

Let $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n$

Consider $\alpha(X^4+Y^4)=\alpha k^{4n}(1+k^4)$

Again consider $(Z^2+W^2)P^4=k^{6n}(k^6+k^4)$.

It follows that

$$\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^4$$

implies that

$\alpha k^{4n}(1+k^4)=(C^2-D^2)k^{6n}(k^6-k^4)$ implies $\alpha(1+k^4)=(C^2-D^2)(k^6+k^4)k^{2n}$. Solve for α, whenever $(1+k^4,D,C)$ becomes a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{x^2-1}\ \text{ if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2-1}\ \text{ if } x \text{ is an odd composite or its Power, for some } p=1,2,3\ldots\right\}$$

$$S_3=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2}\right)^2-1}\ \textit{ if } x \textit{ is geometric power of } 2\right\}$$

$$S_4=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2p^2}\right)^2-1}, \text{otherwise (x is even composite or its power)}\right\}.$$

now I chosen one of the technique of

if r is an even number, then (r, $\left(\frac{r}{2}\right)^2-1$, $\left(\frac{r}{2}\right)^2+1$) is a Pythagorean triplet.

If r is an odd number, then (r, $\frac{r^2-1}{2}$, $\frac{r^2+1}{2}$) is a Pythagorean triplet.

It implies that $(1+k^4,D,C)$ becomes a Pythagorean Triplet depending on *whether* $1+k^4$ is odd or even.

Case 1: If $1+k^4$ is even, then ($1+k^4$, $\left(\frac{1+k^4}{2}\right)^2-1$, $\left(\frac{1+k^4}{2}\right)^2+1$) is a Pythagorean triplet. It follows that $\alpha(1+k^4)=(C^2-D^2)(k^6-k^4)k^n$ and solve for α, whenever $(1+k^4,D,C)$ becomes a Pythagorean Triplet with $C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n$, $D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n$

$C^2-D^2=(1+k^4)^2n^2$ and hence $\alpha=(1+k^4)(k^6+k^4)k^{2n}n^2$.

Hence, we obtain $\alpha(X^4+Y^4)=(C^2-D^2)(Z^2+W^2)P^4$ having integer design of solution is $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n$, $C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n$,

$D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \alpha=(1+k^4)(k^6+k^4)k^{2n}n^2$.

Verification:

Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})=(1+k^4)(k^6+k^4)k^{2n}n^2(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}})=\boldsymbol{k^{6n}}(k^6+k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2\boldsymbol{n^2}$

Consider RHS

$(C^2-D^2)(Z^2+W^2)\boldsymbol{P^4}=(1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{4n}=\boldsymbol{k^{6n}}(k^6+k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2n^2$.

Hence LHS $=$ *RHS*.

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Case 2: If $1+k^4$ is odd, then $\left(1+k^4, \frac{(1+k^4)^2-1}{2}, \frac{(1+k^4)^2+1}{2}\right)$ is a Pythagorean triplet. It follows that $\alpha(1+k^4) = (C^2-D^2)(k^6+k^4)k^n$ and solve for α, whenever $(1+k^4, D, C)$ becomes a Pythagorean Triplet with $C = \left(\frac{(1+k^4)^2+1}{2}\right)n$, $D = \left(\frac{(1+k^4)^2-1}{2}\right)n$. Hence

$C^2-D^2 = (1+k^4)^2n^2$ and hence $\alpha = (1+k^4)(k^6+k^4)k^{2n}n^2$.

Hence**, we obtain**

$$\boldsymbol{\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right) = \left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^4},$$

having an integer design of solution is $\boldsymbol{x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n}$,

$$\boldsymbol{C = \left(\frac{(1+k^4)^2+1}{2}\right)n,\ D = \left(\frac{(1+k^4)^2-1}{2}\right)n,\ \alpha = (1+k^4)(k^6+k^4)k^{2n}n^2}.$$

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4+Y^4}) = (1+k^4)(k^6+k^4)k^{2n}n^2(\boldsymbol{k^{4n}+k^{4n+4}}) = \boldsymbol{k^{6n}}(k^6+k^4)(\boldsymbol{1+k^4})^2\boldsymbol{n^2}$

Consider RHS

$(C^2-D^2)(Z^2+W^2)\boldsymbol{P^4} = (1+k^4)^2n^2(k^{2n+6}+k^{2n+4})k^{4n} = \boldsymbol{k^{6n}}(k^6+k^4)(\boldsymbol{1+k^4})^2n^2$.

Hence LHS $= \boldsymbol{RHS}$**.**

Main result:

From the **proportions 1,2,3,4** the Diophantine equation

$$\boldsymbol{\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)} = \left(\boldsymbol{F_{n+1}^2+(F_n+F_{n+1})^2}\right)^2(C^2-D^2)(Z^2+W^2)P^\beta$$

with $\alpha > 0, \beta > 0$ *and* $x < y < w < z$

Having integer design of solutions for $\beta > 2$ is

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n$

From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1 = \left\{(x,y,z): \frac{z}{y} = 1+\frac{2}{x^2-1} \text{ if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2 = \left\{(x,y,z): \frac{z}{y} = 1+\frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2-1} \text{ if } x \text{ is an odd composite or its Power, for some } p = 1,2,3\ldots\right\}$$

$$S_3 = \left\{(x,y,z): \frac{z}{y} = 1+\frac{2}{\left(\frac{x}{2}\right)^2-1}\ \ if\ x \text{ is geometric power of } 2\right\}$$

$$S_4 = \left\{(x,y,z): \frac{z}{y} = 1+\frac{2}{\left(\frac{x}{2p^2}\right)^2-1}, \text{otherwise (x is even composite or its power)}\right\}.$$

now I chosen one of the technique of

if r is an even number, then $\left(r, \left(\frac{r}{2}\right)^2-1, \left(\frac{r}{2}\right)^2+1\right)$ is a Pythagorean triplet.

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If r is an odd number, then (r, $\frac{r^2-1}{2}$, $\frac{r^2+1}{2}$) is a Pythagorean triplet.

If $1+k^4$ is even then

$$C=\left(\left(\frac{1+k^4}{2}\right)^2+1\right)n,\ D=\left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \alpha=(1+k^4)(k^6+k^4)k^{(\beta-2)n}n^2.$$

and if $1+k^4$ is odd then

$$C=\left(\frac{(1+k^4)^2+1}{2})\right)n, D=\left(\frac{(1+k^4)^2-1}{2}\right)n,\ \alpha=(1+k^4)(k^6+k^4)k^{(\beta-2)n}n^2.$$

III.CONCLUSION

This equation generalizes classical Diophantine problems, blending sums of fourth powers with multiplicative factorizations. While challenging, targeted parametrization and modular analysis can yield solutions. Future work may classify solutions for specific α, β or link to broader number-theoretic frameworks. The parametric framework provides infinite families of solutions by exploiting algebraic identities and modular arithmetic. Future work could explore non-parametric solutions or generalizations to higher exponents.

$$\boldsymbol{\alpha(X^4+Y^4)\left(\left(2F_{n+1}(F_n+F_{n+1})\right)^2+F_n(2F_{n+1}+F_n)^2\right)=\left(F_{n+1}^2+(F_n+F_{n+1})^2\right)^2(C^2-D^2)(Z^2+W^2)P^\beta}$$

With $\alpha>0,\beta>0$ and $X<Y<W<Z,$

Having integer design of solutions **for $\beta>2$** is parameterized by integers k and n, with variables defined as: $x=k^n, y=k^{n+1}, z=k^{n+3}, w=k^{n+2}, p=k^n$, From the References [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12] there is so many methods to generate Pythagorean triplet,

$$S_1=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{x^2-1}\ \text{ if } x \text{ is an odd prime number or its power}\right\}$$

$$S_2=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2-1}\ \text{ if } x \text{ is an odd composite or its Power, for some } p=1,2,3...\right\}$$

$$S_3=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2}\right)^2-1}\ \ if\ x\ is \text{ geometric power of 2}\right\}$$

$$S_4=\left\{(x,y,z):\frac{z}{y}=1+\frac{2}{\left(\frac{x}{2p^2}\right)^2-1}, \text{otherwise (x is even composite or its power)}\right\}.$$

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if r is an even number, then (r, $\left(\frac{r}{2}\right)^2-1$, $\left(\frac{r}{2}\right)^2+1$) is a Pythagorean triplet.

If r is an odd number, then (r, $\frac{r^2-1}{2}$, $\frac{r^2+1}{2}$) is a Pythagorean triplet.

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$$\alpha = (1+k^4)(k^6+k^4)k^{(\beta-2)n}n^2 , \left\{ \begin{array}{c} C = \left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D = \left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \quad if\ 1+k^4\ is\ even \\ C = \left(\frac{(1+k^4)^2+1}{2}\right)n, D = \left(\frac{(1+k^4)^2-1}{2}\right)n, \quad if\ 1+k^4\ is\ odd \end{array} \right\}.$$

for $\boldsymbol{\beta} = \mathbf{1}$ is $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, \alpha = (1+k^4)(k^6+k^4)n^2$,

$$\left\{ \begin{array}{c} C = \left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D = \left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \quad if\ 1+k^4\ is\ even \\ C = \left(\frac{(1+k^4)^2+1}{2}\right)n, D = \left(\frac{(1+k^4)^2-1}{2}\right)n, \quad if\ 1+k^4\ is\ odd \end{array} \right\}.$$

for $\boldsymbol{\beta} = \mathbf{2}$ *is* $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^n, \alpha = (1+k^4)(k^6+k^4)n^2$.

$$\left\{ \begin{array}{c} C = \left(\left(\frac{1+k^4}{2}\right)^2+1\right)n, D = \left(\left(\frac{1+k^4}{2}\right)^2-1\right)n, \quad if\ 1+k^4\ is\ even \\ C = \left(\frac{(1+k^4)^2+1}{2}\right)n, D = \left(\frac{(1+k^4)^2-1}{2}\right)n, \quad if\ 1+k^4\ is\ odd \end{array} \right\}$$

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