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Methods of Synthesis of Regulators Ensuring Quasi-Optimal Speed of Response in Control Systems with Time Delay

XusanovSubanNurullayevich

Docent, PhD, Karshi State Technical University, Karshi, Uzbekistan

ABSTRACT: In this paper, algorithms for the synthesis of quasi-optimal speed regulators intended for controlled objects with time delay are considered. In constructing the control law, approximations of various orders are used. It is shown that starting from the second and higher orders of approximation, it is possible to obtain a unified generalized form of quasi-optimal control representation. A significant advantage of the proposed approach is that its application does not complicate the structure of the strictly optimal control action. In addition, the choice of the approximation order is determined by the required degree of closeness to strictly optimal control, which ensures the achievement of the necessary quality of regulation.

KEY WORDS: synthesis of control algorithms, high-speed regulators, control objects with delay, optimization approaches, approximate representations of functions, development of control strategies, systems with time lag, fast-response regulators.

I.INTRODUCTION

At present, several approaches have been developed for the synthesis of control systems with time delay that ensure a high response speed. The most widely known among them is the method of delay compensation in the time domain within optimal schemes [1–3]. The essence of this approach lies in introducing an anticipatory term of value τ into the argument of the control function. This makes it possible to align the control action of a system with delay with the dynamics of a similar system without delay. From the standpoint of mathematical formalization, this method can be interpreted as constructing in the phase space a control surface that is advanced by τ relative to the switching surface of the system without delay.

There are also methods for synthesizing control laws that are optimal in terms of response speed for systems without time delay, in which the actual coordinates are replaced by their predicted values. These predicted quantities are obtained by solving differential equations that describe the system dynamics without accounting for delay and with given initial conditions. The main drawback of this approach is the need to model the delay process, perform differentiation operations, and adjust a large number of parameters. Many of the known methods for constructing quasi-optimal speed algorithms for controlled objects with delay are based on simplified versions of strictly optimal control laws. Such simplifications facilitate the technical implementation of the regulators, but at the same time often reduce the quality of control: the approximations turn out to be overly rough, and the control laws themselves lack adjustable parameters that would allow the transient processes in quasi-optimal systems to be brought closer to strictly optimal modes [4–9].

II. SIGNIFICANCE OF THE SYSTEM

Control systems with time delay are widely encountered in practical engineering and technological processes. Under such conditions, ensuring high-speed response is a challenging task, as classical optimal control methods often require excessive computational resources or lead to complex structures. The proposed quasi-optimal control approach is therefore significant as a practical solution.

First, this method preserves the simplicity of the strictly optimal control law and does not complicate the system structure. This makes its implementation in real technological processes faster and more reliable.

Second, the ability to select the order of approximation according to the required accuracy provides flexibility in achieving the desired quality of control. Thus, the control process can closely approach strictly optimal results while remaining technically simple to implement.

Third, the applicability of this approach extends to real-world industries such as manufacturing, energy systems, and automated control systems, where delays are unavoidable and must be effectively compensated.

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In summary, the practical significance of the proposed method lies in its ability to ensure high-quality control without increasing system complexity, thereby substantially improving the performance and efficiency of real control systems

III. LITERATURE SURVEY

In recent years, various approaches have been developed for the design of control systems with time delay, with the primary aim of increasing system responsiveness. One of the most widely used techniques is the method of delay compensation in the time domain within optimal schemes [1–3]. The essence of this method is the introduction of an anticipatory term of value τ into the argument of the control function. This approach makes it possible to align the control action of a system with delay with the dynamic properties of a system without delay. From a mathematical perspective, it can be interpreted as constructing, in phase space, a control surface shifted by τ relative to the switching surface of a delay-free system.

In addition, there are methods for constructing control laws that are optimal in terms of response speed for systems without delay. In such cases, the actual coordinates of the object are replaced with their predicted values. These predictions are obtained by solving differential equations that describe the dynamics of the system without delay under known initial conditions. However, the main drawback of this approach is the necessity to model the delay process, perform differentiation operations, and adjust a large number of parameters.

Moreover, many existing methods for designing quasi-optimal speed algorithms for delayed control objects rely on simplified versions of strictly optimal control laws. While this simplification makes the implementation of regulators technically easier, it often leads to a reduction in control quality.

IV. METHODOLOGY

When formulating the problem of synthesizing quasi-optimal speed regulators for control objects with delay in the control channel, it is assumed that the system dynamics can be adequately described by a linear differential equation of a certain type. This approach is based on the classical model of linear systems with constant coefficients, which is widely used in control theory and makes it possible to apply proven analytical methods of analysis and design. Including the delay in the equation reflects the real characteristics of technological and technical processes, where the response to the control action appears with a time lag. This complicates the optimization task but at the same time brings the mathematical model closer to the practical conditions of system operation.

$$\dot{X} = AX + Bu(t-\tau_0),\ \tau_0 \le t < \infty;\ u(t) = \Phi_0(t),\ 0 \le t < \tau_0, \tag{1}$$

where A and B are the state and control matrices of dimensions $n \times n$ and $l \times m$, respectively

Nevertheless, at the subsequent stages of optimization, it becomes expedient to rewrite equation (1) in an alternative form:

$$\dot{X} = AX + Bv_0(t), \tag{2}$$

where

$$v_0(t) = \begin{cases} \Phi_0(t-\tau_0), 0 \le t < \tau_0; \\ u(t-\tau_0), \tau_0 \le t < \infty, \end{cases} \tag{3}$$

the output of the delay element; $\Phi_0(t)$ — the initial function; $u(t)$ — the scalar control.

Equation (2) can be transformed into an equivalent system, free from delayed arguments, by integrating equation (1) subject to condition (3):

$$X(t) = X(0) + A\int_0^{\tau_0} X(\tau)d\tau + B\int_0^{\tau_0} \Phi_0(\tau-\tau_0)d\tau + A\int_{\tau_0}^{t} X(\tau)d\tau + B\int_{\tau_0}^{t} u(\tau-\tau_0)d\tau. \tag{4}$$

Substituting $t = t' + \tau_0$ into (4), where $0 \le t' < \infty$, we obtain:

$$X(t'+\tau_0) = X(0) + A\int_0^{\tau_0} X(\tau)d\tau + B\int_0^{\tau_0} \Phi_0(\tau-\tau_0)d\tau + A\int_{\tau_0}^{t'+\tau_0} X(\tau)d\tau + B\int_{\tau_0}^{t'+\tau_0} u(\tau-\tau_0)d\tau.$$

Define the new state vector $h(t') = X(t+\tau_0)$ and make the change of the integration variable $\tau = \tau' + \tau_0$. As a result of these transformations, we obtain the following expression.

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$$h(t') = X(0) + A\int_0^{\tau_0} X(\tau)d\tau + B\int_0^{\tau_0} \Phi_0(\tau-\tau_0)d\tau + A\int_0^{t'} h(\tau')d\tau + B\int_0^{t'} u(\tau')d\tau .$$

$$h(t^{'}) = X(0) + A\int_0^{\tau_0} X(\tau)d\tau + B\int_0^{\tau_0} \Phi_0(\tau-\tau_0)d\tau + A\int_0^{t^{'}} h(\tau^{'})d\tau + B\int_0^{t^{'}} u(\tau^{'})d\tau^{'}$$

By differentiating this expression with respect to time, we arrive at the following result:

$$\dot{h}(t) = Ah(t) + Bu(t) \tag{5}$$

the initial conditions are specified as follows:

$$h(0) = X(0) + B\int_0^{\tau_0} \Phi(\tau)d\tau + B\int_0^{\tau_0} \Phi_0(\tau-\tau_0)d\tau, \tag{6}$$

where

$$h(t) = X(t+\tau_0) . \tag{7}$$

The optimization problem for the transformed system is formulated under the following conditions:

$$u(t) = 0, \qquad T_y - \tau_0 < t < T_y;$$
$$h(t) = 0, \qquad t = T_y - \tau_0, \tag{8}$$

Here, the time instant T_y is considered a variable quantity and is not predetermined as fixed.

The control that ensures time-optimal performance for system (5) can be determined using known techniques and, in general, is expressed as a function of the state coordinates of the object [10–13]:

$$u(t) = -\sin f\{h(t)\}. \tag{9}$$

By representing the predictive variables in terms of the current values of the object's state coordinates, we obtain the following expression based on the corresponding equation:

$$u(t) = -\sin f\left\{ e^{A\tau_0} X(t) + \int_0^{\tau_0} e^{A(\tau_0 - S_0)} BW_0(t-\tau_0+S_0)dS_0 \right\}. \tag{10}$$

When using approximation approaches to equation (10), it is possible to derive a quasi-optimal control law corresponding to different levels of approximation of the considered function $W_0(t-\tau_0+S_0)$ [1,11]:

in the case of zero-order approximation.

$$u_0(t) = -\sin f\left\{ e^{A\tau_0} X(t) + \left[\int_0^{\tau_0} e^{(\tau_0 - S_0)A} BdS_0 \right] W_0(t-\tau_0) \right\}, \tag{11}$$

for the first-order approximation

$$u_1(t) = -\sin f\left\{ e^{A\tau_0} X(t) + \left[\int_0^{\tau_0} e^{A(\tau_0 - S_0)} B(1-\frac{S_0}{\tau_0})dS_0 \right] W_0(t-\tau_0) + \left[\int_0^{\tau_0} e^{A(\tau_0 - S_0)} B\frac{S_0}{\tau_0}dS_0 \right] W_0(t) \right\}. \tag{12}$$

In a similar way, a quasi-optimal time-response control law for system (1) can be derived for any chosen order of approximation. It has been shown that starting from the second and higher orders of approximation, it is possible to obtain a universal generalized form of representation of the quasi-optimal control law.

$$u_n(t) = -\sin f\left\{ e^{A\tau_0} X(t) + \left[\int_0^{\tau_0/n} e^{A(\tau_0 - S_0)} B(1-\frac{S_0}{\tau_0})dS_0 \right] W_0(t-\tau_0) + \right.$$
$$+\left[\int_{(n-1)\tau_0/n}^{\tau_0} e^{A(\tau_0 - S_0)} B(\frac{nS_0}{\tau_0}-n+1)dS_0 \right] W_0(t) + \sum_{l=1}^{n-1}\left[\int_{(l-1)\tau_0/n}^{l\tau_0/n} e^{A(\tau_0 - S_0)} B(\frac{nS_0}{\tau_0}-l+1)dS_0 + \right.$$
$$\left. + \int \int_{l\tau_0/n}^{(l+1)\tau_0/n} e^{A(\tau_0 - S_0)} B(1+l-\frac{nS_0}{\tau_0})dS_0 \right] + \int_{l\tau_0/n}^{(l+1)\tau_0/n} e^{A(\tau_0 - S_0)} B(1+l-\frac{nS_0}{\tau_0})dS_0 \left] W_0(t-\frac{l}{n}-\tau_0) \right\} \tag{13}$$

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where n – order of approximation;

$$W_0(t) = u(t), \quad 0 < t < \infty;$$

$$W_0(t-\tau_0) = \begin{cases} \Phi_0(t-\tau_0), & 0 \le t < \tau_0; \\ u(t-\tau_0), & \tau_0 \le t < \infty; \end{cases}$$

$$W_0(t - \frac{l}{n} - \tau_0) = \begin{cases} \Phi_0(t - l\tau_0 / n), & 0 \le t < l\tau_0 / n; \\ u_0(t - l\tau_0 / n), & l\tau_0 / n \le t < \infty. \end{cases}$$

For the purpose of simplifying and compactly representing the quasi-optimal control law, let us introduce the following notations:

$$K = e^{A\tau_0};\ K_0^{(\tau)} = \int_0^{\tau_0} e^{A(\tau_0 - S_0)} B dS_0;\ K_{n\,(n\ge1)}^{(\tau)} = \int_0^{\tau_0/n} e^{A(\tau_0 - S_0)} B(1 - \frac{nS_0}{\tau_0}) dS_0;$$

$$K_{n\,(n\ge1)}^{(l)} = \int_{(n-1)\tau_0/n}^{\tau_0} B e^{A(\tau_0 - S_0)} (\frac{nS_0}{\tau_0} - n + 1) dS_0;$$

$$K_{n\,(n\ge1)}^{(l)} = \int_{(l-1)\tau_0/n}^{l\tau_0/n} B e^{A(\tau_0 - S_0)} \times (\frac{nS_0}{\tau_0} - l + 1) dS_0 + \int_{l\tau_0/n}^{(l+1)\tau_0/n} B e^{A(\tau_0 - S_0)} \times (1 + l - \frac{nS_0}{\tau_0}) dS_0. \tag{14}$$

Taking into account (14), the quasi-optimal algorithm (11)–(13) takes the following form:

$$u_0(t) = -\sin f \left\{ KX(t) + K_0^{(\tau)} W_0(t - \tau_0) \right\};$$

$$u_1(t) = -\sin f \left\{ KX(t) + K_1^{(t)} W_0(t) + K_1^{(\tau)} W_0(t - \tau_0) \right\};$$

$$\underset{(n\ge2)}{u_n}(t) = -\sin f \left\{ KX(t) + K_n^{(t)} W_0(t) + K_n^{(\tau)} W_0(t - \tau_0) + \sum_{l=1}^{n-1} K_n^{(l)} W_0(t - l\tau_0 / n) \right\}. \tag{15}$$

The application of a linear approximation of the function $W_0(t - \tau_0 + S_0)$ leads to the fact that, when constructing a control algorithm oriented toward fast response, it is sufficient to take into account the current and previous values of the state coordinates of the object within the specified interval $W_0(t - \tau_0)$. Thanks to this, the implementation of an approximate-to-optimal control law does not cause serious technical difficulties, since the delayed coordinates, such as $W_0(t - \tau_0)$ or $W_0(t - l\tau_0 / n)$, in many practical cases can be determined directly through the delay element.

As the basis for developing such an algorithm, the already known optimal control obtained for an object without time delay is used. Consequently, the proposed method of synthesizing approximate time-optimal control makes it possible to preserve the simplicity of the structure of the strictly optimal control law, which is its significant advantage.
consistency and detect outliers.

V. EXPERIMENTAL RESULTS AND DISCUSSION

To evaluate the effectiveness of the proposed quasi-optimal control algorithms, numerical simulations were carried out on representative control systems with time delay. The main objective of the experiments was to compare the transient responses obtained from the quasi-optimal controllers with those of strictly optimal and simplified control laws.

The results demonstrated that the quasi-optimal controllers provided a response speed close to that of strictly optimal systems while maintaining a simpler structure. The selection of the approximation order was shown to have a direct influence on the accuracy of regulation: higher-order approximations improved the closeness to optimal response but required slightly more computation. At the same time, even low-order approximations ensured satisfactory performance and stability for practical applications.

In addition, the simulations confirmed that the proposed algorithms do not introduce excessive complexity into the system. The implementation remained straightforward, since the delayed coordinates could be directly obtained from the delay element. This result highlights the practical advantage of the approach for real-time applications in technological and engineering processes.

Overall, the experimental results verify that the developed quasi-optimal controllers successfully achieve a balance between response speed and structural simplicity, making them well suited for systems with inherent time delays.

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V. CONCLUSION AND FUTURE WORK

In this paper, algorithms for the synthesis of quasi-optimal speed regulators for control systems with time delay have been studied. The proposed method preserves the structural simplicity of strictly optimal control laws while providing satisfactory transient performance. The use of different orders of approximation makes it possible to adjust the control accuracy according to the desired requirements, ensuring flexibility in practical applications.

The significance of this approach lies in its ability to combine fast response with technical simplicity, making it well suited for technological processes and engineering systems where delays are inevitable. Simulation-based analysis has shown that even low-order approximations can provide stable and efficient regulation, while higher-order approximations allow closer approximation to strictly optimal performance.

For future work, several directions can be considered. First, the method can be extended to nonlinear and uncertain systems, where time delays are more complex and variable. Second, experimental validation on real industrial setups would provide stronger evidence of applicability. Finally, integration of adaptive and intelligent control techniques, such as machine learning-based optimization, may further enhance the robustness and performance of quasi-optimal regulators under varying operating conditions.

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AUTHOR'S BIOGRAPHY

Full name	**XusanovSubanNurullayevich**
Science degree	-
Academic rank	Docent
Institution	Karshi State Technical University, Karshi, Uzbekistan