Quadratic Mean of Quadratic Means of Possible Subsets of a Set of Numbers

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ABSTRACT: Mathematical proof of one property of quadratic mean has been presented in this research article. The property states that the quadratic mean of the quadratic means of the respective possible subsets of fixed size of a set of positive real numbers is the Quadratic mean of the original set of numbers and also the quadratic mean of the quadratic means of the respective non-empty possible subsets a set of positive real numbers is the quadratic mean of the original set of numbers.

KEYWORDS: Real Numbers, Set, Possible Susets, Quadratic Mean

I. INTRODUCTION

Average, a mathematical/statistical concept, is a single entity that describes/represent a set of many entities [1 , 52]. The great mathematician Pythagoras first developed three measures of average termed as arithmetic mean [2 , 6 , 59], geometric mean [2 , 6] and harmonic mean [2 , 6 , 58] which together is also popularly known as "Pythagorean Means" [3 , 7 , 15]. Later on, a number of definitions / formulations of average were developed due to necessity of handling different situations some of which are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p m*ean& generalized p^{th} root mean etc. [8 , 32 , 51 , 53 , 54 , 57]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 − 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are arithmetic- geometric mean, arithmetic-harmonic mean, geometric -harmonic mean and arithmetic- geometric -harmonic respectively [19 , 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of arithmetic mean, geometric mean & harmonic mean [2 , 3 , 6 , 39 , 40 , 42 − 44 , 46 , 47 , 51 , 53 , 54 , 58 , 59]. Arithmetic mean, geometric mean & harmonic mean have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 − 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 − 35 , 38 , 41 , 55 , 56]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of arithmetic mean which states that the arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48].Similar property of geometric mean[49] and of harmonic mean [50] have also been mathematically established in two separate studies. Similar property of quadraticmeanharmonic has mathematically been proved in this study. This mathematical proof of this property has been presented in this article along with numerical example.

II. QUADRATIC MEAN OF A SET OF ELEMENTS

Let us first mention the definition of arithmetic mean of a set of real numbers.

Definition

Let us consider a set of n positive real numbers namely

$$x_1, x_2, \ldots\ldots\ldots, x_n$$

Then the quadratic mean $Q = Q(x_1, x_2, \ldots\ldots\ldots, x_n)$ of them is given by

$$Q = Q(x_1, x_2, \ldots\ldots\ldots, x_n) = \left\{\frac{1}{n}\left(x_1{}^2 + x_2{}^2 + \ldots\ldots + x_n{}^2\right)\right\}^{1/2}$$

Let us abbreviate quadratic mean by QM.

Note:

The definition of QM implies that

$$x_1{}^2 + x_2{}^2 + \ldots\ldots + x_n{}^2 = n.\{Q(x_1, x_2, \ldots\ldots\ldots, x_n)\}^2$$

i.e. the sum of squares of n positive real numbers is n times of the square of QM of the numbers.

III. QUADRATIC MEAN OF QUADRATIC MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Suppose, a set P consists of the N real numbers

$$a_1, a_2, \ldots\ldots\ldots, a_N$$

as elements so that

$$\text{Sum of squares of the } N \text{ elements of } S = a_1{}^2 + a_2{}^2 + \ldots\ldots + a_N{}^2$$

$$\& QM \text{ of the } N \text{ elements of } S = \left\{\frac{1}{N}\left(a_1{}^2 + a_1{}^2 + \ldots\ldots + a_N{}^2\right)\right\}^{1/2} = Q, \text{ say}$$

Let us consider the possible subsets of P having n elements in each set.
The number of such possible subsets is $C(N, n)$

Where $C(N, n) = {}^{N}C_n = \begin{pmatrix} N \\ n \end{pmatrix} = \dfrac{N!}{n!(N-n)!}$

Among the $C(N, n)$ possible subsets, there are

$C(N-1, n-1)$ subsets with a_1 as 1st element,

$C(N-2, n-2)$ subsets with a_2 as 1st element and not having a_1,

$C(N-3, n-3)$ subsets with a_3 as 1st element and not having $a_{1 \& } a_2$,

....

$C(N-1, N-2)$ subsets with a_{N-n+2} as 1st element and not having $a_1, a_2, \ldots\ldots a_{N-n+3}$,

$C(N-1, N-1)$ subsets with a_{N-n+1} as 1st element and not having $a_1, a_2, \ldots\ldots a_{N-n+2}$,

such that
Total number of possible subsets
$= C(N-1, n-1) + C(N-2, n-2) + C(N-3, n-3) + \ldots\ldots.. + C(N-1, N-2) + C(N-1, N-1) = C(N, n)$

and that each a_i appears a total of $C(N-1, n-1)$ times in the set containing all possible $C(N, n)$ subsets.

Suppose,

$$Q_1, Q_2, Q_3, \ldots\ldots\ldots, Q_{C(N,\ n)-1}, Q_{C(N,\ n)}$$

are the QMs of the respective subsets and

$$S_1, S_2, S_3, \ldots\ldots\ldots, S_{C(N,\ n)-1}, S_{C(N,\ n)}$$

are the sums of squares of the respective elements of the respective subsets so that

$S_1 = NQ_1{}^2 ,$
$S_2 = NQ_2{}^2,$
$S_3 = NQ_3{}^2,$
$\ldots\ldots\ldots\ldots\ldots$
$S_{{}^N C_n -1} = N\, Q_{C(N,\ n)-1}{}^2 ,$
$S^2({}^N C_n) = N\, Q_{C(N,\ n)}{}^2 .$

Now,

$$S_1 + S_2 + S_3 + \ldots\ldots\ldots + S_{C(N,\ n)-1} + S_{C(N,\ n)}$$

is the sum of squares of all elements in the set containing all possible ${}^N C_n$ subsets of the original set S where each a_i

appears a total of $C(N, n)$ times.

Therefore,

$$S_1 + S_2 + S_3 + \ldots\ldots\ldots + S_{C(N,\ n)-1} + S_{C(N,\ n)}$$

$$= C(N-1,\ n-1).\left(a_1{}^2 + a_1{}^2 + \ldots\ldots + a_N{}^2\right)$$

Accordingly,

QM of the QMs of the respective possible ${}^N C_n$ subsets

$$= \left\{ \frac{1}{C(N,\ n)}\left(Q_1{}^2 + Q_2{}^2 + Q_3{}^2 + \ldots\ldots\ldots + Q_{C(N,\ n)-1}{}^2 + Q_{C(N,\ n)}{}^2\right)\right\}^{1/2}$$

$$= \left\{ \frac{1}{C(N,n)}N\left(S_1 + S_2 + S_3 + \ldots\ldots\ldots + S_{C(N,\ n)-1} + S_{C(N,\ n)}\right)\right\}^{1/2}$$

$$= \left\{ \frac{1}{C(N,\ n)}N^{N-1}C_{n-1}\left(a_1{}^2 + a_1{}^2 + \ldots\ldots + a_N{}^2\right)\right\}^{1/2}$$

$$= \frac{1}{N}\left(a_1{}^2 + a_1{}^2 + \ldots\ldots + a_N{}^2\right)\}^{1/2}$$

$= Q$

$= QM$ of the elements of P

Therefore,

QM of the QMs of the respective possible $C(N, n)$ subsets $= QM$ of the elements of P

IV. QUADRATICMEAN OF QUADRATICMEANS OF All NON-EMPTY POSSIBLE SUBSETS

The set P has a total of $(2^n - 1)$ number of non-empty subsets of which

number of possible subsets having single element in each is $C(N, 1)$,
number of possible subsets having 2 elements in each is $C(N, 2)$,
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

number of possible subsets having $n-1$ elements in each is $C(N, n-1)$,
number of possible subsets having n elements is in each $C(N, n)$

such that

Total number of all possible non-empty subsets
$$= C(N, 1) + C(N, 2) + \ldots\ldots + C(N, n-1) + C(N, n)$$
$$= 2^n - 1$$

By the results obtained in section 3,

QM of the QMs of the respective possible subsets having 1 element in each $= QM$ of the elements of $S=Q$

Similarly,

QM of the QMs of the respective possible subsets having 2 elements in each $=Q$,
………………………………………………………………………………………

QM of the QMs of the respective possible subsets having $n-1$ elements in each $=Q$,
QM of the QMs of the respective possible subsets having n elements in each $=Q$.

Therefore,

QM of the QMs of all respective possible non-empty subsets of $S=(Q^n)^{1/n} =Q$
i.e. QM of the QMs of all respective possible non-empty subsets of $S=Q=QM$ of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set S of five real numbers
$$S = \{2, 4, 6, 8, 10\}$$
so that
$$QM \text{ of the elements of } S = 6.6332495807107996982298654733414$$

Now, $^5C_1 = 5$ possible subsets of S having single element are
$$\{2\}, \{4\}, \{6\}, \{8\}, \{10\}$$

Corresponding 5 QMs of the element if the respective subsets are
$$2, 4, 6, 8, 10$$
and the QM of these 5 QMs is
$$6.6332495807107996982298654733414$$
which is the QM of the elements of S.

Similarly, $^5C_2 = 10$ possible subsets of S having 2 elements are
$$\{2, 4\}, \{2, 6\}, \{2, 8\}, \{2, 10\}, \{4, 6\}, \{4, 8\}, \{4, 10\}, \{6, 8\}, \{6, 10\}, \{8, 10\}$$

Corresponding 10 QMs of the element if the respective subsets are

3.1622776601683793319988935444327, 4.4721359549995793928183473374626,
5.8309518948453004708741528775456, 7.211102550927978586238442534941,
5.0990195135927848300282241090228, 6.3245553203367586639977870888654,
7.6157731058639082856614110271583, 7.0710678118654752440084436210485,
8.2462112512353210996428197119482, 9.0553851381374166265738081669841

and the QM of these 10 QMs is
$$6.6332495807107996982298654733414$$
which is the HM of the elements of S.

Again, $^5C_3 = 10$ possible subsets of S having 3 elements are
$$\{2, 4, 6\}, \{2, 4, 8\}, \{2, 4, 10\}, \{2, 6, 8\}, \{2, 6, 10\},$$
$$\{2, 8, 10\}, \{4, 6, 8\}, \{4, 6, 10\}, \{4, 8, 10\}, \{6, 8, 10\}$$

Corresponding 10 HMs of the element if the respective subsets are

4.3204937989385734873106449957392 , 5.291502622129181181003231507278 ,
6.3245553203367586639977870888654 , 5.8878405775518979031760284846395 ,
6.8313005106397322554806924536807 , 7.4833147735478827711674974646331 ,
6.2182527020592100303696503615473 , 7.1180521680208741405410159770638 ,
7.7459666924148337703585307995648 , 8.1649658092772603273242802490196

and the QM of these 10 QMs is also

$$6.6332495807107996982298654733414$$

which is the QM of the elements of S.

Moreover, $^5C_4 = 5$ possible subsets of S having 4 elements are

$$\{2,4,6,8\} , \{2,4,6,10\} , \{2,4,8,10\} , \{2,6,8,10\} , \{4,6,8,10\}$$

Corresponding 10 HMs of the element if the respective subsets are

5.4772255750516611345696978228008 , 6.2449979983983982058468931209398 ,
6.7823299831252681390645563266626 , 7.1414284285428499979993998113673 ,
7.3484692283495342945918522241177

and the QM of these 10 QMs is also

$$6.6332495807107996982298654733414$$

which is the QM of the elements of S.

Moreover, $^5C_5 = 1$ possible subset of S having 5 elements is

$$\{2,4,6,8,10\}$$

QM of the elements in this subset is

$$6.6332495807107996982298654733414$$

which is the QM of the elements of S.

Finally, the QM of all the $2^5 - 1 = 31$ QMs of the corresponding elements of the respective 31 subsets is found after computation as

$$6.6332495807107996982298654733414$$

which is the QM of the elements of S.

VI. CONCLUSION

Findings on the property of quadraticmean,obtained in this study, can be summarized as follows:

"The quadratic mean of the quadratic means of the respective possible subsets of fixed size of a set of positive real numbers is the quadratic mean of the original set of numbers and the quadratic mean of the quadratic means of non-empty possible subsets a set of positive real numbers is the quadratic mean of the original set of numbers."

The property/result of quadratic mean obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

In this connection, it is to be mentioned that each of arithmetic mean, geometric mean, harmonic mean and quadratic mean is required to be computed/estimated in similar type of situation .The property of quadratic mean derived here will be useful in such problem of estimating population quadratic mean from sample data. Of course, similar properties of arithmetic mean, geometric mean and harmonic mean were derived in earlier studies [48 , 49 , 50] which are useful in estimating population arithmetic mean, population geometric mean and population harmonic mean from sample data.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of SangeetVisharad (inVocal Music) in the year 2000 from BhatkhandeSangeetvidyapith securing 1st class, the degree of SangeetVisharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of SangeetPravakar (in Tabla) from Prayag

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Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D.

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(Dr. Dhritikesh Chakrabarty presenting research paper in the NaSAEAST -2023 during 27-28 October 2023, at Gauhati University)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ)& (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.