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# Mathematical model of moisture seepage into the soil of sloping fields

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**ABSTRACT**: Soil moisture in sloping fields is maintained by natural precipitation. Therefore, it is important to study the absorption of moisture by the soil through natural precipitation. Information shows that with compacted soil, after autumn field work, water filtration into the soil is significantly lower than evaporation. Thus, 20 hours after precipitation, 1.88 millimeters of precipitation evaporate from the soil, and only 1.8 mm seeps into the soil, after forty hours, 3.68 mm evaporates, and only 2.87 mm seeps into the soil.

**KEY WORDS**: sloping fields, soil, moisture, absorption, evaporation, filtration.

#### I. INTRODUCTION

According to world experts, over the last century alone, 5 million km<sup>2</sup> of land have degraded, become salinized and eroded, and have become completely unsuitable for agriculture due to errors in irrigation and land reclamation [1].

To prevent such situations, it is necessary to study the state of soil moisture absorption under the influence of precipitation on sloping fields.

#### II. SIGNIFICANCE OF THE SYSTEM

The study of literature survey is presented in section III, methodology is explained in section IV, section V covers the experimental results of the study, and section VI discusses the future study and conclusion.

#### III. METHODOLOGY

Immediately after precipitation, simultaneously with the evaporation of water from the field surface, the process of infiltration into the soil occurs. The process of moisture infiltration into the soil is carried out due to the presence of inhomogeneities in the soil in the form of pores of various sizes - from thousandths of a micron to (4...5) mm. During the filtration process, water passes through micropores, and since they are randomly located in the soil, it spreads both in the vertical and horizontal planes and fills them. In what follows, we consider only the movement of water in vertical pores isolated from horizontal pores. We assume that the volume of moisture that the soil can absorb is, in the first approximation, equal to the total volume of micropores not filled with water. Thus, the most important quantitative characteristic of the soil, characterizing its ability to accumulate moisture, is porosity  $\varepsilon$ , defined as the proportion of the soil volume accounted for by pores, or the volume of pores per unit of soil volume.



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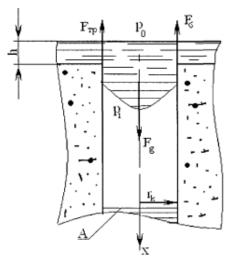


Figure 1. Diagram of forces acting on a moving water layer in a pore channel:  $F_g$  – gravity, N;  $F_a$  – surface tension force, N;  $F_w$  – friction force, N;  $p_0$  – atmospheric pressure in the air, MPa;  $p_1$  – air pressure in the volume between the moving water layer and the groundwater level A, MPa.

Let us consider the movement of water in a separate vertically located micropore, schematically depicted in Figure 1.

The lower boundary of water movement in pores is the surface of impermeable rocks or aquiclude in the form of groundwater.

Water in pores can be in the form of free and bound water. In bound water, molecules are attracted to soil particles and held by them with a force significantly exceeding the intensity of gravity [4]. In free water, gravitational water is distinguished, which is affected only by gravity, and capillary water, which is additionally affected by capillary forces.

#### IV. EXPERIMENTAL RESULTS

The speed of water movement in a pore is the result of the impact of various forces on water: gravity, friction, surface tension, pressure. The force of surface tension is determined by the formula [5].

$$F_{\sigma} = \sigma \cdot l, \tag{1}$$

where  $\sigma$  – the coefficient of surface tension, N/m;

l – the length of the line limiting the interface, m.

It is known from the physics course that if a capillary is placed with one end in a wetted liquid, then it will rise in the capillary to a height  $h_k$ 

$$h_k = \frac{2 \cdot \sigma}{\rho \cdot g \cdot r_k},\tag{2}$$

where g – the acceleration of gravity, m/s;

 $\rho$  – the density of the liquid, kg/m;

 $r_k$  – the radius of the capillary, m.

From formula (2) it follows that with an increase in the radius of the capillary  $r_k$  the lift height decreases. The theoretical value of  $r_k$ , at which h=0, is determined if we take the derivative of (2) with respect to  $r_k$  and equate it to zero. As a result, we obtain that  $r_k \rightarrow \infty$ . In reality, for the conditions of water filtration into the soil ( $\sigma \approx 73 \text{ mJ/m}^2$ ;  $\rho = 1000 \text{ kg/m}^3$ ), depending on the radius, the lift height value calculated according to formula (2) is presented in Table 1.

Table 1 – The value  $h_k$  of the lift height in capillaries of different radii  $r_k$  calculated using formula (2).

Table	1	

	$r_k$ , mm							
	0.001	0.01	0.1	1	10	20	30	40
$h_k$ , mm	13100	2120	271	16.2	1.65	0.642	0.502	0.415



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The data presented in Table 1 show that surface tension forces have a significant effect on the filtration process in pores with radii not exceeding 15 mm. At radii less than 1.5 mm, the effect of surface tension forces is significant, and if the groundwater level is high enough or there is residual moisture from the previous period, the capillary pores are filled with water (capillary water) and, thus, they are not filled with water formed during precipitation (rain, snow). In view of the above, it is necessary to isolate capillary porosity from the total soil porosity, i.e. porosity with pore sizes up to 1 mm.

Table 2, based on data from [6], provides data on the dependence of capillary and non-capillary porosity on the diameter of micropores.

Table 2-Dependence of capillary and non-capillary porosity on the diameter of micropores

		T	able 2						
Porosity, %	$d_k$ , mm								
	0.5	0.5-1	1-2	2-3	3-5				
Complete	46.3	48.9	53.2	58.1	61.4				
Capillary	43.2	23.8	24.3	25.6	24.5				
Non-capillary	1.9	23.1	27.9	34.3	39.6				
Capillary and non-capillary porosity as a percentage of total									
Capillary	93.46	50.78	45.12	40.86	37.49				
Non-capillary	4.72	48.29	53.48	58.24	59.89				

Approximation of the data presented in Table 2 allowed us to obtain the dependence: for total porosity in the form

$$\varepsilon_n = (1400 \cdot r_k)^{1.463} + 36.23 \cdot r_k^{(4\cdot 10^{-6} - r_k^{0.42})},$$
 (3)

for capillary porosity in the form:

$$\varepsilon_k = \frac{0.0095 \cdot (198351.044 \cdot r_k^2 + 1344.047 \cdot r_k + 0.437)}{r_k^{1.125}}.$$
 (4)

for non-capillary at  $r_k \le 0.75$  mm

$$\varepsilon_{nk} = 43600 \cdot r_k - 8.2,\tag{5}$$

 $\varepsilon_{nk} = 43600 \cdot r_k - 8.2,$  and in the area with a radius of micropores  $r_k {\geq 0.75}$  mm

$$\varepsilon_{nk} = 20.63 + 5163 \cdot r_k. \tag{6}$$

The obtained dependencies allow us to determine the total porosity and its components depending on the specific radii of the capillaries.

The process of soil saturation with moisture stops in the following cases:

- the volume of absorbed moisture is equal to the non-capillary volume of the soil;
- there is no water layer on the soil surface, i.e.  $h(t_0) = 0$ .

Provided that the pores are completely filled, the volume of water filling the non-capillary pores is expressed as follows:

$$V_{nk} = \varepsilon_{nk} \cdot V_0. \tag{7}$$

where  $V_0$  is the filtration volume, m<sup>3</sup>.

The filtration process is terminated under the following condition:

$$\varepsilon_{nk} \cdot V_0 = S_f \cdot h_{fmax}. \tag{8}$$

where  $S_f$  is the filtration area, m;  $h_{fmax}$  is maximum decrease in water height on the surface of the soil layer, m.

Taking into account (7), equation (6) takes the form

$$\varepsilon_{nk} \cdot V_0 / S_f = h_{fmax}. \tag{9}$$

Formula (9) allows us to determine the maximum level of reduction of ground water when non-capillary pores are completely filled.

The volumetric flow rate of water is proportional to the filtration area, soil porosity, filtration rate and is expressed by the equation

$$Q = K_1 \cdot \varepsilon \cdot \Omega \cdot u, \tag{10}$$



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where  $K_I$  is the proportionality coefficient;  $\varepsilon$  - volumetric porosity of the soil;  $\Omega$  - filtration area, m; u - filtration speed, m/s.

If we take into account only non-capillary porosity, the equation will be written as

$$Q_{nk} = K_1 \cdot \varepsilon_{nk} \cdot \Omega \cdot u. \tag{11}$$

According to Darcy's law, the filtration rate in a pore is related to the pressure gradient by the formula

$$u = \frac{c \cdot g \cdot \Delta H}{v \cdot L} = \frac{c \cdot g \cdot I}{v},\tag{12}$$

 $u = \frac{c \cdot g \cdot \Delta H}{v \cdot L} = \frac{c \cdot g \cdot I}{v},$  (12) where c is the proportionality coefficient (permeability of the medium), which is a characteristic of the porous medium and does not depend on the size of the sample and the properties of the liquid, m<sup>2</sup>; L - distance between piezometers, m;  $\Delta H$  - pressure difference, m;  $I = \Delta H/L$  - piezometric slope; v - kinematic viscosity of water, m/s.

As a result of water filtration, its volume on the soil surface changes and, therefore, we can write

$$Q_{nk} = dV/dt. (13)$$

The decrease in the water level above the soil per unit of time can be obtained from equation (13) by dividing it by the filtration area.

$$\frac{dh_f}{dt} = \frac{1}{\Omega} \frac{dV}{dt} = K_1 \cdot \varepsilon_{nk} \cdot u. \tag{14}$$

Taking into account the dependence (12), the last formula takes the form

$$\frac{dh_f}{dt} = K_1 \cdot \varepsilon_{nk} \cdot \frac{c \cdot g \cdot l}{v}.$$
 (15)

Integrating expression (15) under the condition of independence of the quantities included in the dependence on time and at t = 0  $h_f = 0$ , we obtain

$$h_f = K_1 \cdot \varepsilon_{nk} \cdot \frac{c \cdot g \cdot l}{v} t. \tag{16}$$

2

 $h_f = K_1 \cdot \varepsilon_{nk} \cdot \frac{c \cdot g \cdot l}{v} t. \tag{16}$  The obtained equation (16) shows that the decrease in the water level above the soil surface is linearly dependent on the porosity, the permeability coefficient of the medium, the magnitude of the hydraulic slope, the filtration time and is inversely proportional to the viscosity of the water.

To illustrate and evaluate the levels of water loss on the soil surface due to evaporation and percolation, graphs of the change in  $h_i = f(t)$  and hf = f(t) were calculated. These graphs are presented in Figure 2.

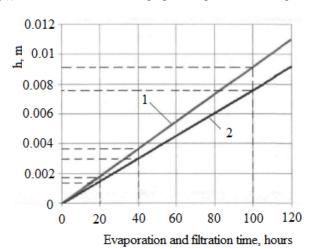


Figure 2. Dependence of evaporation and percolation (filtration) of moisture on time: 1 - evaporation curve; - percolation curve

The calculation of  $h_f = f(t)$  was carried out using formula (16) for the following initial data: permeability coefficient  $c = 1.33 \cdot 10^{-15} \text{ m}^2$ ;  $K_I = 1$ ; water temperature  $t_a = 10^{-0} \text{C}$  (water viscosity v= 1.306·10<sup>-6</sup> m<sup>2</sup>/s), hydraulic slope I=0.01.



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#### IV. CONCLUSION

The graphs presented in Figure 2 show that with compacted soil, after autumn field work, water filtration into the soil is significantly lower than evaporation. Thus, 20 hours after precipitation, 1.88 millimeters of precipitation evaporate from the soil, and only 1.8 mm seeps into the soil; after forty hours, 3.68 mm evaporates, and only 2.87 mm seeps into the soil.

Evaporation of moisture from the soil surface will be more intense with increasing temperature and wind speed.

The obtained results on water evaporation and filtration allow us to conclude that in order to accumulate moisture in the soil, it is necessary to significantly reduce moisture evaporation and create conditions for its effective infiltration into the soil.

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