

Identification of Nonlinear Functions of a Priori Known Form

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ABSTRACT: System identification is a very broad topic, especially for dynamic systems, with various methods depending on the nature of the models being evaluated. It's an iterative process, and sometimes it's necessary to return to previous steps and repeat them. Identification methods and experimental conditions depend on the model's purpose, so they should be clearly defined. If the model is used for control system design, it's important to have an accurate model that accounts for the desired parameters. Identification of linear dynamic objects is also a very complex task, and the difficulties of identifying nonlinear dynamic objects increase disproportionately. In this article, given prior knowledge of the type of nonlinearity, the parameters of "real" nonlinear functions are derived by replacing variables in the original analytical expression and linearization methods are used to obtain a linear model of the object.

KEY WORDS: Model, Object, Dynamic System, Identification, Parameter, A Priori, Experiment, Estimate, Nonlinear, Signal, Differential Equations, Operator Equations, Operator, Modeling, Likelihood, Linearization.

I. INTRODUCTION

To control a dynamic system under uncertainty, it is first necessary to identify this system in order to construct a model of the system and evaluate its parameters. The procedure for system identification is a data-driven search for suitable models. In a dynamic system, knowing its identity is crucial. Identification is a science that involves constructing mathematical models based on observed input and output data [1-3]. It's a very broad topic, depending on the nature of the dynamic systems models being evaluated, with various methods. Identification is an iterative process, and sometimes it's necessary to return to previous stages and repeat the process.

The problem of system identification involves obtaining a mathematical model of a real system based on data obtained from statistics or through scientific or industrial experiments [4,5]. The least-squares method is typically used to solve such problems. A disadvantage of such an identification model is its potential inability to predict the behavior of a real system under conditions different from those used to obtain the statistical data.

At the beginning of the system identification process, the purpose of the model must be determined. Models can be applied in a wide variety of areas, including signal processing, control system design, simulation, and error detection. Designing an identification experiment involves a number of choices, such as the choice of signal to process or measure and the method of processing or measuring it. It also involves some practical considerations. Experimental data can only be modified by new experimental data.

Planning an identification experiment consists of two stages. First, a preliminary identification experiment is conducted to obtain initial data on important system characteristics. Here, transient and impulse responses are analyzed. This information is used to find suitable experiments for the main experiments. Some system characteristics considered in the preliminary experiments include time invariance, linearity, transient response, and frequency response. The main experiments focus on the input signal. Identification yields an accurate model with smaller estimation errors. Systems typically operating with positive feedback are characterized by the fact that the portion of the feedstock material remaining unprocessed to the required condition, passing through the working section of the facility, is returned to the facility's input, creating a recalculation flow. Such systems are widely used in the chemical, mining, pulp and paper, environmental, and other industries. They are complex nonlinear control objects. Their stable output behavior is achieved only for certain values of the system parameters and for input changes within certain limits. Nonlinear systems are typically represented by block-oriented models consisting of various modifications of the Hammerstein and Wiener models or general models, in particular, Volterra and Wiener series and continuous and discrete Kolmogorov-Gabor polynomials [6].

Both static and dynamic processes can exhibit nonlinear characteristics. Identifying linear dynamic objects is also a complex task, and the difficulties of identifying nonlinear dynamic objects are disproportionately greater. The dependence of a nonlinear object's transient response not only on the shape but also on the amplitude of the input signal is one of the main challenges. During active identification, it imposes complex and contradictory requirements on the selection of a test signal. Another major obstacle is the infinite number of nonlinear operators describing the objects. To study nonlinear systems in a phase field and visualize the dynamic processes occurring within them, phase portraits can be constructed. Every dynamic system has its own phase portrait. These portraits depict specific points, called equilibrium points, which help predict the behavior of a dynamic system without solving differential equations.

Depending on the model form, nonlinear differential equations and operator equations with nonlinear operators of various types are used to describe nonlinear objects. This paper [7] examines issues related to the description of objects by functional operators. It proposes a general approach to the problems of active identification of nonlinear dynamic objects with a Hammerstein structure and two-level input signals.

The paper considers problems related to modeling, identification and optimal control of mechanical systems whose dynamics are described by ordinary differential equations nonlinear with respect to the highest derivative [8].

The article considers the issues of using the least squares method in identification problems, studies the features of identifying a dynamic object using the regression and explicit least squares methods, and identifies a dynamic object using the recurrent least squares method [9].

This article [10] examines methods for identifying nonlinear dynamic systems described by Volterra series. Selecting parameters that enable a system to achieve a given behavior is one of the fundamental modeling tasks. If the model structure is predetermined, the solution to this problem is closely related to the task of determining the system parameters. Identification methods for continuous systems are based on the generalized Borel theorem in conjunction with integral transformations. For the study of discrete systems, a discrete analog of the generalized Borel theorem is used in conjunction with discrete transformations. Model examples illustrate the application of the developed methods to modeling systems with given characteristics.

In [11], the problem of structural identification of nonlinear systems is considered using a set of continuous block-oriented models. Elements of this set include the nonlinear Hammerstein and Wiener models, as well as the Wiener-Hammerstein cascade models. The problem of structural identification is considered for periodic input signals with uniformly and absolutely convergent Fourier series, as well as for sinusoidal input signals.

During identification, it will be necessary to find the coefficients of approximation of nonlinearity by a power series and determine the impulse response function of the linear part [12].

The problem of determining maximum likelihood for nonlinear stochastic parametric models is difficult because the likelihood function cannot be solved. A simulation-based pseudo-likelihood estimator for nonlinear stochastic models is presented in [13]. It is based on the first two moments of the model, which are easily estimated using a Monte Carlo method. The resulting estimator is consistent and asymptotically normal. Pseudo-maximum-likelihood, based on a multivariate normal set, solves the problem of minimizing forecast error using an approximate parameterized norm and an implicit linear predictor.

For most linearization methods, not only the rate of solution convergence but also the overall success of the problem solution depend significantly on the choice of the initial approximation for the identified parameters. In the system being identified, the form of the nonlinear equations must be known a priori. They are selected at the stage of hypothesizing the operating law of the modeled system [14].

In nonlinear system identification, one approach can be considered based on the use of direct optimization methods. Software programs can be developed to obtain estimates of two or four parameters using various optimality criteria, including the least-squares method. It is possible to evaluate the function and use a nonlinear function to simulate statistical data. In [15], the consistency of the parameter matrices of the nonlinear difference equation is proven on the basis of a modified nonlinear least squares method. Methods for identifying nonlinear dynamic objects can be divided into those based on the linearization of the mathematical description of the object; the object model is considered essentially nonlinear. In some cases, a linearized model is meaningless. However, identification methods that consider the object as essentially nonlinear are becoming increasingly important.

When the mathematical model of the object under study is considered to be essentially nonlinear and identification consists of finding the characteristics of its linear and nonlinear elements, the structure of the object must be known to some extent [16].

The main difficulty in identifying the parameters of nonlinear models stems from the complexity of calculating the likelihood function. The likelihood function plays a key role in many popular statistical inference methods. For example, the maximum likelihood method is defined as its global maximizer, and Bayesian estimators use it

to construct the posterior distribution. On the other hand, classical methods for estimating forecast errors indirectly use the likelihood function to construct optimal predictors.

II.METHODOLOGY

The parameters of "true" nonlinear functions can be identified with a priori information about the type of nonlinearity. The following techniques can then be used: substituting variables in the original analytical expression, performing linearization, and obtaining a linear model of the object. The diversity of nonlinearities in technological objects, as well as the diversity of operator types, gives rise to a variety of approaches to their identification, often not related to each other in any way.

The object is described by the following expression [17]:

$$y = a_0 + a_1 x_1 x_2^3 + a_2 x_2 e^{-\frac{a_3 x_1^2}{x_3}} + \frac{a_4 x_4}{\sqrt{1 - a_5 - x_5^2}}. \quad (1)$$

New variables need to be introduced for identification:

$$\xi_1 = x_1 x_2^3, \xi_2 = \frac{x_1^2}{x_3}, \xi_3 = x_2, \xi_4 = x_4, \xi_5 = x_5.$$

From here you can get:

$$y = a_0 + a_1 \xi_1 + a_2 \xi_3 \exp(-a_3 \xi_2) + \frac{a_4 \xi_4}{\sqrt{1 - a_5 \xi_5^2}}. \quad (2)$$

We linearize equation (2), assuming that the growth of variables is small:

$$\Delta y = a_1 \Delta \xi_1 - a_2 a_3 \xi_3 \exp(-a_3 \xi_2) \Delta \xi_2 + a_2 \exp(-a_3 \xi_2) \Delta \xi_3 + \frac{a_4}{\sqrt{1 - a_5 \xi_5^2}} \Delta \xi_4 + \frac{a_4 a_5 \xi_4 \xi_5}{\sqrt{(1 - a_5 \xi_5^2)^3}} \Delta \xi_5$$

We introduce the following notations:

$$b_1 = a_1, \quad b_2 = -a_2 a_3 \xi_3 \exp(-a_3 \xi_2), \quad b_3 = a_2 \exp(-a_3 \xi_2),$$

$$b_4 = \frac{a_4}{\sqrt{1 - a_5 \xi_5^2}}, \quad b_5 = \frac{a_4 a_5 \xi_4 \xi_5}{\sqrt{(1 - a_5 \xi_5^2)^3}} \quad (3)$$

from here we get:

$$\Delta y = b_1 \Delta \xi_1 + b_2 \Delta \xi_2 + b_3 \Delta \xi_3 + b_4 \Delta \xi_4 + b_5 \Delta \xi_5 = \sum_{i=1}^5 b_i \Delta \xi_i,$$

$b_1, b_2, \dots, b_5$ can be identified by the linear regression method, $b_5 = b_4 a_5 \xi_4 \xi_5$ can be determined by a_5 , since ξ_4 and ξ_5 are measurable. Substituting the expression for a_5 into formula (3) for b_4 , we obtain a_4 . Term a_1 is directly determined by the value of b_1 according to (3). Term a_2, a_3 can be found from expressions (3) for b_2, b_3 as follows. The value of a_3 is found from expression $b_2 = -a_3 \xi_3 b_3$, where variable ξ_3 is measurable. a_2 is determined by substituting a_3 into expression (3) for b_2 . By analogy with the process identification technique (1) described above, many other types of nonlinear dependencies can be identified.

Exponential dependencies of type $y = ae^{bx}$ can be identified if they are transformed by taking the logarithm to a relationship of the type: $\lg y = \lg a + bx$.

Denoting $\lg y = Y, x = X, \lg a = A$, we get $Y = A + bX$, where A and B are easily calculated using the minimum mean square error method.

Similarly, in processes of type $y = a \cdot x^b$, one can use logarithms, obtaining expression $\lg y = \lg a + b \cdot \lg x$, from which a and b are calculated in the same way as in the previous case.

However, in some cases, the latter method is unsuitable or requires additional information. For example, this method is unsuitable for the following systems:

$$y = a_0 + a_1 \cdot \lg(a_2 + x),$$

in which it is required to identify a_0, a_1, a_2 .

Using the small perturbation method, we obtain:

$$\Delta y = \frac{a_1}{a_2 + x} \cdot \Delta x = b \cdot \Delta x.$$

Here coefficient b can be identified using the mean square criterion $b = \frac{a_1}{a_2 + x}$, however this does not

provide a solution for a_0, a_1, a_2 .

III.CONCLUSION

Of course, if the measurements are noisy, the significance of the derivatives is small, and using second and higher partial derivatives (or second and higher order perturbations) in such situations does not give logically good results in practice. Almost all proposed methods for identifying nonlinear dynamic objects have limited practical application so far. The reliability of the results obtained when identifying nonlinear systems in industrial conditions in the presence of noise and interference depends on the accuracy of measuring the output signals of the systems and the mathematical processing of experimental data.

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