

Square Root Mean of Square Root Means of Possible Subsets of a Set of Numbers

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ABSTRACT: Mathematical proof of one property of **square root mean** has been presented in this research article. The property states that the **square root mean** of the **square root means** of the respective possible subsets of fixed size of a set of positive real numbers is the **square root mean** of the original set of numbers and also the **square root mean** of the **square root means** of the respective non-empty possible subsets a set of positive real numbers is the **square root mean** of the original set of numbers.

KEYWORDS: Real Numbers, Set, Possible Subsets, **Square Root Mean**

I. INTRODUCTION

Several studies had already been done on average [1, 53] which is an entity that describes a set of many entities. The great mathematician Pythagoras first developed three measures of average termed as **arithmetic mean** [2, 6, 60], **geometric mean** [2, 6] and **harmonic mean** [2, 6, 59] which together is also popularly known as “Pythagorean Means” [3, 7, 15]. Later on, a number of definitions / formulations of average were developed due to necessity of handling different situations some of which are **quadratic mean** or **root mean square**, **square root mean**, **cubic mean**, **cube root mean**, **generalized p mean** & **generalized p^{th} root mean** etc. [8, 32]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9, 16, 17, 20]. In another study, four formulations of average were derived from the three Pythagorean means which are **arithmetic-geometric mean**, **arithmetic-harmonic mean**, **geometric-harmonic mean** and **arithmetic-geometric-harmonic** respectively [19, 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of **arithmetic mean**, **geometric mean** & **harmonic mean** [2, 3, 6, 39, 40, 42 – 44, 46, 47, 59, 60]. **Arithmetic mean**, **geometric mean** & **harmonic mean** have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7, 15, 21 – 31, 36, 37] and in developing the statistical concept of expectation [5, 33 – 35, 38, 41, 56, 57]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of **arithmetic mean** which states that the **arithmetic mean** of the **arithmetic means** of the respective possible subsets of fixed size of a set of real numbers is the **arithmetic mean** of the original set of numbers and also the **arithmetic mean** of the **arithmetic means** of the respective non-empty possible subsets a set of real numbers is the **arithmetic mean** of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48]. Similar properties of **geometric mean**, **harmonic mean** and **quadratic mean** have also been mathematically established in three separate studies [49, 50, 51]. The Similar property of **square root** has mathematically been proved in this study. The property states that the **square root mean** of the **square root means** of the respective possible subsets of fixed size of a set of positive real numbers is the **square root mean** of the original set of numbers

and also the **square root mean** of the **square root means** of the respective non-empty possible subsets a set of positive real numbers is the **square root mean** of the original set of numbers.

II. SQUARE ROOT MEAN OF A SET OF ELEMENTS

Let us first mention the definition of **square root mean** of a set of real numbers.

Definition

Let us consider a set of n positive real numbers namely

$$x_1, x_2, \dots, x_n$$

Then the **square root mean** $R = R(x_1, x_2, \dots, x_n)$ of them is given by

$$R = R(x_1, x_2, \dots, x_n) = \left\{ \frac{1}{n} (x_1^{1/2} + x_2^{1/2} + \dots + x_n^{1/2}) \right\}^2$$

Let us abbreviate **square root SRM**.

Note:

The definition of **SRM** implies that

$$x_1^{1/2} + x_2^{1/2} + \dots + x_n^{1/2} = n \{ R(x_1, x_2, \dots, x_n) \}^{1/2}$$

i.e. the **sum of squares roots** of n positive real numbers is n times of the **squares root** of the **SRM** of the numbers.

III. SQUARE ROOT MEAN OF SQUARE ROOT MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Suppose, a set S consists of the N real numbers

$$a_1, a_2, \dots, a_N$$

as elements so that

$$\text{Sum of squares roots of the } N \text{ elements of } S = a_1^{1/2} + a_2^{1/2} + \dots + a_N^{1/2}$$

$$\& \text{ SRM of the } N \text{ elements of } S = \left\{ \frac{1}{N} (a_1^{1/2} + a_2^{1/2} + \dots + a_N^{1/2}) \right\}^2 = R, \text{ say}$$

Let us consider the possible subsets of S having n elements in each set.

The number of such possible subsets is $C(N, n)$

$$\text{where } C(N, n) = {}^N C_n = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Among the $C(N, n)$ possible subsets, there are

$$C(N-1, n-1) \text{ subsets with } a_1 \text{ as } 1^{\text{st}} \text{ element,}$$

$$C(N-2, n-2) \text{ subsets with } a_2 \text{ as } 1^{\text{st}} \text{ element and not having } a_1,$$

$$C(N-3, n-3) \text{ subsets with } a_3 \text{ as } 1^{\text{st}} \text{ element and not having } a_1 \& a_2,$$

.....

$$C(N-1, N-2) \text{ subsets with } a_{N-n+2} \text{ as } 1^{\text{st}} \text{ element and not having } a_1, a_2, \dots, a_{N-n+3},$$

$$C(N-1, N-1) \text{ subsets with } a_{N-n+1} \text{ as } 1^{\text{st}} \text{ element and not having } a_1, a_2, \dots, a_{N-n+2},$$

such that

Total number of possible subsets
 $= C(N-1, n-1) + C(N-2, n-2) + C(N-3, n-3) + \dots + C(N-1, N-2) + C(N-1, N-1) = C(N, n)$

and that each a_i appears a total of $C(N-1, n-1)$ times in the set containing all possible $C(N, n)$ subsets. Suppose,

$$R_1, R_2, R_3, \dots, R_{C(N, n)-1}, R_{C(N, n)}$$

are the *SRMs* of the respective subsets and

$$S_1, S_2, S_3, \dots, S_{C(N, n)-1}, S_{C(N, n)}$$

are the *sums of squares roots* of the respective elements of the respective subsets so that

$$\begin{aligned} S_1 &= N R_1^{1/2}, \\ S_2 &= N R_2^{1/2}, \\ S_3 &= N R_3^{1/2}, \\ &\dots \\ S_{C(N, n)-1} &= N R_{C(N, n)-1}^{1/2}, \\ S_{C(N, n)} &= N R_{C(N, n)}^{1/2}. \end{aligned}$$

Now,

$$S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)}$$

is the *sum of squares* of all elements in the set containing all possible $C(N, n)$ subsets of the original set S

where each a_i appears a total of $C(N-1, n-1)$ times.

Therefore,

$$\begin{aligned} &S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)} \\ &= C(N-1, n-1) (a_1^{1/2} + a_1^{1/2} + \dots + a_N^{1/2}) \end{aligned}$$

Accordingly,

$$\begin{aligned} &\text{SRM of the SRMs of the respective possible } {}^N C_n \text{ subsets} \\ &= \left\{ \frac{1}{C(N, n)} (R_1^{1/2} + R_2^{1/2} + R_3^{1/2} + \dots + R_{C(N, n)-1}^{1/2} + R_{C(N, n)}^{1/2}) \right\}^2 \\ &= \left\{ \frac{1}{C(N, n)} N (S_1 + S_2 + S_3 + \dots + S_{C(N, n)-1} + S_{C(N, n)}) \right\}^2 \\ &= \left\{ \left\{ \frac{1}{C(N, n)} N C(N-1, n-1) (a_1^{1/2} + a_1^{1/2} + \dots + a_N^{1/2}) \right\} \right\}^2 \\ &= \frac{1}{N} (a_1^{1/2} + a_1^{1/2} + \dots + a_N^{1/2})^2 \\ &= R \\ &= \text{SRM of the elements of } S \end{aligned}$$

Therefore,

$$\text{SRM of the SRMs of the respective possible } C(N, n) \text{ subsets} = \text{SRM of the elements of } S$$

IV. SQUARE ROOT MEAN OF SQUARE ROOT MEANS OF ALL NON-EMPTY POSSIBLE SUBSETS

Now, the set S has a total of $(2^n - 1)$ number of non-empty subsets of which

number of possible subsets having single element in each is $C(N, 1)$,
 number of possible subsets having 2 elements in each is $C(N, 2)$,

 number of possible subsets having $n - 1$ elements in each is $C(N, n - 1)$,
 number of possible subsets having n elements in each is $C(N, n)$

such that

$$\begin{aligned} &\text{Total number of all possible non-empty subsets} \\ &= C(N, 1) + C(N, 2) + \dots + C(N, n - 1) + C(N, n) \\ &= 2^n - 1 \end{aligned}$$

By the results obtained in section 3,

SRM of the SRM s of the respective possible subsets having 1 element in each = SM of the elements of S
 = R

Similarly,

SRM of the SRM s of the respective possible subsets having 2 elements in each = R ,

 SRM of the SRM s of the respective possible subsets having $n - 1$ elements in each = R ,
 SRM of the SRM s of the respective possible subsets having n elements in each = R .

Therefore,

SRM of the SRM s of all respective possible non-empty subsets of S = R
 i.e. SRM of the SRM s of all respective possible non-empty subsets of S = R = SRM of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set S of five real numbers

$$S = \{2, 4, 6, 8, 10\}$$

so that

$$SRM \text{ of the elements of } S = 5.621079646637481669054459075562$$

Now, ${}^5C_1 = 5$ possible subsets of S having single element are

$$\{2\}, \{4\}, \{6\}, \{8\}, \{10\}$$

Corresponding 5 SRM s of the elements in the respective subsets are

$$2, 4, 6, 8, 10$$

and the SRM of these 5 SRM s is

$$5.621079646637481669054459075562$$

which is the SRM of the elements in S .

Similarly, ${}^5C_2 = 10$ possible subsets of S having 2 elements are

$$\{2, 4\}, \{2, 6\}, \{2, 8\}, \{2, 10\}, \{4, 6\}, \{4, 8\}, \{4, 10\}, \{6, 8\}, \{6, 10\}, \{8, 10\}$$

Corresponding 10 SRM s of the elements in the respective subsets are

2.9142135623730950488016887242095 , 3.7320508075688772935274463415059 , 4.5 ,
5.2360679774997896964091736687313 , 4.9494897427831780981972840747059 ,
5.8284271247461900976033774484194 , 6.6622776601683793319988935444327 ,
6.9641016151377545870548926830117 , 7.8729833462074168851792653997824 ,
8.9721359549995793928183473374626

and the *SRM* of these 10 *SRM*s is

5.621079646637481669054459075562

which is the *SRM* of the elements in S .

Again, ${}^5C_3 = 10$ possible subsets of S having 3 elements are

$\{2, 4, 6\}$, $\{2, 4, 8\}$, $\{2, 4, 10\}$, $\{2, 6, 8\}$, $\{2, 6, 10\}$,
 $\{2, 8, 10\}$, $\{4, 6, 8\}$, $\{4, 6, 10\}$, $\{4, 8, 10\}$, $\{6, 8, 10\}$

Corresponding 10 *SRM*s of the elements in the respective subsets are

3.8203351612111779735672973957429 , 4.3300625276085711761800294100574 ,
4.8055818666850062565376693054994 , 4.9760677434251697247032617886745 ,
5.4849342805671483889403935155642 , 6.0925350811108307063233426694195 ,
5.8853415478520545701580240916165 , 6.4376669996262108068335302306316 ,
7.0945958844062883655202748134732 , 7.915209296153222606690002409003

and the *SRM* of these 10 *SRM*s is also

5.621079646637481669054459075562

which is the *SRM* of the elements in S .

Moreover, ${}^5C_4 = 5$ possible subsets of S having 4 elements are

$\{2, 4, 6, 8\}$, $\{2, 4, 6, 10\}$, $\{2, 4, 8, 10\}$, $\{2, 6, 8, 10\}$, $\{4, 6, 8, 10\}$

Corresponding 10 *SRM*s of the elements in the respective subsets are

4.7220707131522737812961723179632 , 5.091770774150184088528437938342 ,
5.5282805699467583919078701808139 , 6.0693349253533544637472813576235 ,
6.8123538610106245982130151219537

and the *SRM* of these 10 *SRM*s is also

5.621079646637481669054459075562

which is the *SRM* of the elements in S .

Moreover, ${}^5C_5 = 1$ possible subset of S having 5 elements is

$\{2, 4, 6, 8, 10\}$

SRM of the elements in this subset is

5.621079646637481669054459075562

which is the *SRM* of the elements in S .

Finally,

the *SRM* of all these $2^5 - 1 (= 31)$ *SRM*s of the corresponding elements of the respective 31 subsets is found after computation as

5.621079646637481669054459075562

which is the *SRM* of the elements in S .

VI. CONCLUSION

Findings on the property of *quadratic mean*, obtained in this study, can be summarized as follows:

“The square root mean of the square root means of the respective possible subsets of fixed size of a set of positive real numbers is the square root mean of the original set of numbers and the square root mean of the square root means of non-empty possible subsets a set of positive real numbers is the square root mean of the original set of numbers.”

The property/result on square root mean obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class & 1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (in Vocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag

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(Dr. Dhritikesh Chakrabarty presenting research paper titled *“Tendency of Rainfall by Probabilistic Approach: Application in Indian Scenario”* in the NaSAEAST -2023 during 27-28 October 2023, at Gauhati University)

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education &



Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.