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Scaling behavior and variance in a coupled map lattice model for changing parametric values for Edward-Wilkinson growth class

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ABSTRACT: We provide a coupled map lattice model based on coupled diffusive maps. We study a coupled map lattice with a non-linear neighbor coupling. We see a power-law increase in roughness with time, followed by saturation. We observe the typical scaling corresponding to the Edwards-Wilkinson class. By changing the parameter (delta) in our model's function, we perform the conventional study of finite-size scaling.

KEYWORDS: Coupled Map Lattice, Finite Size Scaling, Edwards-Wilkinson class

I. INTRODUCTION

Interfaces are prevalent in nature, with every object having some form of surface, such as biological cell membranes and clouds. The appearance of these interfaces can vary significantly with observation scale ballistic deposition (BD), random deposition (RD), and random deposition with surface relaxation (RDSR) [1], and roughness can be described using scaling exponents. Common physics governs these systems, characterized by an elastic interface moving through a disordered medium, where medium irregularities result in roughness [1]. Various deposition processes, including atomic deposition, molecular beam epitaxy, ballistic deposition (BD), random deposition (RD), and random deposition with surface relaxation (RDSR) [1] can be analysed using similar methods.

II. DEVELOPING THE STOCHASTIC GROWTH EQUATION

The random deposition (RD) model is simple, and we can derive all key quantities exactly from its microscopic growth rules. One useful way to study these growth processes is to use a stochastic growth equation that fits the specific mechanism. To demonstrate this approach, we can create a differential equation to represent RD. The aim is to describe how the height (h(x, t)) changes over time (t) at each point (x) on a (d)-dimensional surface. More generally, the growth dynamics can be represented using a continuum equation.

$$\frac{h(x,t)}{t} = r(x,t) \qquad (1)$$

Here, (r(x, t)) represents the number of particles arriving per unit time at position (x) and time (t). Since particles land in random positions, the flux varies. To address this randomness in the model, (ϕ) is split into two components. This changes the equation to the following form:

$$\frac{h(x,t)}{t} = F + \eta(x,t) \qquad (2)$$

Here, F is the average number of particles arriving at site x. Random fluctuations in the deposition process are represented by η(x,t), which is an uncorrelated random number that has zero configurational average ⟨η(x,t)⟩=0 [7].
We assert that the motion equation should be the simplest compatible with the system's symmetries [5]. Initially, we identify the equilibrium equation for an interface, which signifies that no external field influences it, allowing two balanced areas to coexist without one growing at the expense of the other. Examples of such interfaces include magnetic materials and immiscible fluid mixtures, where the condition (F = 0) signifies equilibrium.

To obtain the growth equation, we consider the following basic symmetries of the problem:

i) the equation maintains its invariance after the transformation, or invariance under time translation (t) $t \to t + \delta_h$

ii) translation invariance along the growth direction $h \to h + \delta_h$ [5]

iii) translation invariance in the direction perpendicular to the growth direction $x \to x + \delta_x$ [5]

iv) rotation and inversion symmetry along the growth direction [5] which means we rule out all the odd-ordered derivatives of vectors such as ∇h, $\nabla(\nabla^2 h)$, etc that are not included in the in the coordinates.

v) Up/down symmetry for h: Since the fluctuations at the interface are comparable in relation to the average interface height, we do not take even powers of h into consideration $(\nabla h)^2$, $(\nabla h)^4$.

In this final form of the growth equation, we consider all terms that can be formed from the combinations of powers of $(\nabla^n h)$. All those terms that violate at least one of the symmetries mentioned above are eliminated. In this way, we find,

$$\frac{\partial h(x,t)}{\partial t} = (\nabla^2 h) + (\nabla^4 h) + (\nabla^{2n} h) + \cdots + (\nabla^{2k} h) + (\nabla h)^{2j} + \eta(x,t) \qquad (3)$$

Here, (n), (j), and (k) can take any positive integer values. We focus on the long-time limit ($(t \to \infty)$), the large-distance limit ($(x \to \infty)$), and the overall behavior of the functions that define the interface. These elements determine the scaling properties that matter. This situation is known as the hydrodynamic limit.

In scaling analysis, higher-order derivatives are deemed less significant than lower-order derivatives regarding the growth equation's scaling behavior. Specifically, in the hydrodynamic limit, the $(\nabla^4 h)$ term becomes negligible compared to the $(\nabla^2 h)$ term, indicating that $(\nabla^4 h)$ decreases faster. This leads to a linear stochastic equation that can be solved precisely to determine scaling exponents. The Edwards–Wilkinson (EW) equation provides the simplest description of equilibrium interface fluctuations.

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h + \eta(x,t) \qquad (4)$$

When nonlinear terms are added to the growth equation, the predictions of the linear model no longer apply. Kardar, Parisi, and Zhang (KPZ) suggested extending the EW framework by including these nonlinear contributions. While the KPZ equation cannot be derived strictly, its structure is based on (i) physical reasons that support including nonlinear effects and (ii) considerations of symmetry. BD shows lateral growth, meaning the interface moves along its local normal direction. When a particle attaches to the surface, the resulting height increase (∂h) occurs in the direction normal to the interface [5]. According to the Pythagorean relation, this height increment is expressed as:

$$\partial h = [(\nu\, \partial t)^2 + (\nu\, \partial t \nabla h)^2]^{1/2} = \nu\, \partial t [1 + (\nabla h)^2]^{1/2} \qquad (5)$$

If $|\nabla h| << 1$, the expansion of the above equation gives

$$\frac{\partial h(x,t)}{\partial t} = \nu + \frac{\nu}{2} (\nabla h)^2 + \cdots \qquad (6)$$

suggesting a non-linear term $(\nabla h)^2$ that must be present in the growth equation. Adding this term to the EW equation, we get the KPZ equation:

$$\frac{\partial h(x,t)}{\partial t} = \nu(\nabla^2 h) + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t) \qquad (7)$$

The first term on the RHS describes relaxation of the interface caused by a surface tension ν. The KPZ equation is the simplest growth equation that has the following symmetry principles.

i) the equation maintains its invariance after the transformation, or invariance under time translation (t) $t \to t + \delta_h$

ii) translation invariance along the growth direction $h \to h + \delta_h$ [5]

iii) translation invariance in the direction perpendicular to the growth direction $x \to x + \delta_x$ [5]

iv) rotation and inversion symmetry along the growth direction [5] which means we rule out all the odd ordered derivatives in the coordinates excluding vectors such as ∇h, $\nabla(\nabla^2 h)$, etc.

v)the interface fluctuations are dissimilar with respect to the mean interface height, so we also consider the even powers of h, terms such as $(\nabla h)^2$.

The model we work upon is as follows:

$$x_i(t) = f\big(x_i(t)\big) + \frac{\epsilon}{2}\big[f\big(x_{i-1}(t)\big) - 2f\big(x_i(t)\big) + f\big(x_{i+1}(t)\big)\big] \qquad (8)$$

where,

$$f\big(x_i(t)\big) = x_i(t) - \mu \sin\big[\big(2\pi x_i(t)\big)\big] \qquad (9)$$

We consider the system for $\mu = 1$ and $\epsilon = 0.1$. This function is known to show diffusive behavior and was initially analyzed in [3]. We want to explore the possibility that the coupled diffusive maps show behavior in EW class. Naturally, we study variance given by $\rho(t) = \sum_{i=1}^{N}\big(x_i(t) - \bar{x}(t)\big)^2$ where $\bar{x}(t) = \frac{1}{N}\sum_i x_i(t)$.

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As seen in the above equation, we come across a new parameter δ. We vary this parameter in steps of 0.1 starting from 0.1 and see the behavior of the function every time. Again $0 \leq \delta \leq 1$. Now we present few plots depicting scaling. Every time we double the number of lattice sites from a previous value considered, for a particular value of δ chosen. As seen from inset of the first graph, n1h means the number of lattice sites is one hundred. nc4k implies the number of configurations to be four thousand and d.1 indicates that the value of delta is 0.1. For others, analogy applied. We update at position x as the function of time as follows.

$$x_i = x_i + \sqrt{\epsilon\delta}\left[\sin\left(2\pi(x_{i+1} + x_{i-1})\right)\right](x_{i+1} + x_{i-1})$$

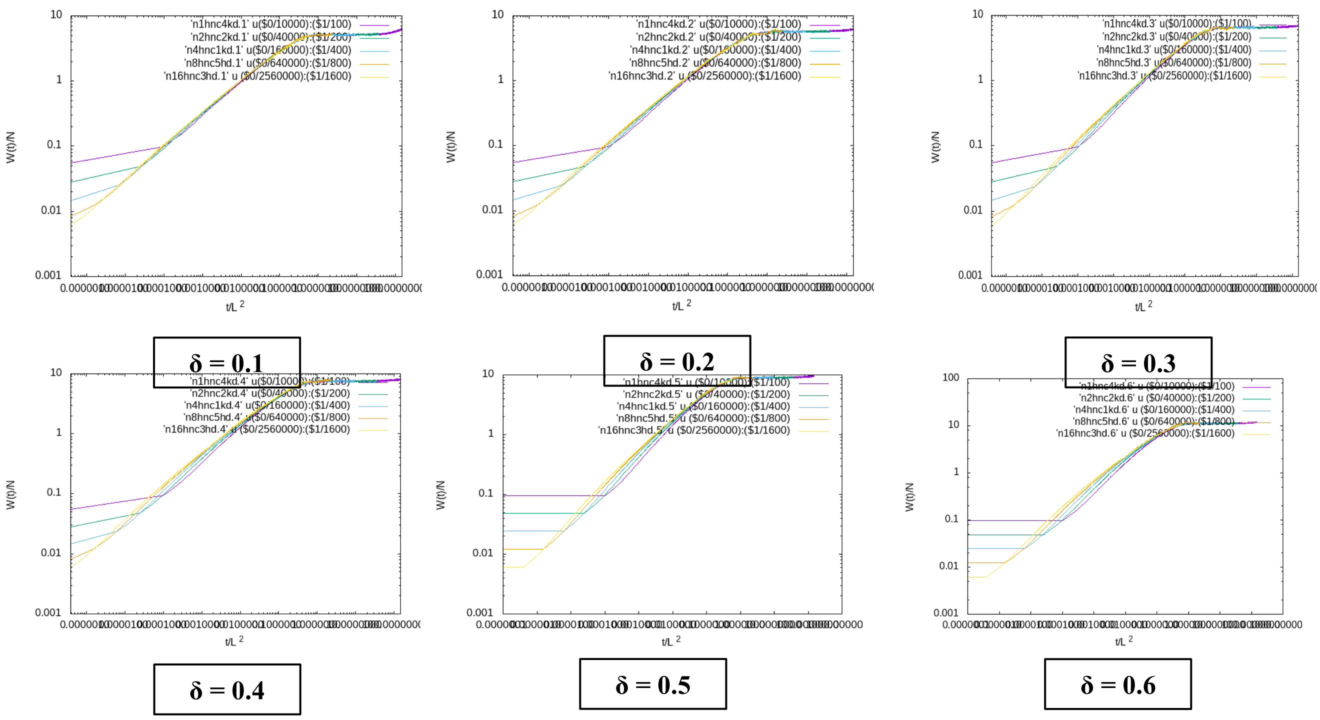

Figure 1. Plots depicting scaling for different values of delta (δ).

III. CONCLUSION

From the figures we see that for a particular value of δ, graphs for different number of lattice sites have different variance. This is called the 'width'. Greater the number of lattice sites, higher is the variance. All the graphs remain constant for a certain time and then increase with time. Ultimately, all the graphs saturate at a certain point.

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