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Combined Set of Several Sets of Observations: Quadratic Mean

Dhritikesh Chakrabarty

Independent Researcher, Ex Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: A formula has been derived for computing quadratic mean of a set of observations, which is a combined set of several sets of observations, when the observations are unknown but the respective quadratic means of the sets and the respective numbers of observations in the sets are known. This article is based on the derivation of the formula along with numerical example.

KEYWORDS*:* Observations, Several Sets, Combined Set, Quadratic Mean, Formula

I. INTRODUCTION

Many researches/studies had been done on average [1 , 47] which is an entity that describes a set of many entities. Pythagoras, the pioneer of developing measures of average, derived three measures of average namely arithmetic mean [2 , 6 , 54], geometric mean [2 , 6] and harmonic mean [2 , 6 , 53] popularly known as "Pythagorean Means" [3 , 7 , 15]. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p* mean & generalized p^{th} root mean etc. [8 , 32]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are arithmetic-geometric mean, arithmetic-harmonic mean, geometric-harmonic mean and arithmetic-geometric-harmonic respectively [19 , 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of arithmetic mean, geometric mean & harmonic mean [2 , 3 , 6 , 39 , 40 , 42 – 44 , 53 , 54]. Arithmetic mean, geometric mean & harmonic mean have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 – 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 – 35 , 38 , 41 , 50 , 51]. It is to be mentioned that quadratic mean [4 , 46 , 48 , 52], which focuses on the effective magnitude or "strength" of a set of values, is also very important and useful in various fields like Physics, Engineering & Signal Processing, Statistics, Error Analysis, Forestry and in many others [46 , 48 , 49 , 52]. For this reason, it is also important to identify the properties satisfied by quadratic mean. For this reason, some basic properties of quadratic mean have been identified in a recent study [45].

There are many situations where quadratic means of several sets of observations are available but the observations are not available and it is required to find out the quadratic mean of all the observations of the sets combined together [44]. Attempt has here been made on deriving a formula for computing quadratic mean of a set of observations which is a combined set of a number of sets of observations when the observations are unknown but the respective quadratic means of the sets and the respective numbers of observations in the sets are known. Derivation of the formula, along with numerical example, has been presented in this article.

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Vol. 12, Issue 5, May 2025

II. QUADRATIC MEAN

Quadratic mean of a Set of Numbers:

Definition

Let us consider a list of N real numbers or values namely

$$a_1 , a_2 , \ldots\ldots , a_N$$

Quadratic mean of them, denoted by $Q(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by

$$Q(a_1 , a_2 , \ldots\ldots , a_N) = \{ \frac{1}{N} ({a_1}^2 + {a_2}^2 + \ldots\ldots + {a_N}^2)\}^{1/2} \quad (2.1)$$

where the absolute (positive) square root is taken [4 , 46 , 48 , 52].

Note (2.1):

Arithmetic mean of $a_1 , a_2 , \ldots\ldots , a_N$, denoted by $A(a_1 , a_2 , \ldots\ldots , a_N)$, is defined by1

$$A(a_1 , a_2 , \ldots\ldots , a_N) = \frac{1}{N} (a_1 + a_2 + \ldots\ldots + a_N)$$

which implies,

$$Q(a_1 , a_2 , \ldots\ldots , a_N) = \{A({a_1}^2 , {a_2}^2 , \ldots\ldots , {a_N}^2)\}^{1/2} \quad (2.2)$$

Thus, quadratic mean of a set of numbers can also be defined as the absolute square root of arithmetic mean of squares of the numbers.

Note (2.2):

The definition of quadratic mean implies that

$${a_1}^2 + {a_2}^2 + \ldots\ldots + {a_N}^2 = N.(Q(a_1 , a_2 , \ldots\ldots , a_N)^2$$

i.e. Sum of squares of $a_1 , a_2 , \ldots\ldots , a_N = N.(Q(a_1 , a_2 , \ldots\ldots , a_N)^2$ (2.3)

Thus if $q(a)$ is the quadratic mean of $a_1 , a_2 , \ldots\ldots , a_N$,

then

Sum of squares of $a_1 , a_2 , \ldots\ldots , a_N = N.\{q(a)\}^2$

i.e. Sum of squares of the N numbers = N times of the square of the quadratic mean of the numbers

Note (2.3):

As per the definition quadratic mean from first principle [8 , 18], quadratic mean of the numbers

$$a_1 , a_2 , \ldots\ldots , a_N$$

is the value $q(a)$ such that

$${a_1}^2 + {a_2}^2 + \ldots\ldots + {a_N}^2 = \{q(a)\}^2 + \{q(a)\}^2 + \ldots\ldots + \{q(a)\}^2 \quad (2.4)$$

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International Journal of AdvancedResearch in Science, Engineering and Technology
Vol. 12, Issue 5, May 2025

This also implies the equation (2.3).

III. QUADRATIC MEAN OF COMBINED SET

Suppose, there are *k* sets namely

$$S_1 , S_2 , \ldots\ldots\ldots , S_k$$

containing

$$n_1 , n_2 , \ldots\ldots\ldots , n_k$$

real valued observations respectively such that quadratic means of the respective sets of observations are

$$q_1 , q_2 , \ldots\ldots\ldots , q_k$$

respectively.

If the observations in the *k* sets are combined together to form a single set S, then this set will contain

$$n_1 + n_2 + \ldots\ldots\ldots + n_k$$

observations.

By equation (2.3),

Sum of squares of observations in the set $S_1 = n_1 {q_1}^2$,

Sum of squares of observations in the set $S_2 = n_2 {q_2}^2$,

.....

Sum of squares of observations in the set $S_k = n_k {q_k}^2$.

This implies,

Sum of squares of observations in the set $S = n_1 {q_1}^2 + n_2 {q_2}^2 + \ldots\ldots\ldots + n_k {q_k}^2$

Therefore by equation (2.1), quadratic mean q of the observations in the set S will be

$$q = \left(\frac{n_1 q_1 2 + n_2 q_2 2 + \ldots\ldots\ldots + n_k q_k 2}{n_1 + n_2 + \ldots\ldots\ldots + n_k} \right)^{1/2} \qquad (3.1)$$

This is the formula for computing quadratic mean of a combined set of several sets of observations when the observations are unknown but the quadratic means of the sets and the numbers of observations in the respective sets are known.

This formula has been stated in the form of a theorem below:

***Theorem*:**

If $q_1 , q_2 , \ldots\ldots\ldots , q_k$ are quadratic means of respective sets having $n_1 , n_2 , \ldots\ldots\ldots , n_k$ observations respectively then the quadratic mean of the set of all observations in the k sets combined together is given by

$$q = \left(\frac{n_1 q_1 2 + n_2 q_2 2 + \ldots\ldots\ldots + n_k q_k 2}{n_1 + n_2 + \ldots\ldots\ldots + n_k} \right)^{1/2}$$

ISSN: 2350-0328
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Vol. 12, Issue 5, May 2025

***Corollary*:**

In particular, if q_x & q_y is the quadratic means of

$$x_1 , x_2 , \ldots\ldots\ldots , x_m$$

&

$$y_1 , y_2 , \ldots\ldots\ldots , y_n$$

respectively,

then the quadratic mean q_{xy} of

$$x_1 , x_2 , \ldots\ldots\ldots , x_m , y_1 , y_2 , \ldots\ldots\ldots , y_n$$

is given by

$$q_{xy} = \left(\frac{m q_{xy}2 + n\, q_y 2}{m + n} \right)^{1/2}$$

Note:

The formula given by equation (3.1), stated in the theorem, can also be expressed as

$$q = (w_1 q_1{}^2 + w_2 q_2{}^2 + \ldots\ldots\ldots + w_k q_k{}^2)^{1/2} \qquad (3.2)$$

where

$$w_1 = \frac{n_1}{n_1 + n_2 + \ldots\ldots\ldots + n_k} \ ,$$

$$w_2 = \frac{n_2}{n_1 + n_2 + \ldots\ldots\ldots + n_k} \ ,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$w_k = \frac{n_k}{n_1 + n_2 + \ldots\ldots\ldots + n_k} \ .$$

This formula can sometimes be more convenient in computational works.

IV. NUMERICAL EXAMPLE

Let us consider the following three sets of numbers

$$\{1 , 3 , 5 , 7 , 9\} , \{2 , 4 , 6 , 8 , 10 , 12\} \ \& \ \{3 , 6 , 9 , 12 , 15 , 18 , 21\}$$

As per the formula given by equation (2.1) in ***Definition***,

Quadratic mean of the numbers in the 1st set = 5.744562646538028659850611468 2189,

Quadratic mean of the numbers in the 2nd set = 7.788880963698615073774789928 4054

& **Quadratic mean** of the numbers in the 3rd set = 13.416407864998738178455042012388 .

Now, the combined set of the 1st set and the 2nd set is

$$\{1 , 3 , 5 , 7 , 9 , 2 , 4 , 6 , 8 , 10 , 12\}$$

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Vol. 12, Issue 5, May 2025

so that by equation (2.1),

Quadratic mean of the numbers in this combined set = 6.9347609252885633208766775403114

Applying the formula given by equation (3.1), the value of **quadratic mean** of the numbers in this combined set is also found as 6.9347609252885633208766775403114 .

Similarly, the combined set of the 1st set and the 3rd set is

{1 , 3 , 5 , 7 , 9 , 3 , 6 , 9 , 12 , 15 , 18 , 21}

so that by equation (2.1),

Quadratic mean of the numbers in this combined set = 10.897247358851683880592454959649

Applying the formula given by equation (3.1), the value of **quadratic mean** of the numbers in the combined set is also found as 10.897247358851683880592454959649 .

Also, the combined set of the 2nd set and the 3rd set is

{2 , 4 , 6 , 8 , 10 , 12 , 3 , 6 , 9 , 12 , 15 , 18 , 21}

so that by equation (2.1),

Quadratic mean of the numbers in this combined set = 11.176899253508413147976354627433

Applying the formula given by equation (3.1), the value of **quadratic mean** of the numbers in the combined set is also found as. 11.176899253508413147976354627433

Moreover, the combined set of all the three sets is

{1 , 3 , 5 , 7 , 9 , 2 , 4 , 6 , 8 , 10 , 12 , 3 , 6 , 9 , 12 , 15 , 18 , 21}

so that by equation (2.1),

Quadratic mean of the numbers in this combined set = 9.969397619158786912151531249173

Applying the formula given by equation (3.1), the value of **quadratic mean** of the numbers in the combined set is also found as 9.969397619158786912151531249173 .

V. CONCLUSION

The type of situation, where respective quadratic means of several sets of observations obtained from independent studies are available but the observations are not available and it is required to find out the quadratic mean of all the observations of the sets combined together, arises in many research studies specially in meta analysis in clinical trials [1 , 4 , 5]. The formula derived here is a solution of this type of problem.

In this connection, it is to be mentioned that each of arithmetic mean, geometric mean & harmonic mean is required to be computed in similar type of situation. Of course, the formula for computing arithmetic mean in this situation is an established one while the formula for computing quadratic mean has been derived here. However, the formula for computing geometric mean in this situation is yet to be found out. This is a problem of research to be studied at this stage.

Similar types of situations/problems may also arise in many researches/studies in the case of determining other measures of average. Accordingly, similar attempts are required to be made on searching for the solutions of the respective problems.

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International Journal of AdvancedResearch in Science, Engineering and Technology

Vol. 12, Issue 5, May 2025

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ISSN: 2350-0328
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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag

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Vol. 12, Issue 5, May 2025

(Dr. Dhritikesh Chakrabarty, 2nd from the right, in the 5th Canadian Conference in Applied Statistics / Interdisciplinary Mathematical & Statistical Techniques (Statistics 2011 Canada / IMST 2011– FIM XX) held at Concordia University, Montreal, Quebec, Canada, during July 1 – 4, 2011)

Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

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Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.