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To model and solve optimization Problems under Vagueness using Interval Type-2 Fuzzy Linear Programming

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ABSTRACT: Interval Type-2 Fuzzy Linear Programming (IT2FLP) provides a robust framework to address uncertainty in optimization problems. This study investigates IT2FLP with a focus on vagueness in the Objective Function Vector (OFV), Resource Vector (RsV) and Technological Coefficients (TCs). We examine their individual and combined effects on optimization, proposing novel solution methods tailored to each scenario. Practical applications, including furniture production and logistics optimization, illustrate the methods effectiveness. Results emphasize IT2FLP's adaptability in addressing real – world uncertainties.

KEY WORDS: Interval Type-2 Fuzzy Sets, Linear Programming, Optimization under Uncertainty, α-Cuts, Fuzzy Decision-Making.

I.INTRODUCTION

Optimization under uncertainty is essential in manufacturing, construction, and logistics. Traditional linear programming assumes precise inputs, which rarely hold in real-world scenarios. To address this, fuzzy set theory, particularly Interval Type-2 Fuzzy Sets (IT2FSs), provides a robust framework for handling uncertainty.
Interval Type-2 Fuzzy Linear Programming (IT2FLP) accommodates vagueness in three key components of an optimization problem:

1. **Objective Function Vector (OFV):** Accounts for uncertainties in profits or costs.
2. **Resource Vector (RsV):** Captures fluctuations in resource availability.
3. **Technological Coefficients (TCs):** Models imprecision in input-output relationships.

This study demonstrates IT2FLP's effectiveness through practical cases:

- A furniture company maximizing profits amid fluctuating costs (OFV uncertainty).
- A construction firm allocating resources across projects with uncertain requirements (TC vagueness).
- A logistics company optimizing fleet operations under uncertain costs, travel times, and resource availability (combined uncertainty).

Using α-cuts, fuzzy problems are converted into crisp equivalents for systematic solution exploration. This research advances fuzzy optimization by proposing new methods for handling combined uncertainties. It also sets the stage for future studies integrating IT2FLP with multi-objective optimization and machine learning for dynamic decision-making.

II. LITERATURE SURVEY

Optimization under uncertainty has been widely studied, with fuzzy set theory playing a key role in handling imprecise data. Traditional linear programming assumes precise parameters, which is often unrealistic. Bellman and Zadeh (1970) introduced decision-making in a fuzzy environment, later expanded by Zimmerman (1978) to fuzzy linear programming (FLP). The advancement of fuzzy set theory led to Interval Type-2 Fuzzy Sets (IT2FSs), which model uncertainty more effectively through upper and lower membership functions.
Interval Type-2 Fuzzy Linear Programming (IT2FLP) extends these concepts, allowing vagueness in the objective function vector, resource vector, and technological coefficients. Studies by Sargolzaei and Nehi (2024) have explored

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IT2FLP applications in resource allocation, while Nehi and Sargolzaei (2024) developed multi-objective algorithms. IT2FLP has proven effective in manufacturing, construction, and logistics, with α-cuts facilitating solution transformations. Future research could integrate machine learning for dynamic optimization and explore applications in sustainable resource management and resilient logistics.

III. METHODOLOGY

This study employs Interval Type-2 Fuzzy Linear Programming (IT2FLP) to optimize decision-making under uncertainty. The optimization problem is formulated with uncertainty in three key components: the objective function vector (OFV) representing fluctuating profits or costs, the resource vector (RsV) capturing variable resource availability, and the technological coefficients (TCs) modeling imprecise input-output relationships. To handle these uncertainties, IT2FSs with upper and lower membership functions are used, and the α-cut method converts the fuzzy problem into crisp equivalents by defining confidence levels ($\alpha \in [0,1]$). Best-case and worst-case scenarios are formulated and solved using interval programming. The simplex method is applied iteratively at different α levels to balance precision and robustness. To validate the methodology, IT2FLP is tested on three real-world case studies: maximizing profits in furniture production under OFV uncertainty, optimizing resource allocation in construction under TC vagueness, and minimizing transportation costs in logistics under combined OFV, RsV, and TC uncertainties. Interval-based solutions are generated and analyzed across different confidence levels.

IV.THE IT2FLP PROBLEM WITH VAGUENESS IN OFV

In this section, the MF of the OFV is initially articulated, followed by the introduction of a novel methodology aimed at resolving the IT2FLP problem when confronted with vagueness in the OFV. The $\tilde{\tilde{c}}_j$, j=1,2,…..,n are the IT2FSs,which are defined by the UMF,$\bar{\mu}_{\tilde{\tilde{c}}_j}$ and LMF, $\underline{\mu}_{\tilde{\tilde{c}}_j}$.

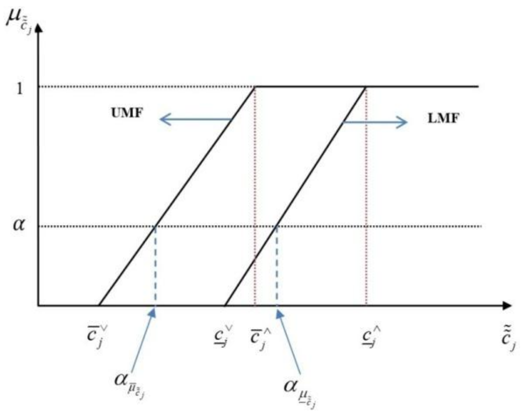

Figure 2. The MF of the interval type-2 OFV

Since the objective function is a maximization, thus the UMF is equal to:

$$\bar{\mu}_{\tilde{\tilde{c}}_j}(c_j, \bar{c}_j^{\vee}, \bar{c}_j^{\wedge}) = \begin{cases} 1, & c_j \geq \bar{c}_j^{\wedge}, \\ \dfrac{c_j - \bar{c}_j^{\vee}}{\bar{c}_j^{\wedge} - \bar{c}_j^{\vee}}, & \bar{c}_j^{\vee} \leq c_j \leq \bar{c}_j^{\wedge}, \\ 0, & c_j \leq \bar{c}_j^{\vee}. \end{cases}$$

And its LMF is expressed as below:

$$\underline{\mu}_{\tilde{\tilde{c}}_j}(c_j, \underline{c}_j^{\vee}, \underline{c}_j^{\wedge}) = \begin{cases} 1, & c_j \geq \underline{c}_j^{\wedge}, \\ \dfrac{c_j - \underline{c}_j^{\vee}}{\underline{c}_j^{\wedge} - \underline{c}_j^{\vee}}, & \underline{c}_j^{\vee} \leq c_j \leq \underline{c}_j^{\wedge}, \\ 0, & c_j \leq \underline{c}_j^{\vee}. \end{cases}$$

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Solving method

Here, we introduce a method for solving the IT2FLP problem with vagueness in the OFV. The IT2FLP problem with vagueness in the OFV is considered, which is a dual-mode IT2FLP problem with vagueness in the RsV,

$$\max \sum_{j=1}^{n} \tilde{\tilde{c}}_j x_j$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\ldots,m, \quad x_j \ge 0, \quad j = 1,2,\ldots,n,$$

Which $\tilde{\tilde{a}}_j$, j=1,2,…,n are interval type-2 fuzzy OFVs. As shown, by applying the α-cut, the interval $[\alpha_{\bar{\mu}_{\tilde{\tilde{c}}_j}}, \alpha_{\underline{\mu}_{\tilde{\tilde{c}}_j}}]$ is obtained. Considering Figure2, we have

$\left[\alpha_{\bar{\mu}_{\tilde{\tilde{c}}_j}}, \alpha_{\underline{\mu}_{\tilde{\tilde{c}}_j}}\right] = \left[\frac{c_j - \bar{c}_j^{\vee}}{\bar{c}_j^{\wedge} - \bar{c}_j^{\vee}}, \frac{c_j - \underline{c}_j^{\vee}}{\underline{c}_j^{\wedge} - \underline{c}_j^{\vee}}\right]$. Now, the above problem is rewritten as follows:

$$\max \sum_{j=1}^{n} \left[\frac{c_j - \bar{c}_j^{\vee}}{\bar{c}_j^{\wedge} - \bar{c}_j^{\vee}}, \frac{c_j - \underline{c}_j^{\vee}}{\underline{c}_j^{\wedge} - \underline{c}_j^{\vee}}\right] x_j$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\ldots,m, \quad x_j \ge 0, \quad \alpha \in [0,1] \quad j = 1,2,\ldots,n,$$

Which the above equation is equivalent to

$$\max \sum_{j=1}^{n} \left[\bar{c}_i^{\vee} + \alpha(\bar{c}_i^{\wedge} - \bar{c}_i^{\vee}), \underline{c}_i^{\vee} + \alpha(\underline{c}_i^{\wedge} - \underline{c}_i^{\vee})\right] x_j$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\ldots,m, \quad x_j \ge 0, \quad \alpha \in [0,1] \quad j = 1,2,\ldots,n,$$

Since the objective function is maximization, the optimal solution for the above problem. given the interval programming is as follows:

$$\max \sum_{j=1}^{n} \left[\underline{c}_i^{\vee} + \alpha(\underline{c}_i^{\wedge} - \underline{c}_i^{\vee})\right] x_j$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\ldots,m, \quad x_j \ge 0, \quad \alpha \in [0,1] \quad j = 1,2,\ldots,n.$$

The above problem is a nonlinear that is solved for different values of $\alpha \in [0,1]$. In this method, in addition to α values, the optimal value of the objective function and optimal solutions are also calculated, which is one of the advantages of the method. Therefore, for different values of $\alpha \in [0,1]$, a table of optimal solutions and the optimal value of the objective function corresponding to each α value are obtained, and the DM can choose a value from among the obtained solutions.

Numerical Example

A furniture company produces three types of furniture: chairs, tables, and cabinets. The profit margins for these items are uncertain due to fluctuating raw material costs and market demand. How can the company maximize its profit using interval type-2 fuzzy linear programming (IT2FLP) while considering vagueness in profit margins?

1. **Objective Function Profit Margins:**
   Lower bounds (c^):[15, 20, 30] (profits for chairs, tables, cabinets, respectively).
   Upper bounds (c^): [25, 35, 50].
2. **Resource Constraints:**
   The company has limited resources (wood, labour, and paint), modelled as:
   Technological coefficient matrix A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 3 & 6 \\ 2 & 1 & 3 \end{bmatrix}$ (Each column represents chairs, tables, and cabinets).
   Resource availability (b): [120, 150, 90] (total units of wood, labour, and paint available).

Solution Using IT2FLP

1. **Model the Problem:**
   $$\max \sum_{j=1}^{n} \left[\underline{c}_i^{\vee} + \alpha(\underline{c}_i^{\wedge} - \underline{c}_i^{\vee})\right] x_j$$
   $$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\ldots,m, \quad x_j \ge 0, \quad \alpha \in [0,1] \quad j = 1,2,\ldots,n.$$
2. **Optimize Using α-Cuts:** Solve for different confidence levels (α) to obtain solutions.

Solution

The optimization results for various α values are summarized below:

For α=1 the simplex method is as follows:

Max z = $25x_1 + 35x_2 + 50x_3$

Subject to

$3x_1+2x_2+5x_3 \le 120$

$4x_1+3x_2+6x_3 \le 150$

$2x_1+1x_2+3x_3 \le 90$

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x_1,x_2, x_3 ≥0;
The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate and solve this by simplex method, in 3rd iteration the optimal solution is arrived with the value of variables as:
x_1=0,x_2=3.75, x_3=22.5 Max z = 1256.25
Similarly, calculations made for each α and shown in the following table:

α	x_1^* (Chairs)	x_2^* (tables)	x_3^* (cabinets)	z^*(profit)
0.0	0	3.75	22.5	750
0.1	0	3.75	22.5	800.625
0.2	0	3.75	22.5	851.25
0.3	0	3.75	22.5	901.875
0.4	0	3.75	22.5	952.5
0.5	0	3.75	22.5	1003.125
0.6	0	3.75	22.5	1053.75
0.7	0	3.75	22.5	1104.375
0.8	0	3.75	22.5	1155
0.9	0	3.75	22.5	1205.625
1.0	0	3.75	22.5	1256.25

Result:
At α=1, the company achieves the maximum profit of $\boldsymbol{z}^*$=1256.25 by producing **($\boldsymbol{x_1^*}, \boldsymbol{x_2^*}, \boldsymbol{x_3^*}$)=(0,3.75,22.5)**. However, the company can choose other α levels based on its confidence in the profit margin estimates.

V.THE IT2FLP PROBLEM WITH VAGUENESS IN THE TCs

In this section, we review the MFs of interval type-2 fuzzy TCs and propose a method to solve such problems. In this study, the MFs of the TCs have a triangular shape. The main structure of an IT2FLP problem with vagueness in the TCs is as follows:

$$\max \sum_{j=1}^{n} c_j x_j$$

s.t.$\sum_{j=1}^{n} \tilde{\tilde{a}}_{ij} x_j \leq b_i \quad i = 1,2,\dots,m, \quad x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

Where (for i=1,2,…,m and j=1,2,……n), $c_j x_j \in \mathbb{R}^m$, and $\tilde{\tilde{a}}_{ij} \in \mathbb{R}^{m\times n}$ are the IT2FSs. We examine the MF of interval type-2 fuzzy TCs with imprecision of the vagueness type. The MF representing this FS is shown in the Definition 2.5 (see Figure 1). Then for each α- cut of two ILP problems,

α_{z_1}:max$\sum_{j=1}^{n} c_j x_j$

s.t $\sum_{j=1}^{n} \left[\alpha_{\tilde{\tilde{a}}_{ij}^{\vee}}, \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\vee}}\right] x_j \leq b_i \quad i = 1,2,\dots,m, \ x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

and

α_{z_2}: max$\sum_{j=1}^{n} c_j x_j$

s.t.$\sum_{j=1}^{n} \left[\alpha_{\tilde{\tilde{a}}_{ij}^{\wedge}}, \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\wedge}}\right] x_j \leq b_i \quad i = 1,2,\dots,m, \quad x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

To solve the above programming problems, the best and worst objective function values are displayed with ${}_{b}^{\alpha}z_i$ and ${}_{w}^{\alpha}z_i$, respectively, where i=1,2.

${}_{b}^{\alpha}z_1$: max $\sum_{j=1}^{n} c_j x_j$

s.t $\sum_{j=1}^{n} \alpha_{\tilde{\tilde{a}}_{ij}^{\vee}} x_j \leq b_i \quad i = 1,2,\dots,m, \ x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

and

${}_{b}^{\alpha}z_1$: max $\sum_{j=1}^{n} c_j x_j$

s.t $\sum_{j=1}^{n} \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\vee}} x_j \leq b_i \quad i = 1,2,\dots,m, \ x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

and

${}_{b}^{\alpha}z_2$: max $\sum_{j=1}^{n} c_j x_j$

s.t.$\sum_{j=1}^{n} \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\wedge}} x_j \leq b_i \quad i = 1,2,\dots,m, \quad x_j \geq 0, \quad j = 1,2,\dots\dots,n,$

and

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${}_{b}^{\alpha}z_2$: max $\sum_{j=1}^{n} c_j x_j$
s.t $\sum_{j=1}^{n} \alpha_{\tilde{\bar{a}}_{ij}^{\wedge}} x_j \leq b_i$ $i = 1,2, \dots, m,$ $x_j \geq 0,$ $j = 1,2, \dots\dots, n.$
Next, we propose a new approach for solving the IT2FLP problem with vagueness in TCs,

5.1 The new solving method.

Consider the IT2FLP problem with vagueness in the TCs.
max $\sum_{j=1}^{n} c_j x_j$
s.t $\sum_{j=1}^{n} \tilde{\tilde{a}}_{ij} x_j \leq b_i$ $i = 1,2, \dots, m,$
$x_j \geq 0,$ $j = 1,2, \dots\dots, n,$
For j= 1,2,….,n, $\tilde{\tilde{a}}_{ij}$ are interval type- 2 fuzzy TCs. As the MF of the constraints
max α
s.t.$\alpha(z_b - z_w) - \sum_{j=1}^{n} c_j x_j + z_w \leq 0,$
$\sum_{j=1}^{n} \left(\underline{a}_{ij}^{\wedge} + \underline{\Delta}_{ij}\alpha\right) x_j \leq b_i,$ i= 1,2, ……, m,
$\alpha \in [0,1]$, $x_j \geq 0,$ $j = 1, 2, \dots, n.$
the problem is considered as follows:
max $\sum_{j=1}^{n} c_j x_j$
s.t $\mu_{\tilde{\tilde{G}}_i}(x) \geq \alpha$,i= 1, 2, …., m,
$\alpha \in [0,1]$, $x_j \geq 0,$ $j = 1, 2, \dots, n.$
which the above problem is equivalent to:
max $\sum_{j=1}^{n} c_j x_j$
s.t.$\sum_{j=1}^{n} \left(a_{ij} + \Delta_{ij}\alpha\right) x_j - b_i \leq 0$ i =1, 2, ….., m,
$\alpha \in [0,1]$, $x_j \geq 0,$ $j = 1, 2, \dots, n.$
for the upper and lower MFs, suppose that $\tilde{\tilde{a}}_{ij} \in \left[\frac{\bar{a}_{ij}^{\vee} + \underline{a}_{ij}^{\vee}}{2}, \frac{\bar{a}_{ij}^{\wedge} + \underline{a}_{ij}^{\wedge}}{2}\right]$ and $\Delta_{ij} \in \left[\underline{\Delta}_{ij}, \overline{\Delta}_{ij}\right]$. Then the above problem is displayed as an interval non-LP problem:
max $\sum_{j=1}^{n} c_j x_j$
s.t.$\sum_{j=1}^{n} \left(\left[\frac{\bar{a}_{ij}^{\vee} + \underline{a}_{ij}^{\vee}}{2}, \frac{\bar{a}_{ij}^{\wedge} + \underline{a}_{ij}^{\wedge}}{2}\right] + \left[\underline{\Delta}_{ij}, \overline{\Delta}_{ij}\right]\alpha\right) x_j - b_i \leq 0$ i =1, 2, ….., m,
$\alpha \in [0,1]$, $x_j \geq 0,$ $j = 1, 2, \dots, n.$
According to the interval programming, the optimal problem of the above is expressed as follows:
max $\sum_{j=1}^{n} c_j x_j$
s.t.$\sum_{j=1}^{n} \left(\left[\frac{\bar{a}_{ij}^{\vee} + \underline{a}_{ij}^{\vee}}{2}\right] + \left[\underline{\Delta}_{ij}\right]\alpha\right) x_j - b_i \leq 0$ i =1, 2, ….., m,
$\alpha \in [0,1]$, $x_j \geq 0,$ $j = 1, 2, \dots, n.$
The above problem is also a nonlinear. As a result, it is solved for $\alpha \in [0,1]$ to solve the IT2FLP problem with vagueness in the TCs.

5.2 Numerical Example

A construction company is planning three projects (A, B, C), and the resource requirements for these projects are uncertain due to fluctuating costs and labor availability. How can the company allocate resources optimally to maximize its total return using interval type-2 fuzzy linear programming (IT2FLP) with vagueness in the technological coefficients (TCs)?

- Objective Function Returns (c): [60, 90, 40] (returns for projects A, B, and C).

2. Technological Coefficient Matrix ($\bar{a}^{\vee}$,$\underline{a}^{\vee}$): $\bar{a}^{\vee} = \begin{bmatrix} 4 & 3 & 5 \\ 8 & 4 & 7 \\ 3 & 2 & 4 \end{bmatrix}$ $\underline{a}^{\vee} = \begin{bmatrix} 6 & 5 & 7 \\ 10 & 6 & 9 \\ 4 & 3 & 5 \end{bmatrix}$
3. Resource Constraints (b): Total available resources b = [100, 150, 80]

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Solution Using IT2FLP

1. Model the Problem:
   Maximize the return: $Z = 60x_1 + 90x_2 + 40x_3$

Subject to the constraints:

$\sum_{j=1}^{n}\left(\left[\frac{\bar{a}_{ij}^{\vee}+\underline{a}_{ij}^{\vee}}{2}\right] + [\underline{\Delta}_{ij}]\alpha\right)x_j - b_i \le 0$ i =1, 2, ….., m,

$\alpha \in [0,1]$, $x_j \ge 0$, $j = 1, 2, \dots, n.$

2. Optimization Approach: Apply α-cuts to reduce the fuzzy problem into linear programming sub-problems.

Solution

The optimization results for varying α levels are summarized as:

For α=0 the simplex method is as follows:

Max z = $60x_1 + 90x_2 + 40x_3$

Subject to

$5x_1+4x_2+6x_3 \le 100$

$9x_1+5x_2+8x_3 \le 150$

$3.5x_1+2.5x_2+4.5x_3 \le 80$

$x_1, x_2, x_3 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate and solve this by simplex method, in 2nd iteration the optimal solution is arrived with the value of variables as:

x_1=0, x_2=25, x_3=0 Max z = 2250

Similarly, calculations made for each α and shown in the following table:

α	x_1^* (project - A)	x_2^* (project - B)	x_3^* (project - C)	z^*(profit)
0.0	0	25	0	2250
0.1	0	24.3902	0	2195.12
0.2	0	23.8095	0	2142.8571
0.3	0	23.2558	0	2093.0233
0.4	0	22.7273	0	2045.4545
0.5	0	22.2222	0	2000
0.6	0	21.7391	0	1956.5217
0.7	0	21.2766	0	1914.8936
0.8	0	20.8333	0	1875
0.9	0	20.4082	0	1836.7347
1.0	0	20	0	1800

Result:

At α=0 (highest confidence in the data), the company maximizes its return to z* = 2250 by allocating resources to projects A, B, and C as (x_1^*, x_2^*, x_3^*) = (0, 25, 0). The company can choose lower α value to account for higher uncertainty in the technological coefficients, balancing risk and resource allocation.

VI.THE IT2FLP PROBLEM WITH VAGUENESS IN OFV, RsV AND TCs

The IT2FLP problem with vagueness in OFV, RsV and TCs is as follows:

$\max \sum_{j=1}^{n} \tilde{\tilde{c}}_j x_j$

s.t $\sum_{j=1}^{n} \tilde{\tilde{a}}_{ij} x_j \le \tilde{\tilde{b}}_i$ $i = 1,2, \dots, m,$ $x_j \ge 0,$ $j = 1,2, \dots, n,$

where $\tilde{\tilde{c}}_j$,$\tilde{\tilde{a}}_{ij}$ and $\tilde{\tilde{b}}_i$are IT2FSs. They represent the interval type – 2 fuzzy of the OFV, TCs and RsV with an impression of vagueness type, respectively. Then the MF of such problems is introduced and a new solving method is suggested.

By applying α – cut on IT2FSs, the left and right bounds are obtained from the ILP problems. To solve the above problem, we need eight ILP equivalents for sixteen LP problems. Using the following theorem, we show that the MF of the above problem can be obtained only by solving two – LP problems related to each α– cut. Given the theorem which states that,

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For each i= 1,2, …., m, the interval inequality $\sum_{j=1}^{n}\left[\alpha_{\tilde{\tilde{a}}_{ij}^{\vee}}, \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\vee}}\right] x_j \leq \left[\alpha_{\tilde{\tilde{b}}_i^{\vee}}, \alpha_{\tilde{\tilde{b}}_i^{\wedge}}\right]$ and the interval inequality $\sum_{j=1}^{n}\left[\alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\wedge}}, \alpha_{\tilde{\tilde{a}}_{ij}^{\wedge}}\right] x_j \leq \left[\alpha_{\underline{\tilde{\tilde{b}}}_i^{\vee}}, \alpha_{\underline{\tilde{\tilde{b}}}_i^{\wedge}}\right]$ are the biggest and smallest feasible areas, respectively.

and ILP problems, obtaining the objective function in the largest and smallest feasible area is required to calculate the best and worst objective function values. Therefore, instead of eight ILP problems related to each α – cut, the MF of the objective function can be obtained only by solving two ILP problems:

$$\begin{aligned} \max \quad & \sum_{j=1}^{n}\left[\alpha_{\tilde{\tilde{c}}_j^{\vee}}, \alpha_{\tilde{\tilde{c}}_j^{\wedge}}\right] x_j \\ \text{s.t} \quad & \sum_{j=1}^{n}\left[\alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\wedge}}, \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\vee}}\right] x_j \leq \left[\alpha_{\tilde{\tilde{b}}_i^{\vee}}, \alpha_{\tilde{\tilde{b}}_i^{\wedge}}\right], \qquad i = 1, 2, \ldots, m, \\ & x_j \geq 0, \qquad j = 1,2, \ldots, n. \end{aligned}$$

and

$$\begin{aligned} \max \quad & \sum_{j=1}^{n}\left[\alpha_{\underline{\tilde{\tilde{c}}}_j^{\wedge}}, \alpha_{\underline{\tilde{\tilde{c}}}_j^{\vee}}\right] x_j \\ \text{s.t} \quad & \sum_{j=1}^{n}\left[\alpha_{\tilde{\tilde{a}}_{ij}^{\vee}}, \alpha_{\tilde{\tilde{a}}_{ij}^{\wedge}}\right] x_j \leq \left[\alpha_{\underline{\tilde{\tilde{b}}}_i^{\wedge}}, \alpha_{\underline{\tilde{\tilde{b}}}_i^{\vee}}\right], \qquad i = 1, 2, \ldots, m, \\ & x_j \geq 0, \qquad j = 1,2, \ldots, n. \end{aligned}$$

6.1.Theorem

For each α – cut, the UMF of the objective function is obtained by solving two LP problems:

$$\begin{aligned} z_b^{\alpha}: \quad & \max \sum_{j=1}^{n} \alpha_{\tilde{\tilde{c}}_j^{\wedge}} x_j \\ \text{s.t} \quad & \sum_{j=1}^{n} \alpha_{\underline{\tilde{\tilde{a}}}_{ij}^{\wedge}} x_j \leq \alpha_{\tilde{\tilde{b}}_i^{\wedge}}, \qquad i = 1, 2, \ldots, m, \\ & x_j \geq 0, \qquad j = 1,2, \ldots, n. \end{aligned}$$

and

$$\begin{aligned} z_w^{\alpha}: \quad & \max \sum_{j=1}^{n} \alpha_{\underline{\tilde{\tilde{c}}}_j^{\wedge}} x_j \\ \text{s.t} \quad & \sum_{j=1}^{n} \alpha_{\tilde{\tilde{a}}_{ij}^{\wedge}} x_j \leq \alpha_{\underline{\tilde{\tilde{b}}}_i^{\wedge}}, \qquad i = 1, 2, \ldots, m, \\ & x_j \geq 0, \qquad j = 1,2, \ldots, n. \end{aligned}$$

which the above problems are the best and worst LP problems, respectively.

6.2. The new solving method

In this subsection, a new method for solving such problems is proposed by integrating the methods applied in the preceding sections as well as all the rules used in interval programming, the following problem can be used to solve the IT2FLP problem with vagueness in OFV,TCs and RsV:

$$\begin{aligned} & \max \sum_{j=1}^{n}\left(\underline{c}_i^{\vee} + \alpha\left(\underline{c}_i^{\wedge} - \underline{c}_i^{\vee}\right)\right) x_j \\ & \text{s.t.} \sum_{j=1}^{n}\left(\left[\frac{\bar{a}_{ij}^{\vee} + \underline{a}_{ij}^{\vee}}{2}\right] + \left[\underline{\Delta}_{ij}\right]\alpha\right) x_j + \underline{\Delta}_i \alpha - \overline{\Delta}_i - \overline{b}_i^{\wedge} \leq 0 \qquad \text{i} = 1, 2, \ldots, \text{m}, \\ & \alpha \in [0,1], \quad x_j \geq 0, \qquad j = 1, 2, \ldots, n. \end{aligned}$$

The above problem is a nonlinear, and an arbitrary value to $\alpha \in [0,1]$ is assigned to solve it.

6.3 Numerical Example

A logistics company wants to optimize its fleet operations to minimize transportation costs. However, the costs and travel times are uncertain due to fluctuating fuel prices and traffic conditions. How can the company determine the optimal allocation of vehicles across routes using interval type-2 fuzzy linear programming (IT2FLP) with vagueness in both the objective function vector (OFV), technological coefficients (TCs), and resource constraints?

1. **Objective Function Costs ($\boldsymbol{c}^{\vee}, \boldsymbol{c}^{\wedge}$):** Costs associated with three routes (x_1, x_2, x_3):
   Lower bounds ($\boldsymbol{c}^{\vee}$): [200, 300, 150].
   Upper bounds ($\boldsymbol{c}^{\wedge}$): [250, 350, 200].
2. **Technological Coefficient Matrix ($\boldsymbol{a}^{\vee}, \boldsymbol{a}^{\wedge}$):** Resource requirements (fuel consumption) for each route:
   Lower bounds ($\boldsymbol{a}^{\vee}$): $a^{\vee} = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

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Upper bounds ($\boldsymbol{a}^{\wedge}$): $a^{\wedge} = \begin{bmatrix} 4 & 5 & 3 \\ 6 & 7 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

3. **Resource Constraints ($\boldsymbol{b}^{\vee}$, $\boldsymbol{b}^{\wedge}$):** Total available resources (fuel, drivers, and vehicles):
   Lower bounds($\boldsymbol{b}^{\vee}$): [100, 150, 60].
   Upper bounds ($\boldsymbol{b}^{\wedge}$): [120, 170, 80].

Solution Using IT2FLP

1. **Model the Problem:**
   Minimize the total cost:

$$\max \quad \sum_{j=1}^{n}\left(\underline{c}_i^{\vee} + \alpha\left(\underline{c}_i^{\wedge} - \underline{c}_i^{\vee}\right)\right)x_j$$

$$\text{s.t.}\sum_{j=1}^{n}\left(\left[\frac{\overline{a}_{ij}^{\vee}+\underline{a}_{ij}^{\vee}}{2}\right] + \left[\underline{\Delta}_{ij}\right]\alpha\right)x_j + \underline{\Delta}_i\alpha - \overline{\Delta}_i - \overline{b}_i^{\wedge} \leq 0 \qquad i = 1, 2, \ldots, m,$$

$$\alpha \in [0,1], \quad x_j \geq 0, \qquad j = 1, 2, \ldots, n.$$

2. **Optimization Approach:**
   Apply α-cuts to solve the problem for varying α values, transforming it into a series of linear programming problems.

Solution

The optimization results for varying α\alphaα values are summarized in the table below:
For α=0 the simplex method is as follows:
Max z = 200x_1 + 300x_2 + 150x_3
Subject to
3.5x_1+4.5x_2+2.5x_3≤140
5.5x_1+6.5x_2+3.5x_3≤210
2.5x_1+3.5x_2+1.5x_3≤90
x_1,x_2, x_3 ≥0;
The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate and solve this by simplex method. In 3rd iteration the optimal solution is arrived with the value of variables as:
x_1=0, x_2=7.5, x_3=42.5 Max z = 8625

Similarly, calculations made for each α and shown in the following table:

α	x_1^* (Route 1)	x_2^* (Route 2)	x_3^* (Route 3)	z^*(Total Cost)
0.0	0	7.5	42.5	8625
0.1	0	5.3	43.7	8390
0.2	0	3.2	44.8	8160
0.3	0	1.2	45.8	7935
0.4	0	0	45.26	7694.74
0.5	0	0	42.5	7437.5
0.6	0	0	40	7200
0.7	0	0	37.72	6979.55
0.8	0	0	35.65	6773.91
0.9	0	0	33.75	6581.25
1.0	0	0	32	6400

Result:
At α=1, the company achieves the minimum total cost of z* = 8625 by allocating vehicles to routes as $\boldsymbol{x_1^*}$= 0, $\boldsymbol{x_2^*}$= 7.5, and$\boldsymbol{x_3^*}$ **= 42.5**. The company can adjust the α value based on the confidence level in cost and resource data, balancing uncertainty and efficiency.

VII. RESULT

Three real-life examples were analyzed:

1. A **furniture production company** addressing uncertainties in profit margins (vagueness in OFV).
2. A **construction company** optimizing resource allocation under uncertain technological coefficients (vagueness in TCs).
3. A **logistics company** minimizing transportation costs with combined uncertainties in costs, technological coefficients, and resource availability (vagueness in OFV, RsV, and TCs).

The results highlight IT2FLP's effectiveness in modelling and solving problems with multi-dimensional uncertainty. The proposed solution methodologies, based on α-cuts and interval programming, enable decision-makers to navigate uncertainty systematically by exploring optimal solutions at varying confidence levels. This approach bridges the gap between theoretical developments and practical applications, making IT2FLP a versatile tool for diverse industries.

VIII. CONCLUSION AND FUTURE WORK

This study explores the application of Interval Type-2 Fuzzy Linear Programming (IT2FLP) to address optimization problems under uncertainty, focusing on vagueness in the Objective Function Vector (OFV), Resource Vector (RsV), and Technological Coefficients (TCs). Through detailed theoretical formulations and practical examples, the study demonstrates IT2FLP's ability to provide robust and adaptive solutions for real-world scenarios.

Future research could extend these methods to multi-objective optimization, integrate IT2FLP with advanced computational techniques such as machine learning, and explore its application in emerging fields like sustainable resource management and resilient logistics networks.

By advancing the theoretical and practical aspects of IT2FLP, this study establishes a foundation for its broader adoption in tackling complex optimization challenges under uncertainty.

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