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Arithmetic Mean of Arithmetic Means of Possible Subsets of a Set of Real Numbers

Dhritikesh Chakrabarty

Independent Researcher, Ex Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: One more mathematical property of arithmetic mean which states that the arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, has mathematically been proved since no research publication on the proof of this property has been found available.

KEYWORDS: Real Numbers, Set, Possible Susets, Arithmetic Mean

I. INTRODUCTION

Many researches/studies had been done on average [1 , 48] which is an entity that describes a set of many entities. Pythagoras, the pioneer of developing measures of average, derived three measures of average namely arithmetic mean [2 , 6 , 55], geometric mean [2 , 6] and harmonic mean [2 , 6 , 54] popularly known as "Pythagorean Means" [3 , 7 , 15]. Later on, a number of definitions / formulations of average had been derived due to necessity of handling different situations. Some of them are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p mean* & generalized p^{th} root mean etc. [8 , 32]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 − 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are arithmetic-geometric mean, arithmetic-harmonic mean, geometric-harmonic mean and arithmetic-geometric-harmonic respectively [19 , 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of arithmetic mean, geometric mean & harmonic mean [2 , 3 , 6 , 39 , 40 , 42 – 44 , 46 , 54 , 55]. Arithmetic mean, geometric mean & harmonic mean have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 − 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 – 35 , 38 , 41 , 51 , 52]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of arithmetic mean which states that the arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, has here been mathematically proved since no research publication on the proof of this property has been found available.

II. ARITHMETIC MEAN OF A SET OF ELEMENTS

Let us first mention the definition of arithmetic mean of a set of real numbers.

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Vol. 12, Issue 6, June 2025

Definition

Let us consider a set of n real numbers namely

$$x_1 , x_2 , \ \dots\dots\dots \ , x_n$$

Then the *arithmetic mean* $\boldsymbol{A} = \boldsymbol{A}(x_1 , x_2 , \ \dots\dots\dots \ , x_n)$ of them is given by

$$\boldsymbol{A} = \boldsymbol{A}(x_1 , x_2 , \ \dots\dots\dots \ , x_n) = \frac{1}{n}(x_1 + x_2 + \dots\dots\dots + x_n)$$

Note:

The definition of *arithmetic mean* implies that

$$(x_1 + x_2 + \dots\dots\dots + x_n) = n\,\boldsymbol{A}(x_1 , x_2 , \ \dots\dots\dots \ , x_n)$$

i.e. the sum of n real numbers is n times of the *arithmetic mean* of the numbers.

III. ARITHMETIC MEAN OF ARITHMETIC MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Suppose, a set S consists of the N real numbers

$$a_1 , a_2 , \ \dots\dots\dots \ , a_N$$

as elements so that

$$\text{Sum of the } N \text{ elements of } S = a_1 + a_2 + \dots\dots\dots + a_N$$

$$\text{\& Arithmetic Mean of the } N \text{ elements of } S = \frac{1}{\mathrm{N}}(a_1 + a_2 + \dots\dots\dots + a_N)$$

Let us consider the possible subsets of S having n elements in each set.
The number of such possible subsets is ${}^{N}C_n$

where $\quad {}^{N}C_n = N(N-1)\,(N-2)\,\dots\dots\,(N-n+1) = \dfrac{N!}{n!(N-n)!}$

Among the ${}^{N}C_n$ possible subsets, there are

$${}^{N-1}C_{n-1} \text{ number of subsets having } a_1 \text{ as } 1^{\text{st}} \text{ element,}$$

$${}^{N-2}C_{n-2} \text{ number of subsets having } a_2 \text{ as } 1^{\text{st}} \text{ element and not having } a_1 ,$$

$${}^{N-3}C_{n-3} \text{ number of subsets having } a_3 \text{ as } 1^{\text{st}} \text{ element and not having } a_1 \ \& \ a_2 ,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$${}^{N-1}C_{N-2} \text{ number of subsets having } a_{N-n+2} \text{ as } 1^{\text{st}} \text{ element and not having } a_1 , a_2 , \ \dots\dots\dots \ a_{N-n+3} ,$$

$${}^{N-1}C_{N-1} \text{ number of subsets having } a_{N-n+1} \text{ as } 1^{\text{st}} \text{ element and not having } a_1 , a_2 , \ \dots\dots\dots \ a_{N-n+2} ,$$

such that

Total number of possible subsets

$$= {}^{N-1}C_{n-1} + {}^{N-2}C_{n-2} + {}^{N-3}C_{n-3} + \ \dots\dots\dots\dots + {}^{N-1}C_{N-2} + {}^{N-1}C_{N-1}$$

$$= {}^{N}C_n$$

and that each a_i appears a total of ${}^{N-1}C_{n-1}$ times in the set containing all possible ${}^{N}C_n$ subsets.

Suppose,

$$\overline{A}(1) \ , \ \overline{A}(2) \ , \ \overline{A}(3) \ , \ \dots\dots\dots \ , \ \overline{A}(N_{C_n} - 1) \ , \ \overline{A}(N_{C_n})$$

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are the arithmetic means of the respective subsets
and

$$S(1)\ ,\ S(2)\ ,\ S(3)\ ,\ \ldots\ldots\ldots\ ,\ S(N_{C_n}-1)\ ,\ S(N_{C_n})$$

are the sums of the respective elements of the respective subsets
so that

$$S(1)=n\,\overline{A}(1)\ ,$$
$$S(2)=n\,\overline{A}(2)\ ,$$
$$S(3)=n\,\overline{A}(3)\ ,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$S(N_{C_n}-1)=n\,\overline{A}(N_{C_n}-1)\ ,$$
$$S(N_{C_n})=n\,\overline{A}(N_{C_n})$$

Now,

$$S(1)\ +\ S(2)\ +\ S(3)\ +\ \ldots\ldots\ldots\ +\ S(N_{C_n}-1)\ +\ S(N_{C_n})$$

is the sum of all elements in the set containing all possible ${}^{N}C_{n}$ subsets of the original set S where each a_i appears a total of ${}^{N-1}C_{n-1}$ times.
Therefore,

$$S(1)\ +\ S(2)\ +\ S(3)\ +\ \ldots\ldots\ldots\ +\ S(N_{C_n}-1)\ +\ S(N_{C_n})$$
$$=\ {}^{N-1}C_{n-1}\,(a_1+a_2+\ldots\ldots\ldots+a_N)$$

Accordingly,

AM of the AMs of the respective possible ${}^{N}C_{n}$ subsets

$$=\ \frac{1}{N_{C_n}}\{\overline{A}(1)\ +\ \overline{A}(2)\ +\ \overline{A}(3)\ +\ \ldots\ldots\ldots+\ \overline{A}(N_{C_n}-1)\ +\ \overline{A}(N_{C_n})\}$$

$$=\ \frac{1}{N_{C_n}}\ \frac{1}{n}\{S(1)\ +\ S(2)\ +\ S(3)\ +\ \ldots\ldots\ldots\ +\ S(N_{C_n}-1)\ +\ S(N_{C_n})\}$$

$$=\ \frac{1}{N_{C_n}}\ \frac{1}{n}\ {}^{N-1}C_{n-1}\,(a_1+a_2+\ldots\ldots\ldots+a_N)$$

$$=\ \frac{1}{N}\ (a_1+a_2+\ldots\ldots\ldots+a_N)$$

= AM of the elements of S

IV. ARITHMETIC MEAN OF ARITHMETIC MEANS OF All NON-EMPTY POSSIBLE SUBSETS

Now, the set S has a total of 2^n non-empty subsets of which

number of possible subsets having single element in each is ${}^{N}C_{1}$,
number of possible subsets having 2 elements in each is ${}^{N}C_{2}$,

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number of possible subsets having $n-1$ elements in each is ${}^{N}C_{n-1}$,
number of possible subsets having n elements is in each ${}^{N}C_{n}$

such that

Total number of all possible non-empty subsets

$$= {}^{N}C_1 + C_2 + {}^{N}C_3 + \ldots\ldots\ldots. + {}^{N}C_{n-1} + {}^{N}C_n$$
$$= 2^n - 1$$

By the results obtained in section 3,

AM of the AMs of the respective possible subsets having single element in each
= AM of the elements of S ,

AM of the AMs of the respective possible subsets having 2 elements in each
= AM of the elements of S ,

………………………………………………………………………………………

AM of the AMs of the respective possible subsets having $n-1$ elements in each
= AM of the elements of S ,

AM of the AMs of the respective possible subsets having n elements in each
= AM of the elements of S .

Therefore,

AM of the AMs of all respective possible non-empty subsets of S = AM of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set P of five real numbers

$$P = \{2 , 4 , 6 , 8 , 10\}$$

so that

AM of the elements of $S = 6$

Now, $5\ C_1 = 5$ possible subsets of P having single element are

$$\{2\} , \{4\} , \{6\} , \{8\} , \{10\}$$

Corresponding 5 AMs of the element if the respective subsets are

$$2 , 4 , 6 , 8 , 10$$

and the AM of these 5 AMs is 6 which is the AM of the elements of S.

Similarly, ${}^{5}C_2 = 10$ possible subsets of P having 2 elements are

$$\{2 , 4\} , \{2 , 6\} , \{2 , 8\} , \{2 , 10\} , \{4 , 6\} , \{4 , 8\} , \{4 , 10\} , \{6 , 8\} , \{6 , 10\} , \{8 , 10\}$$

Corresponding 10 AMs of the element if the respective subsets are

$$3 , 4 , 5 , 6 , 5 , 6 , 7 , 7 , 8 , 9$$

and the AM of these 10 AMs is also 6 which is the AM of the elements of S.

Again, ${}^{5}C_{3} = 10$ possible subsets of P having 3 elements are

$$\{2 , 4 , 6\} , \{2 , 4 , 8\} , \{2 , 4 , 10\} , \{2 , 6 , 8\} , \{2 , 6 , 10\} , \{2 , 8 , 10\} , \{4 , 6 , 8\} , \{4 , 6 , 10\} , \{4 , 8 , 10\} , \{6 , 8 , 10\}$$

Corresponding 10 AMs of the element if the respective subsets are

$$4 , 14/3 , 16/3 , 16/3 , 6 , 20/3 , 6 , 20/3 , 22/3 , 8$$

and the AM of these 10 AMs is also 6 which is the AM of the elements of S.

Moreover, ${}^{5}C_{4} = 5$ possible subsets of P having 4 elements are

$$\{2 , 4 , 6 , 8\} , \{2 , 4 , 6 , 10\} , \{2 , 4 , 8 , 10\} , \{2 , 6 , 8 , 10\} , \{4 , 6 , 8 , 10\}$$

Corresponding 10 AMs of the element if the respective subsets are

$$5 , 5.5 , 6 , 6.5 , 6 , 7$$

and the AM of these 10 AMs is also 6 which is the AM of the elements of S.

Moreover, ${}^{5}C_{5} = 1$ possible subset of P having 5 elements is

$$\{2 , 4 , 6 , 8 , 10\}$$

AM of the elements in this subset is 6 which is the AM of the elements of S.

Finally, the AM of all the 31 AMs of the corresponding elements of the respective 31 subsets is

$$(2 + 4 + 6 + 8 + 10 + 3 + 4 + 5 + 6 + 5 + 6 + 7 + 7 + 8 + 9 + 4 + 14/3 + 16/3 + 16/3 + 6 + 20/3 + 6 + 20/3 + 22/3 + 8 + 5 + 5.5 + 6 + 6.5 + 6 + 7 + 6)/31$$

This is also equal to 6 which is the AM of the elements of S.

VI. CONCLUSION

Findings of the study can be summarized as follows:

"The arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers

and

the arithmetic mean of the arithmetic means of non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers."

The property/result of arithmetic mean obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

In this connection, it is to be mentioned that each of arithmetic mean, geometric mean harmonic mean & quadratic mean is required to be computed/estimated in similar type of situation. Therefore, there is necessity of study on whether these measures of average possess similar properties like arithmetic mean.

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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for

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securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

(Dr. Dhritikesh Chakrabarty, at the middle, with some faculties of a school in Gunotsav ceremony of the school on October10 of 2018)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists

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Vol. 12, Issue 6, June 2025

and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.