

Geometric Mean of Geometric Means of Possible Subsets of a Set of Numbers

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ABSTRACT: One mathematical property of **geometric mean** which states that the **geometric mean** of the **geometric means** of the respective possible subsets of fixed size of a set of positive real numbers is the **geometric mean** of the original set of numbers and also the **geometric mean** of the **geometric means** of the respective non-empty possible subsets a set of positive real numbers is the **geometric mean** of the original set of numbers, has mathematically been established in this article.

KEYWORDS: Real Numbers, Set, Possible Subsets, **Geometric Mean**

I. INTRODUCTION

There had been several studies on average [1, 50] which is an entity that describes a set of many entities. The great mathematician Pythagoras first developed three measures of average termed as **arithmetic mean** [2, 6, 57], **geometric mean** [2, 6] and **harmonic mean** [2, 6, 56] which together is also popularly known as “Pythagorean Means” [3, 7, 15]. Later on, a number of definitions / formulations of average were developed due to necessity of handling different situations some of which are **quadratic mean** or **root mean square**, **square root mean**, **cubic mean**, **cube root mean**, **generalized p mean** & **generalized p^{th} root mean** etc. [8, 32]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9, 16, 17, 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are **arithmetic-geometric mean**, **arithmetic-harmonic mean**, **geometric-harmonic mean** and **arithmetic-geometric-harmonic** respectively [19, 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of **arithmetic mean**, **geometric mean** & **harmonic mean** [2, 3, 6, 39, 40, 42 – 44, 46, 47, 56, 57]. **Arithmetic mean**, **geometric mean** & **harmonic mean** have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7, 15, 21 – 31, 36, 37] and in developing the statistical concept of expectation [5, 33 – 35, 38, 41, 53, 54]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of **arithmetic mean** which states that the **arithmetic mean** of the **arithmetic means** of the respective possible subsets of fixed size of a set of real numbers is the **arithmetic mean** of the original set of numbers and also the **arithmetic mean** of the **arithmetic means** of the respective non-empty possible subsets a set of real numbers is the **arithmetic mean** of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48]. Similar property of **geometric mean** which states that the **geometric mean** of the **geometric means** of the respective possible subsets of fixed size of a set of positive real numbers is the **geometric mean** of the original set of numbers and also the **geometric mean** of the **geometric means** of the respective non-empty possible subsets a set of positive real numbers is the **geometric mean** of the original set of numbers, has mathematically been established in this article.

II. GEOMETRIC MEAN OF A SET OF ELEMENTS

Let us first mention the definition of arithmetic mean of a set of real numbers.

Definition

Let us consider a set of n positive real numbers namely

$$x_1, x_2, \dots, x_n$$

Then the **geometric mean** $G = G(x_1, x_2, \dots, x_n)$ of them is given by

$$G = G(x_1, x_2, \dots, x_n) = (\prod_{i=1}^n x_i)^{1/n}$$

Let us abbreviate **geometric mean** by GM .

Note:

The definition of GM implies that

$$\prod_{i=1}^n x_i = \{ G(x_1, x_2, \dots, x_n) \}^n$$

i.e. the **product** of n positive real numbers is n^{th} power of the GM of the numbers.

III. GEOMETRIC MEAN OF GEOMETRIC MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Suppose, a set S consists of the N real numbers

$$a_1, a_2, \dots, a_N$$

as elements so that

$$\text{Product of the } N \text{ elements of } S = \prod_{i=1}^N a_i$$

$$\text{ \& } GM \text{ of the } N \text{ elements of } S = (\prod_{i=1}^N a_i)^{1/N} = G, \text{ say}$$

Let us consider the possible subsets of S having n elements in each set.

The number of such possible subsets is ${}^N C_n$

$$\text{where } {}^N C_n = N(N-1)(N-2) \dots (N-n+1) = \frac{N!}{n!(N-n)!}$$

Among the ${}^N C_n$ possible subsets, there are

$${}^{N-1} C_{n-1} \text{ number of subsets with } a_1 \text{ as 1}^{st} \text{ element,}$$

$${}^{N-2} C_{n-2} \text{ number of subsets with } a_2 \text{ as 1}^{st} \text{ element and not having } a_1,$$

$${}^{N-3} C_{n-3} \text{ number of subsets with } a_3 \text{ as 1}^{st} \text{ element and not having } a_1 \text{ \& } a_2,$$

$$\dots\dots\dots$$

$${}^{N-1} C_{N-2} \text{ number of subsets with } a_{N-n+2} \text{ as 1}^{st} \text{ element and not having } a_1, a_2, \dots, a_{N-n+3},$$

$${}^{N-1} C_{N-1} \text{ number of subsets with } a_{N-n+1} \text{ as 1}^{st} \text{ element and not having } a_1, a_2, \dots, a_{N-n+2},$$

such that

Total number of possible subsets

$$= {}^{N-1} C_{n-1} + {}^{N-2} C_{n-2} + {}^{N-3} C_{n-3} + \dots + {}^{N-1} C_{N-2} + {}^{N-1} C_{N-1} = {}^N C_n$$

and that each a_i appears a total of ${}^{N-1} C_{n-1}$ times in the set containing all possible ${}^N C_n$ subsets.

Suppose,

$$\overline{G}(1), \overline{G}(2), \overline{G}(3), \dots, \overline{G}({}^N C_n - 1), \overline{G}({}^N C_n)$$

are the GM s of the respective subsets and

$$P(1), P(2), P(3), \dots, P({}^N C_n - 1), P({}^N C_n)$$

are the sums of the respective elements of the respective subsets so that

$$P(1) = \{ \overline{G}(1) \}^n,$$

$$\begin{aligned} P(2) &= \{ \overline{G}(2) \}^n, \\ P(3) &= \{ \overline{G}(3) \}^n, \\ &\dots\dots\dots \\ P({}^N C_n - 1) &= \{ \overline{G}({}^N C_n - 1) \}^n, \\ P({}^N C_n) &= \{ \overline{G}({}^N C_n) \}^n \end{aligned}$$

Now,

$$P(1) \cdot P(2) \cdot P(3) \cdot \dots\dots\dots P({}^N C_n - 1) \cdot P({}^N C_n)$$

is the **product** of all elements in the set containing all possible ${}^N C_n$ subsets of the original set S where each a_i appears a total of ${}^{N-1} C_{n-1}$ times.

Therefore,

$$\begin{aligned} &P(1) \cdot P(2) \cdot P(3) \cdot \dots\dots\dots P({}^N C_n - 1) \cdot P({}^N C_n) \\ &= (a_1 \cdot a_2 \cdot \dots\dots\dots a_N)^{{}^{N-1} C_{n-1}} \end{aligned}$$

Accordingly,

$$\begin{aligned} &\text{GM of the GMs of the respective possible } {}^N C_n \text{ subsets} \\ &= \{ \overline{G}(1) \cdot \overline{G}(2) \cdot \overline{G}(3) \cdot \dots\dots\dots \overline{G}({}^N C_n - 1) \cdot \overline{G}({}^N C_n) \}^R, \text{ where } R = \frac{1}{{}^N C_n} \\ &= \{ P(1) \cdot P(2) \cdot P(3) \cdot \dots\dots\dots P({}^N C_n - 1) \cdot P({}^N C_n) \}^{R/n} \\ &= \{ a_1 \cdot a_2 \cdot \dots\dots\dots a_N \}^{R/Nn} \\ &= \{ a_1 \cdot a_2 \cdot \dots\dots\dots a_N \}^{1/N} \\ &= \text{GM of the elements of } S \end{aligned}$$

Therefore,

$$\text{GM of the GMs of the respective possible } {}^N C_n \text{ subsets} = \text{GM of the elements of } S$$

IV. GEOMETRIC MEAN OF GEOMETRIC MEANS OF ALL NON-EMPTY POSSIBLE SUBSETS

Now, the set S has a total of $(2^n - 1)$ number of non-empty subsets of which

$$\begin{aligned} &\text{number of possible subsets having single element in each is } {}^N C_1, \\ &\text{number of possible subsets having 2 elements in each is } {}^N C_2, \\ &\dots\dots\dots \\ &\text{number of possible subsets having } n-1 \text{ elements in each is } {}^N C_{n-1}, \\ &\text{number of possible subsets having } n \text{ elements in each is } {}^N C_n \end{aligned}$$

such that

$$\text{Total number of all possible non-empty subsets} = {}^N C_1 + {}^N C_2 + {}^N C_3 + \dots\dots\dots + {}^N C_{n-1} + {}^N C_n = 2^n - 1$$

By the results obtained in section 3,

$$\text{GM of the GMs of the respective possible subsets having 1 element in each} = \text{GM of the elements of } S = G$$

Similarly,

$$\begin{aligned} &\text{GM of the GMs of the respective possible subsets having 2 elements in each} = G, \\ &\dots\dots\dots \end{aligned}$$

GM of the GM s of the respective possible subsets having $n-1$ elements in each = G ,
 GM of the GM s of the respective possible subsets having n elements in each = G .

Therefore,

GM of the GM s of all respective possible non-empty subsets of $S = (G^n)^{1/n} = G$
 i.e. GM of the GM s of all respective possible non-empty subsets of $S = G = GM$ of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set S of five real numbers

$$S = \{2, 4, 6, 8, 10\}$$

so that

$$GM \text{ of the elements of } S = 5.2103421693947037846515338478572$$

Now, ${}^5C_1 = 5$ possible subsets of S having single element are

$$\{2\}, \{4\}, \{6\}, \{8\}, \{10\}$$

Corresponding 5 GM s of the element if the respective subsets are

$$2, 4, 6, 8, 10$$

and the GM of these 5 GM s is

$$5.2103421693947037846515338478572$$

which is the GM of the elements of S .

Similarly, ${}^5C_2 = 10$ possible subsets of P having 2 elements are

$$\{2, 4\}, \{2, 6\}, \{2, 8\}, \{2, 10\}, \{4, 6\}, \{4, 8\}, \{4, 10\}, \{6, 8\}, \{6, 10\}, \{8, 10\}$$

Corresponding 10 GM s of the element if the respective subsets are

$$2.8284271247461900976033774484194, 3.4641016151377545870548926830117, 4.0, \\ 4.4721359549995793928183473374626, 4.8989794855663561963945681494118, \\ 5.6568542494923801952067548968388, 6.3245553203367586639977870888654, \\ 6.9282032302755091741097853660235, 7.7459666924148337703585307995648, \\ 8.9442719099991587856366946749251$$

and the GM of these 10 GM s is

$$5.2103421693947037846515338478572$$

which is the GM of the elements of S .

Again, ${}^5C_3 = 10$ possible subsets of S having 3 elements are

$$\{2, 4, 6\}, \{2, 4, 8\}, \{2, 4, 10\}, \{2, 6, 8\}, \{2, 6, 10\}, \\ \{2, 8, 10\}, \{4, 6, 8\}, \{4, 6, 10\}, \{4, 8, 10\}, \{6, 8, 10\}$$

Corresponding 10 GM s of the element if the respective subsets are

$$3.6342411856642793177824235126545, 4.0, 4.3088693800637674435185871330387, \\ 4.5788569702133274712321688477587, 4.9324241486609402029832226463092, \\ 5.428835233189813143036178939359, 5.7689982812296335292865532431204, \\ 6.2144650119077177337553248550448, 6.8399037867067879574124354901754, \\ 7.8297352823377271908498721208233$$

and the GM of these 10 GM s is also

$$5.2103421693947037846515338478572$$

which is the GM of the elements of S .

Moreover, ${}^5C_4 = 5$ possible subsets of S having 4 elements are

$$\{2, 4, 6, 8\}, \{2, 4, 6, 10\}, \{2, 4, 8, 10\}, \{2, 6, 8, 10\}, \{4, 6, 8, 10\}$$

Corresponding 10 GM s of the element if the respective subsets are

4.4267276788012863696351610937613 , 4.6806946386414318769182819376333,
5.0297337187317416332710620186354 , 5.5663153674274812042150129057089 ,
6.6195018392937462100666768761159

and the GM of these 10 GM s is also

5.2103421693947037846515338478572

which is the GM of the elements of S .

Moreover, ${}^5C_5 = 1$ possible subset of S having 5 elements is

$\{2, 4, 6, 8, 10\}$

GM of the elements in this subset is

5.2103421693947037846515338478572

which is the GM of the elements of S .

Finally, the GM of all the $2^5 - 1 = 31$ GM s of the corresponding elements of the respective 31 subsets is found after computation as

5.2103421693947037846515338478572

which is the GM of the elements of S .

VI. CONCLUSION

Findings of the study can be summarized as follows:

“The geometric mean of the geometric means of the respective possible subsets of fixed size of a set of positive real numbers is the geometric mean of the original set of numbers and the geometric mean of the geometric means of non-empty possible subsets a set of positive real numbers is the geometric mean of the original set of numbers.”

The property/result of **geometric mean** obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

In this connection, it is to be mentioned that each of **arithmetic mean**, **geometric mean**, **harmonic mean** and **quadratic mean** is required to be computed/estimated in similar type of situation. Therefore, there is necessity of study on whether **harmonic mean** and **quadratic mean** possess similar property like **arithmetic mean** which was derived in earlier study [48] and like **geometric mean** which has been derived here.

REFERENCES

- [1] Bakker Arthur (2003): “The Early History of Average Values and Implications for Education”, *Journal of Statistics Education*, 11(1), 17 – 26.
- [2] Bullen P. S. (2003): “*The Arithmetic, Geometric and Harmonic Means*”, *Handbook of Means and Their Inequalities*. Dordrecht: Springer Netherlands. 60 – 174. doi:10.1007/978-94-017-0399-4_2. ISBN 978-90-481-6383-0.
- [3] Cantrell David W. “*Pythagorean Means*”. *MathWorld*.
- [4] Chai T. (2022): “Root Mean Square”, In: Daya Sagar, B.S., Cheng, Q., McKinley, J., Agterberg, F. (eds) *Encyclopedia of Mathematical Geosciences*. Encyclopedia of Earth Sciences Series. Springer, Cham. https://doi.org/10.1007/978-3-030-26050-7_280-1.
- [5] Chattamvelli R., Shanmugam R. (2024): “Mathematical Expectation”, In: *Random Variables for Scientists and Engineers*, Synthesis Lectures on Engineering, Science, and Technology. Springer, Cham. https://doi.org/10.1007/978-3-031-58931-7_1.
- [6] Coggeshall F. (1886): “The Arithmetic, Geometric, and Harmonic Means”, *The Quarterly Journal of Economics*, 1(1), 83–86. <https://doi.org/10.2307/1883111>. <https://www.jstor.org/stable/1883111>.
- [7] Dhritikesh Chakrabarty (2016): “Pythagorean Mean: Concept behind the Averages and Lot of Measures of Characteristics of Data”, *NaSAEAST- 2016, Abstract ID: CMAST_NaSAEAST (Inv)-1601*, 2016. DOI: 10.13140/RG.2.2.27022.57920.
- [8] Dhritikesh Chakrabarty (2017): “Objectives and Philosophy behind the Construction of Different Types of Measures of Average”, *NaSAEAST- 2017, Abstract ID: CMAST_NaSAEAST (Inv)- 1701*. DOI: 10.13140/RG.2.2.23858.17606.
- [9] Dhritikesh Chakrabarty (2018): “General Technique of Defining Average”, *NaSAEAST- 2018, Abstract ID: CMAST_NaSAEAST-1801 (I)*. DOI: 10.13140/RG.2.2.22599.88481.
- [10] Dhritikesh Chakrabarty (2018): “Derivation of Some Formulations of Average from One Technique of Construction of Mean”, *American Journal of Mathematical and Computational Sciences*, 3(3), 62 – 68. <http://www.aascit.org/journal/ajmcs>.
- [11] Dhritikesh Chakrabarty (2018): “One Generalized Definition of Average: Derivation of Formulations of Various Means”, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 7(3), 212 – 225. www.jecet.org. DOI: 10.24214/jecet.C.7.3.21225.
- [12] Dhritikesh Chakrabarty (2018): “ f_H - Mean: One Generalized Definition of Average”, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 7(4), 301 – 314. www.jecet.org. DOI: 10.24214/jecet.C.7.4.30114.
- [13] Dhritikesh Chakrabarty (2018): “Generalized f_G - Mean: Derivation of Various Formulations of Average”, *American Journal of Computation*,



- Communication and Control, 5(3), 101 – 108. <http://www.aascit.org/journal/ajmcs>.
- [14] Dhritikesh Chakrabarty (2019): “One Definition of Generalized f_G - Mean: Derivation of Various Formulations of Average”, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 8(2), 051 – 066. www.jecet.org. DOI: 10.24214/jecet.C.8.2.05166.
- [15] Dhritikesh Chakrabarty (2019): “Pythagorean Geometric Mean: Measure of Relative Change in a Group of Variables”, *NaSAEAST- 2019, Abstract ID: CMAST_NaSAEAST-1902 (I)*. DOI: 10.13140/RG.2.2.29310.77124 .
- [16] Dhritikesh Chakrabarty (2019): “One General Method of Defining Average: Derivation of Definitions/Formulations of Various Means”, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Sec. C, 8(4), 327 – 338, www.jecet.org. DOI: 10.24214/jecet.C.8.4.32738 .
- [17] Dhritikesh Chakrabarty (2019): “A General Method of Defining Average of Function of a Set of Values”, *Aryabhatta Journal of Mathematics & Informatics*, 11(2), 269 – 284. www.abjni.com .
- [18] Dhritikesh Chakrabarty (2020): “Definition / Formulation of Average from First Principle”, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Sec C, 9(2), 151 – 163. www.jecet.org . DOI: 10.24214/jecet.C.9.2.15163.
- [19] Dhritikesh Chakrabarty (2021): “Four Formulations of Average Derived from Pythagorean Means”, *International Journal of Mathematics Trends and Technology*, 67(6), 97 – 118. <http://www.ijmtjournal.org> . doi:10.14445/22315373/IJMTT-V67I6P512 .
- [20] Dhritikesh Chakrabarty (2021): “Recent Development on General Method of Defining Average: A Brief Outline”, *International Journal of Advanced Research in Science, Engineering and Technology*, 8(8), 17947 – 17955. www.ijarset.com.
- [21] Dhritikesh Chakrabarty (2021): “Measuremental Data: Seven Measures of Central Tendency”, *International Journal of Electronics and Applied Research*, 8(1), 15 – 24. www.eses.net.in . DOI: 10.33665/IJEAR.2021.v08i01.002 .
- [22] Dhritikesh Chakrabarty (2022): “AGM, AHM, GHM & AGH: Measures of Central Tendency of Data”, *International Journal of Electronics and Applied Research*, 9(1), 1 – 26. http://eses.net.in/online_journal.html .
- [23] Dhritikesh Chakrabarty (2022): “Logical Derivation of AHM as a Measure of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 10, 2022. DOI: 10.13140/RG.2.2.28852.01929.
- [24] Dhritikesh Chakrabarty (2022): “Logical Derivation of Arithmetic-Geometric Mean as a Measure of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 11, 2022. DOI: 10.13140/RG.2.2.22141.13282.
- [25] Dhritikesh Chakrabarty (2022): “Logical Derivation of Geometric-Harmonic Mean as a Measure of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 12, 2022. DOI: 10.13140/RG.2.2.35562.90565.
- [26] Dhritikesh Chakrabarty (2022): “Logical Derivation of Arithmetic-Geometric-Harmonic Mean as a Measure of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 13, 2022. DOI: 10.13140/RG.2.2.11235.94245.
- [27] Dhritikesh Chakrabarty (2022): “Geometric Mean of Arithmetic Mean and Harmonic Mean: A Measure of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 14, 2022. DOI: 10.13140/RG.2.2.18785.68968.
- [28] Dhritikesh Chakrabarty (2022): “Second Derivation of AGM, AHM, GHM & AGHM as Measures of Central Tendency”, Unpublished Research Paper, Uploaded in Research Gate on June 16, 2022. DOI: 10.13140/RG.2.2.12074.80329.
- [29] Dhritikesh Chakrabarty (2022): “Arithmetic-Geometric Mean and Central Tendency of Sex Ratio”, Unpublished Research Paper, Uploaded in Research Gate on June 17, 2022. DOI: 10.13140/RG.2.2.20463.41123
- [30] Dhritikesh Chakrabarty (2022): “Arithmetic-Harmonic Mean and Central Tendency of Sex Ratio”, Unpublished Research Paper, Uploaded in Research Gate on July 27, 2022. DOI: 10.13140/RG.2.2.27174.29761.
- [31] Dhritikesh Chakrabarty (2022): “Central Tendency of Sex Ratio in India: Estimate by AGM”, Unpublished Research Paper, Uploaded in Research Gate on August 21, 2022. DOI: 10.13140/RG.2.2.30529.74088.
- [32] Dhritikesh Chakrabarty (2022): “A Brief Review on Formulation of Average”, Unpublished Research Paper, Uploaded in Research Gate on September 03, 2022. DOI: 10.13140/RG.2.2.17107.96807/1.
- [33] Dhritikesh Chakrabarty (2024): “Idea of Arithmetic, Geometric and Harmonic Expectations”, *Partners Universal International Innovation Journal (PUIJ)*, (ISSN: 2583-9675), 02(01), 119 – 124. www.puij.com . DOI: 10.5281/zenodo.10680751.
- [34] Dhritikesh Chakrabarty (2024): “Arithmetic, Geometric and Harmonic Expectations: Expected Rainy Days in India”, *Partners Universal International Research Journal (PUIRJ)*, (ISSN: 2583-5602), 03(01), 119 – 124. www.puirj.com . DOI:10.5281/zenodo.10825829.
- [35] Dhritikesh Chakrabarty (2024): “Beautiful Multiplicative Property of Geometric Expectation”, *Partners Universal International Innovation Journal (PUIJ)*, (ISSN: 2583-9675), 02(02), 92 – 98. www.puij.com . DOI: 10.5281/zenodo.10999414.
- [36] Dhritikesh Chakrabarty (2024): “Average: A Basis of Measures of Dispersion of Data”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 11(7), 22053 – 22061. www.ijarset.com .
- [37] Dhritikesh Chakrabarty (2024): “Measure of Variation in Data of Ratio Type: Standard Multiplicative Deviation”, *Partners Universal International Research Journal (PUIRJ)*, (ISSN: 2583-5602), 03(03), 110 – 119. www.puirj.com. DOI:10.5281/zenodo.13827583.
- [38] Dhritikesh Chakrabarty (2024): “Rhythmic Additive Property of Harmonic Expectation”, *Partners Universal International Innovation Journal (PUIJ)*, (ISSN: 2583-9675), 02(05), 37 – 42. www.puij.com. DOI:10.5281/zenodo.13995073.
- [39] Dhritikesh Chakrabarty (2024): “Additive Property of Harmonic Mean”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 11(10), 22389 – 22396. www.ijarset.com .
- [40] Dhritikesh Chakrabarty (2024): “Multiplicative Property of Geometric Mean”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 11(11), 22534 – 22541. www.ijarset.com .
- [41] Dhritikesh Chakrabarty (2024): “Additive Property of Harmonic Expectation From That of Arithmetic Expectation”, *Partners Universal International Innovation Journal (PUIJ)*, 2(6), 24 – 30. www.puij.com . <https://doi.org/10.5281/zenodo.14629929> .
- [42] Dhritikesh Chakrabarty (2024): “Additive Property of Harmonic Mean from that of Arithmetic Mean”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 11(12), 22668 – 22767. www.ijarset.com .
- [43] Dhritikesh Chakrabarty (2025): “Multiplicative Property of Geometric Mean: Second Proof”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 12(1), 22771 – 22778. www.ijarset.com.
- [44] Dhritikesh Chakrabarty (2025): “Combined Set of Several Sets of Observations: Harmonic Mean”, *Partners Universal International Innovation Journal (PUIJ)*, (ISSN: 2583-9675), 3(1), 49 – 53. www.puij.com . DOI:10.5281/zenodo.14949601.
- [45] Dhritikesh Chakrabarty (2025): “Some Properties of Quadratic Mean”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 12(4), 23303 – 23310. www.ijarset.com.
- [46] Dhritikesh Chakrabarty (2025): “Combined Set of Several Sets of Observations: Quadratic Mean”, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 12(5), 23444 – 23452. www.ijarset.com.
- [47] Dhritikesh Chakrabarty (2025): “Combined Set of Several Sets of Observations: Geometric Mean”, *Partners Universal International Innovation Journal (PUIJ)*, 3(3), 69 – 73. www.puij.com. <https://doi.org/10.5281/zenodo.15757409>.



- [48] Dhritikesh Chakrabarty (2025): "Arithmetic Mean of Arithmetic Means of Possible Subsets of a Set of Real Numbers", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN: 2350 – 0328), 12(6), 23540 – 23548. www.ijarset.com.
- [49] HELM (2008): "The Mean Value and the Root-Mean-Square Value", Workbook 14, Section 14.2, 10 – 19. <https://www.sheffield.ac.uk/media/download>.
- [50] Miguel de Carvalho (2016): "Mean, what do you Mean?", *The American Statistician*, , 70, 764 – 776.
- [51] Oliviero Carugo (2007): "Statistical Validation of the Root-Mean-Square-Distance, A Measure of Protein Structural Proximity", *Protein Engineering, Design and Selection*, 20(1), 33 – 37. <https://doi.org/10.1093/protein/gzl051>.
- [52] Pavel Polasek (1979): "The Significance of the Root Mean Square Velocity Gradient and Its Calculation in Devices for Water Treatment", *Water SA*, 5(4), 196 – 207.
- [53] Pfeiffer P.E. (1990): "Mathematical Expectation", In: *Probability for Applications. Springer Texts in Statistics*, Springer, New York, NY. https://doi.org/10.1007/978-1-4615-7676-1_15.
- [54] Yadav S. K., Singh S., Gupta R. (2019): "Random Variable and Mathematical Expectation", In: *Biomedical Statistics*, Springer, Singapore. https://doi.org/10.1007/978-981-32-9294-9_26.
- [55] Weisstein, Eric W. "Root-Mean-Square". [MathWorld](http://MathWorld.com).
- [56] Weisstein Eric W (2003): "Harmonic Mean", mathworld.wolfram.com.
- [57] Weisstein Eric W (2020): "Arithmetic Mean", mathworld.wolfram.com.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class & 1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (in Vocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag

Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.



(Dr. Dhritikesh Chakrabarty, at the middle, distributing certificate to the paper presenters in NaSAEAST-2017 organized by Electronic Scientists and Engineers' Society (ESES) Assam, during June 26 – 28, 2017)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.