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Cube Root Mean of Cube Root Means of Possible Subsets of a Set of Numbers

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ABSTRACT: Relation between the cube root mean of the elements of a set of real numbers and the cube root means of the respective elements of the respective possible subsets of the set has mathematically been derived in this study. Derivation of the relation has been presented, in this article, along with numerical example.

KEYWORDS: Set, Real Numbers, Possible Subsets, Cube Root Mean

I. INTRODUCTION

Several researches had already been done on average [1 , 55] since the introduction of its concept. As per the history of development of theory of average is known, Pythagoras was the first mathematician to introduce the concept of measure of average and consequently to develop three measures of average termed as arithmetic mean [2 , 6 , 61], geometric mean [2 , 6] and harmonic mean [2 , 6 , 61] which together is also popularly known as "Pythagorean Means" [3 , 7 , 15]. Later on, a number of definitions / formulations of average were developed due to necessity of handling different situations some of which are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p* mean & generalized p^{th} root mean etc. [4 , 8 , 18 , 32 , 45 , 54 , 56 , 57 , 60]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. In another study, four formulations of average were derived from the three Pythagorean means which are arithmetic- geometric mean, arithmetic- harmonic mean, geometric - harmonic mean and arithmetic-geometric - harmonic respectively [19 , 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on the properties of arithmetic mean, geometric mean, harmonic mean & quadratic mean [2 , 3 , 6 , 39 , 40 , 42 – 47 , 61 , 62]. Arithmetic mean, geometric mean & harmonic mean have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 – 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 – 35 , 38 , 41 , 58 , 59]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of arithmetic mean which states that the arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48]. Similar properties of geometric mean, harmonic mean, quadratic mean, square root mean and cubic mean have also been mathematically established in separate studies [49 , 50 , 51 , 52 , 53]. The Similar property of cube root mean has mathematically been derived in this study. Derivation of the property has been presented, in this article, along with numerical example.

II. SET OF REAL NUMBERS: CUBE ROOT MEAN

Let us first mention the definition of cube root mean of a set of real numbers.

Definition: Let us consider a set of n positive real numbers namely

$$x_1 , x_2 , \ \ldots\ldots\ldots \ , x_n$$

Then the cube root mean $C_R = C(x_1 , x_2 , \ \ldots\ldots\ldots \ , x_n)$ of them is given by

$$C_R = C_R(x_1 , x_2 , \ \ldots\ldots\ldots \ , x_n) \ = \ \{\frac{1}{n}(x_1{}^{1/3} + x_2{}^{1/3} + \ldots\ldots + x_n{}^{1/3})\}^3$$

Let us abbreviate cube root mean by CRM.

Note: The definition of CRM implies that

$$x_1{}^{1/3} + x_2{}^{1/3} + \ldots\ldots + x_n{}^{1/3} \ = n.\{C_R(x_1 , x_2 , \ \ldots\ldots\ldots \ , x_n)\}^{1/3}$$

i.e. the *sum of cube roots of n positive real numbers is* n times of the *cube root* of the CRM of the numbers.

III. POSSIBLE SUBSETS OF FIXED SIZE: CUBE ROOT MEAN OF CUBE ROOT MEANS

Suppose, a set S consists of the N real numbers

$$a_1 , a_2 , \ \ldots\ldots\ldots \ , a_N$$

as elements so that

$$\textit{Sum of cube roots of the } N \text{ elements of } S = a_1{}^{1/3} + a_2{}^{1/3} + \ldots\ldots + a_N{}^{1/3}$$

& CRM of the N elements of $S = \{\frac{1}{N}(a_1{}^{1/3} + a_2{}^{1/3} + \ldots\ldots + a_N{}^{1/3})\}^3 = \boldsymbol{R_0}$, say

Let us consider the possible subsets of S having n elements in each set.
The number of such possible subsets is $C(N , n)$

where $\quad C(N , n) = {}^N C_n = \binom{N}{n} = \dfrac{N!}{n!(N-n)!}$

Among the $C(N , n)$ possible subsets, there are

$C(N - 1 , n - 1)$ subsets with a_1 as 1st element,

$C(N - 2 , n - 2)$ subsets with a_2 as 1st element and not having a_1 ,

$C(N - 3 , n - 3)$ subsets with a_3 as 1st element and not having a_1 & a_2 ,

……………………………………………………………………………………

$C(N - 1 , N - 2)$ subsets with a_{N-n+2} as 1st element and not having $a_1 , a_2 , \ \ldots\ldots\ldots a_{N-n+3}$,

$C(N - 1 , N - 1)$ subsets with a_{N-n+1} as 1st element and not having $a_1 , a_2 , \ \ldots\ldots\ldots a_{N-n+2}$,

such that

Total number of possible subsets

$= C(N - 1 , n - 1) + C(N - 2 , n - 2) + C(N - 3 , n - 3) + \ \ldots\ldots\ldots.. + C(N - 1 , N - 2) + C(N - 1 , N - 1) \ = C(N , n)$

and that each a_i appears a total of $C(N - 1 , n - 1)$ times in the set containing all possible $C(N , n)$ subsets.
Suppose,

$$C_1 , C_2 , C_3 , \ \ldots\ldots\ldots \ , C_{C(N,\ n)-1} , C_{C(N,\ n)}$$

are the CRMs of the respective subsets and

$$S_1 , S_2 , S_3 , \ \ldots\ldots\ldots \ , S_{C(N,\ n)-1} , S_{C(N,\ n)}$$

are the *sums of cube roots* of the respective n elements in the respective subsets so that

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$S_1 = n\, C_1^{1/3}$,
$S_2 = n\, C_2^{1/3}$,
$S_3 = n\, C_3^{1/3}$,
………………
$S_{C(N,\ n)-1} = n\, C_{C(N,\ n)-1}^{\ 1/3}$,
$S_{C(N,\ n)} = n\, C_{C(N,\ n)}^{\ 1/3}$.

Now,

$$S_1 + S_2 + S_3 + \ \ldots\ldots\ldots \ + S_{C(N,\ n)-1} + S_{C(N,\ n)}$$

is the sum of cube roots of all elements in the set containing the elements of all possible $C(N , n)$ subsets of the original set S where each a_i appears a total of $C(N - 1 , n - 1)$ times.

Therefore,

$$S_1 + S_2 + S_3 + \ldots\ldots\ldots + S_{C(N,\ n)-1} + S_{C(N,\ n)} = C(N - 1 , n - 1)\, (a_1^{1/3} + a_1^{1/3} + \ldots\ldots\ldots + a_N^{1/3})$$

Accordingly,

CRM of the *CRM*s of the respective elements of the respective possible $C(N , n)$ subsets

$$= \{\frac{1}{C(N,n)}\, (C_1^{1/3} + C_2^{1/3} + C_3^{1/3} + \ \ldots\ldots\ldots \ + C_{C(N,\ n)-1}^{\ 1/3} + C_{C(N,\ n)}^{\ 1/3})\}^3$$

$$= \{\frac{1}{n\, C(N,n)}\, \frac{1}{n}\, (S_1 + S_2 + S_3 + \ \ldots\ldots\ldots \ + S_{C(N,\ n)-1} + S_{C(N,\ n)}\ )\}^{1/3}$$

$$= \{\{\frac{1}{n\, C(N,n)}\, \frac{1}{n}\, C(N - 1 , n - 1)\, (a_1^3 + a_1^3 + \ldots\ldots + a_N^3)\}^{1/3}$$

$$= \frac{1}{N}\, (a_1^{1/3} + a_1^{1/3} + \ldots\ldots + a_N^{1/3})\}^3$$

$= \boldsymbol{R_0} =$ *CRM* of the elements of S

Therefore,

CRM of the *CRM*s of the respective possible $C(N , n)$ subsets = *CRM* of the elements of S

IV. POSSIBLE NON-EMPTY SUBSETS: CUBE ROOT MEAN OF CUBE ROOT MEANS

Now, the set S has a total of $(2^n - 1)$ number of non-empty subsets of which

number of possible subsets having single element in each is $C(N , 1)$,
number of possible subsets having 2 elements in each is $C(N , 2)$,
……………………………………………………
number of possible subsets having $n - 1$ elements in each is $C(N , n - 1)$,
number of possible subsets having n elements is in each $C(N , n)$

such that

Total number of all possible non-empty subsets

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$= C(N, 1) + C(N, 2) + \ldots\ldots\ldots + C(N, n-1) + C(N, n) = 2^n - 1$

By the results obtained in **section III**,

CRM of the *CRM*s of the respective elements of the respective possible subsets having 1 element in each = *CRM* of the elements of S = $\boldsymbol{R_0}$

Similarly,

CRM of the *CRM*s of the respective elements of the respective possible subsets having 2 elements in each = $\boldsymbol{R_0}$,

……………………………………………………………………………………

CRM of the *CRM*s of the respective elements of the respective possible subsets having n -1 elements in each = $\boldsymbol{R_0}$,

CRM of the *CRM*s of the respective elements of the respective possible subsets having n elements in each = $\boldsymbol{R_0}$.

Therefore,

CRM of the *CRM*s of the respective elements of the respective non-empty possible subsets of S = $\boldsymbol{R_0}$

i.e. *CRM* of the *CRM*s of the respective elements of the respective non-empty possible subsets of S is *SRM* of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set S of five real numbers

$$S = \{2 , 4 , 6 , 8 , 10\}$$

so that

CRM of the elements of S = 5.48693606828904711082979002010 2

Now, ${}^5C_1$ = 5 possible subsets of S having single element are

{2} , {4} , {6} , {8} , {10}

Corresponding 5 *CRM*s of the elements in the respective subsets are

2 , 4 , 6 , 8 , 10

and the *CRM* of these 5 *CRM*s is 5.48693606828904711082979002010 2 .
which is the *CRM* of the elements in S.

Similarly, ${}^5C_2$ = 10 possible subsets of S having 2 elements are

{2 , 4} , {2 , 6} , {2 , 8} , {2 , 10} , {4 , 6} , {4 , 8} , {4 , 10} , {6 , 8} , {6 , 10} , {8 , 10}

Corresponding 10 *CRM*s of the elements in the respective subsets are

2.8854915763973044796391871849129 , 3.6417500450194843726387713513535 ,
4.3304323638184593532145951403716 , 4.9754952636671722911449221745113 ,
4.9326274094866702570683092782 41 , 5.7709831527946089592783743698258 ,
6.5488368364264698829734342274445 , 6.9521263259191795012427750987977 ,
7.8305424732491067651760275344 52 , 8.9628436602574097519464976129686

and the *CRM* of these 10 *CRM*s is 5.48693606828904711082979002010 2 .
which is the *SRM* of the elements in S.

Again, ${}^5C_3$ = 10 possible subsets of S having 3 elements are

{2 , 4 , 6} , {2 , 4 , 8} , {2 , 4 , 10} , {2 , 6 , 8} , {2 , 6 , 10} ,
{2 , 8 , 10} , {4 , 6 , 8} , {4 , 6 , 10} , {4 , 8 , 10} , {6 , 8 , 10}

Corresponding 10 *CRM*s of the elements in the respective subsets are

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3.7586814578227166215357290219254 , 4.2183428423734437902613797615142 ,
4.6345113219374139959693285738216 , 4.8469485814569240979319313263235 ,
5.3028461906463610610954483096441 , 5.8786363592993781492098219651469 ,
5.8466623997400544783864810901348 , 6.3623275490646030238620043500507 ,
7.0111382191136532036318912082579 , 7.8867596803493307885933938775 81

and the CRM of these 10 CRMs is also 5.4869360682890471108297900201 02 . 2
which is the CRM of the elements in S.

Moreover, ${}^{5}C_{4} = 5$ possible subsets of S having 4 elements are

{2 , 4 , 6 , 8} , {2 , 4 , 6 , 10} , {2 , 4 , 8 , 10} , {2 , 6 , 8 , 10} , {4 , 6 , 8 , 10}

Corresponding 10 CRMs of the elements in the respective subsets are

4.6249979001582671452259839532691 , 4.9540304248710920715212900451501 ,
5.3634111441664626407715514852456 , 5.9088223572165210051799588535178 ,
6.7484733287212306157704446754503

and the CRM of these 10 CRMs is also 5.4869360682890471108297900201 02 .
which is the CRM of the elements in S.

Moreover, ${}^{5}C_{5} = 1$ possible subset of S having 5 elements is

{2 , 4 , 6 , 8 , 10}

CRM of the elements in this subset is 5.4869360682890471108297900201 02 .
which is the CRM of the elements in S.
Finally,
the CRM of all these $2^{5} - 1 (= 31)$ CRMs of the corresponding elements of the respective 31 subsets is found after computation as 5.4869360682890471108297900201 02 which is the CRM of the elements in S.

VI. CONCLUSION

Findings on the property of cube root mean, obtained in this study, can be summarized as follows:

"The cube root mean of the cube root means of the respective possible subsets of fixed size of a set of real numbers is the cube root mean of the original set of numbers and the cube root mean of the cube root means of non-empty possible subsets a set of positive real numbers is the cube root mean of the original set of numbers."

This property of cube root mean is a relation between the cube root mean of the elements of a set of real numbers and the cube root means of the respective elements of the respective possible subsets of the set.

The property/result on cube root mean obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

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(Dr. Dhritikesh Chakrabarty with his cousin sister Mrs. Minu Sarma, a lovely house wife and permanent resident of Tezpur, Assam, in a holy moment of their meet on February 07 of 2016 at the residence of Mrs. Devi)

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town

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University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.