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Harmonic Mean of Harmonic Means of Possible Subsets of a Set of Numbers

Dhritikesh Chakrabarty

Independent Researcher, Ex Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: Mathematical proof of one property of harmonic mean has been presented in this article. The property states that the harmonic mean of the harmonic means of the respective possible subsets of fixed size of a set of non-zero real numbers is the Harmonic mean of the original set of numbers and also the harmonic mean of the harmonic means of the respective non-empty possible subsets a set of positive real numbers is the harmonic mean of the original set of numbers.

KEYWORDS: Set, Real Numbers, Possible Susets, Harmonic Mean

I. INTRODUCTION

There had been several studies on average [1 , 51] which is an entity that describes a set of many entities. The great mathematician Pythagoras first developed three measures of average termed as arithmetic mean [2 , 6 , 58], geometric mean [2 , 6] and harmonic mean [2 , 6 , 57] which together is also popularly known as "Pythagorean Means" [3 , 7 , 15]. Later on, a number of definitions / formulations of average were developed due to necessity of handling different situations some of which are quadratic mean or root mean square, square root mean , cubic mean, cube root mean, generalized *p* mean & generalized p^{th} root mean etc. [8 , 32 , 50 , 52 , 53 , 56]. In addition to these, generalized definitions of average had also been developed for deriving measures of average [10 – 14]. Moreover, one general method had been identified for defining average of a set of values of a variable as well as a generalized method of defining average of a function of a set (or of a list) of values [9 , 16 , 17 , 20]. Recently, four formulations of average have been derived from the three Pythagorean means which are arithmetic- geometric mean, arithmetic-harmonic mean, geometric -harmonic mean and arithmetic- geometric -harmonic respectively [19 , 32].

Each of the measures of average is to carry its own properties of whose some are known. Several studies have already been done on properties of arithmetic mean, geometric mean & harmonic mean [2 , 3 , 6 , 39 , 40 , 42 – 44 , 46 , 47 , 50 , 52 , 53 , 57 , 58]. Arithmetic mean, geometric mean & harmonic mean have been found to be widely in developing most of the statistical measures of characteristics of data like central tendency, dispersion etc. [7 , 15 , 21 – 31 , 36 , 37] and in developing the statistical concept of expectation [5 , 33 – 35 , 38 , 41 , 54 , 55]. However, more properties of these means are yet to be identified due to their importance in mathematical/statistical analysis of numerical data. One more mathematical property of arithmetic mean which states that the arithmetic mean of the arithmetic means of the respective possible subsets of fixed size of a set of real numbers is the arithmetic mean of the original set of numbers and also the arithmetic mean of the arithmetic means of the respective non-empty possible subsets a set of real numbers is the arithmetic mean of the original set of numbers, was mathematically established in a recent study since no research publication on the proof of this property had been found available [48]. Similar property of geometric mean has also been mathematically established in another study [49]. Similar property of harmonic mean has mathematically been proved in this study. The property states that the harmonic mean of the harmonic means of the respective possible subsets of fixed size of a set of non-zero real numbers is the Harmonic mean of the original set of numbers and also the harmonic mean of the harmonic means of the respective non-empty possible subsets a set of positive real numbers is the harmonic mean of the original set of numbers.

II. HARMONIC MEAN OF A SET OF ELEMENTS

Let us first mention the definition of harmonic mean of a set of real numbers.

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Definition

Let us consider a set of n non-zero real numbers namely

$$x_1 , x_2 , \dots\dots\dots , x_n$$

Then the harmonic mean $H = H(x_1 , x_2 , \dots\dots\dots , x_n)$ of them is given by

$$H = H(x_1 , x_2 , \dots\dots\dots , x_n) = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Let us abbreviate harmonic mean by HM.

Note:

The definition of HM implies that

$$\sum_{i=1}^{n} \frac{1}{x_i} = n \cdot \frac{1}{H(x_1 , x_2 , \dots\dots\dots , x_n)}$$

i.e. the sum of the reciprocals of n non-zero real numbers is n times of the reciprocal of the HM of the numbers.

III. HARMONIC MEAN OF HARMONIC MEANS OF POSSIBLE SUBSETS OF FIXED SIZE

Suppose, a set S consists of the N non-zero real numbers

$$a_1 , a_2 , \dots\dots\dots , a_N$$

as elements so that

$$\text{Sum of the reciprocals of the } N \text{ elements of } S = \sum_{i=1}^{N} \frac{1}{a_i}$$

$$\& \ HM \text{ of the } N \text{ elements of } S = \frac{N}{\sum_{i=1}^{N} \frac{1}{a_i}} = H, \text{ say}$$

By the above **Note**,

$$\text{Sum of the reciprocals of the } N \text{ elements of } S = \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots\dots\dots + \frac{1}{a_N} = \frac{N}{H}$$

Let us consider the possible subsets of S having n elements in each set.

The number of such possible subsets is ${}^{N}C_{n}$

$$\text{where} \quad {}^{N}C_{n} = C(N , n) = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

Among the ${}^{N}C_{n}$ possible subsets, there are

$${}^{N-1}C_{n-1} \text{ subsets with } a_1 \text{ as } 1^{\text{st}} \text{ element,}$$

$${}^{N-2}C_{n-2} \text{ subsets with } a_2 \text{ as } 1^{\text{st}} \text{ element and not having } a_1 ,$$

$${}^{N-3}C_{n-3} \text{ subsets with } a_3 \text{ as } 1^{\text{st}} \text{ element and not having } a_1 \ \& \ a_2 ,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$${}^{N-1}C_{N-2} \text{ subsets with } a_{N-n+2} \text{ as } 1^{\text{st}} \text{ element and not having } a_1 , a_2 , \dots\dots\dots a_{N-n+3} ,$$

$${}^{N-1}C_{N-1} \text{ subsets with } a_{N-n+1} \text{ as } 1^{\text{st}} \text{ element and not having } a_1 , a_2 , \dots\dots\dots a_{N-n+2} ,$$

such that

Total number of possible subsets

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$$= {}^{N-1}C_{n-1} + {}^{N-2}C_{n-2} + {}^{N-3}C_{n-3} + \ldots\ldots\ldots.. + {}^{N-1}C_{N-2} + {}^{N-1}C_{N-1} = {}^{N}C_{n}$$

and that each a_i appears a total of ${}^{N-1}C_{n-1}$ times in the set containing all possible ${}^{N}C_{n}$ subsets.

Suppose,

$$H(1)\,,H(2)\,,H(3)\,,\ \ \ldots\ldots\ldots\,,\ H({}^{N}C_{n}-1)\,,H({}^{N}C_{n})$$

are the HMs of the elements in the respective subsets and

$$S'(1)\,,S'(2)\,,S'(3)\,,\ \ \ldots\ldots\ldots\,,\ S'({}^{N}C_{n}-1)\,,S'({}^{N}C_{n})$$

are the Sums of the reciprocals of the elements in the respective subsets so that

$$S'(1) = \frac{n}{H(1)}\ ,$$
$$S'(2) = \frac{n}{H(2)}\ ,$$
$$S'(3) = H(3)\ ,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$S'({}^{N}C_{n}-1) = \frac{n}{H({}^{N}C_{n}-1)}\ ,$$
$$S'({}^{N}C_{n}) = H({}^{N}C_{n})\ .$$

Now,

$$S'(1) + S'(2) + S'(3) + \ldots\ldots\ldots\ + S'({}^{N}C_{n}-1) + S'({}^{N}C_{n})$$

is the sum of the reciprocals of all elements in the set containing all possible ${}^{N}C_{n}$ subsets of the original set S where each a_i appears a total of ${}^{N-1}C_{n-1}$ times.

Therefore,

$$S'(1) + S'(2) + S'(3) + \ldots\ldots\ldots\ + S'({}^{N}C_{n}-1) + S'({}^{N}C_{n})$$
$$= {}^{N-1}C_{n-1}\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots\ldots\ldots + \frac{1}{a_N}\right)$$
$$= {}^{N-1}C_{n-1}\,\frac{N}{H}$$

Accordingly,

HM of the HMs of the respective possible ${}^{N}C_{n}$ subsets

$$= \frac{{}^{N}C_{n}}{\frac{1}{H(1)} + \frac{1}{H(2)} + \frac{1}{H(3)} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{H({}^{N}C_{n}-1)} + \frac{1}{H({}^{N}C_{n})}}$$

But

$$\frac{1}{H(1)} + \frac{1}{H(2)} + \frac{1}{H(3)} + \ldots\ldots\ldots\ldots\ldots\ldots.. + \frac{1}{H({}^{N}C_{n}-1)} + \frac{1}{H({}^{N}C_{n})}$$
$$= \frac{1}{n}\left\{S'(1) + S'(2) + S'(3) + \ldots\ldots\ldots\ + S'({}^{N}C_{n}-1) + S'({}^{N}C_{n})\right\}$$

$= \frac{1}{n}\{ {}^{N-1}C_{n-1} \frac{N}{H} \}$

This implies,

$$HM \text{ of the } HMs \text{ of the respective possible } {}^{N}C_{n} \text{ subsets} = \frac{{}^{N}C_{n}}{\frac{1}{n}\{ {}^{N-1}C_{n-1} \frac{N}{H} \}} = H$$

Therefore,

$$HM \text{ of the } HMs \text{ of the respective possible } {}^{N}C_{n} \text{ subsets} = HM \text{ of the elements of } S$$

IV. HARMONIC MEAN OF HARMONIC MEANS OF All NON-EMPTY POSSIBLE SUBSETS

Now, the set S has a total of $(2^N - 1)$ number of non-empty subsets of which

number of possible subsets having single element in each is ${}^{N}C_{1}$,

number of possible subsets having 2 elements in each is ${}^{N}C_{2}$,

…………………………………………………

number of possible subsets having $N-1$ elements in each is ${}^{N}C_{N-1}$,

number of possible subsets having N elements is in each ${}^{N}C_{N}$.

such that

$$\text{Total number of all possible non-empty subsets} = {}^{N}C_{1} + {}^{N}C_{2} + {}^{N}C_{3} + \dots\dots + {}^{N}C_{N-1} + {}^{N}C_{N} = 2^N - 1$$

By the results obtained in **section 3**,

HM of the HMs of the respective possible subsets having 1 element in each = HM of the elements of S = H

Similarly,

HM of the HMs of the respective possible subsets having 2 elements in each = H ,

HM of the HMs of the respective possible subsets having 3 elements in each = H ,

………………………………………………………………………………

HM of the HMs of the respective possible subsets having $N-1$ elements in each = H ,

HM of the HMs of the respective possible subsets having N elements in each = H .

Thus,

HMs of all the $2^N - 1$ non-empty subsets are H i.e. a constant number.

Therefore,

HM of the HMs of all respective possible non-empty subsets of S = HM of N number of H = H

i.e. HM of the HMs of all respective possible non-empty subsets of S = HM of the elements of S

V. NUMERICAL EXAMPLE

Let us consider the following set S of five real numbers

$$S = \{2 , 4 , 6 , 8 , 10\}$$

so that

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HM of the elements of S = 4.379562043795620437956204379562

Now, ${}^5C_1$ = 5 possible subsets of S having single element are

{2} , {4} , {6} , {8} , {10}

Corresponding 5 HMs of the element if the respective subsets are

2 , 4 , 6 , 8 , 10

and the HM of these 5 HMs is

4.379562043795620437956204379562

which is the HM of the elements of S.

Similarly, ${}^5C_2$ = 10 possible subsets of P having 2 elements are

{2 , 4} , {2 , 6} , {2 , 8} , {2 , 10} , {4 , 6} , {4 , 8} , {4 , 10} , {6 , 8} , {6 , 10} , {8 , 10}

Corresponding 10 HMs of the element if the respective subsets are

2.666666666666666666666666666667 , 3 , 3.2 , 3.333333333333333333333333333333 ,
4.800000000000000000000000000001 , 5.333333333333333333333333333333 ,
5.714285714285714285714285714286 , 6.857142857142857142857142857143 ,
7.500000000000000000000000000002 , 8.888888888888888888888888888889

and the HM of these 10 HMs is

4.379562043795620437956204379562

which is the HM of the elements of S.

Again, ${}^5C_3$ = 10 possible subsets of S having 3 elements are

{2 , 4 , 6} , {2 , 4 , 8} , {2 , 4 , 10} , {2 , 6 , 8} , {2 , 6 , 10} ,
{2 , 8 , 10} , {4 , 6 , 8} , {4 , 6 , 10} , {4 , 8 , 10} , {6 , 8 , 10}

Corresponding 10 HMs of the element if the respective subsets are

3.272727272727272727272727272727 , 3.428571428571428571428571428571 4 ,
3.529411764705882352941176470588 3 , 3.789473684210526315789473684210 5 ,
3.913043478260869565217391304347 8 , 4.137931034482758620689655172413 7 ,
5.538461538461538461538461538461 4 , 5.806451612903225806451612903225 9 ,
6.315789473684210526315789473684 3 , 7.659574468085106382978723404255 1

and the HM of these 10 HMs is also

4.379562043795620437956204379562

which is the HM of the elements of S.

Moreover, ${}^5C_4$ = 5 possible subsets of S having 4 elements are

{2 , 4 , 6 , 8} , {2 , 4 , 6 , 10} , {2 , 4 , 8 , 10} , {2 , 6 , 8 , 10} , {4 , 6 , 8 , 10}

Corresponding 10 HMs of the element if the respective subsets are

3.84 , 3.934426229508196721311475409836 , 4.102564102564102564102564102564 1 ,
4.485981308411214953271028037383 1 , 6.233766233766233766233766233766 1

and the HM of these 10 HMs is also

4.379562043795620437956204379562

which is the HM of the elements of S.

Moreover, ${}^5C_5$ = 1 possible subset of S having 5 elements is

$$\{2, 4, 6, 8, 10\}$$

HM of the elements in this subset is

$$4.379562043795620437956204379562$$

which is the HM of the elements of S.

Finally, the HM of all the $2^5 - 1 = 31$ HMs of the corresponding elements of the respective 31 subsets is found after computation as

$$4.379562043795620437956204379562$$

which is the HM of the elements of S.

VI. CONCLUSION

Findings on the property of harmonic mean, obtained in this study, can be summarized as follows:

"The harmonic mean of the harmonic means of the respective possible subsets of fixed size of a set of positive real numbers is the harmonic mean of the original set of numbers and the harmonic mean of the harmonic means of non-empty possible subsets a set of positive real numbers is the harmonic mean of the original set of numbers."

The property/result of harmonic mean obtained here is hoped to be useful for analysis of data specially on estimation based on sample from population.

In this connection, it is to be mentioned that each of arithmetic mean, Harmonic mean harmonic mean and quadratic mean is required to be computed/estimated in similar type of situation. Therefore, there is necessity of study on whether quadratic mean possess similar property like that of arithmetic mean derived in earlier study [48], of geometric mean derived in another study [49] and of harmonic mean derived in this study.

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AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag

Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty, currently an independent researcher, served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years.

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(Dr. Dhritikesh Chakrabarty delivering speech in the NaSAEAST -2023 during 27-28 October 2023, at

Gauhati University)

Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 280 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS), (3) Partners Universal International Research Journal (PUIRJ) & (4) International Journal of Advanced Research in Science, Engineering and Technology (IJARSET). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held. Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.