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f (R, T) Modified Gravity Cosmological Model with Variable Deceleration Parameter

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ABSTRACT: In f (R, T) modified gravity theory, the LRS Bianchi type-I cosmological model is examined. The deceleration parameter (q) is a linear function of the Hubble parameter (H), and f (R, T) = R + 2f (T) is used to solve Einstein's field equations in f (R,T) gravity. An early deceleration phase gives way to a late temporal acceleration phase as the cosmos progresses from its initial solitary condition. We discovered that at late times, the model's jerk parameter (j) approaches that of the ΛCDM model. We also go over the model's geometrical and physical characteristics.

KEYWORDS: Space time, f(R,T) theory, LRS Bianchi type I, cosmological term.

I. INTRODUCTION

Modern cosmology provides the relevant experimental evidence about the acceleration of our universe and has reached a new vision to establish revolutionary advancements on account of the current accelerated expansion. Recent observational studies, including the supernovae cosmology project, have provided the main evidence for the cosmic acceleration of the universe and several observations like those of the distant supernovae, large scale structure (LSS), fluctuations of the cosmic microwave background radiation, the Wilkinson microwave anisotropy probe, the Sloan digital sky survey (SDSS) and the Chandra X-ray observatory suggest that our universe is undergoing an accelerated expansion. The observations have indicated a change with time from an early deceleration phase to a late time acceleration phase. There are promising approaches confirmed by the cosmological research community to deliberate the cosmic expansion of the universe.

A negative gravity-like matter known as dark energy (DE) dominates the first theory of the expanding universe and is thought to be the cause of the universe's accelerating rate of expansion. According to recent cosmological findings, the universe is composed of 4.9% baryonic matter, 26.8% dark matter, and 68.3% DE. Other representations, such as quintessence, phantom, and essence, do exist, but the cosmological constant is a strong contender for DE.

The second method involves using more generic actions to characterize the gravitational field and generalizing Einstein's gravity model of general relativity. One method, for instance, substitutes an arbitrary function of the Ricci scalar for the conventional Einstein-Hilbert action. Explaining the universe's late-time acceleration and the effective cause associated with DE is a basic theoretical issue to gravitational theories. Several different theories of gravity, including f (R), f (G), f (T), and f (R, T) gravity, have emerged as a result of changes to the action. In f (T) gravity, the cosmic evolution for DE models was examined.

The f (R) modified theory yields both cosmic inflation and an explanation of DE that includes the current cosmic acceleration. Recently, researchers have looked into anisotropic cosmological models in f (R, T) gravity with varying deceleration parameters. The fundamental formalism of the f (R, T) gravity field equations and a few crucial reviews are covered in this study. Assuming that $f\ (R,T)\ =\ R\ +\ 2f\ (T)$ and that the deceleration parameter (q) is a linear function of the Hubble parameter (H), the field equations in f (R, T) gravity are solved. (General relativity with a non-standard matter field is analogous to this situation.) An early deceleration phase gives way to a late temporal acceleration phase as the cosmos progresses from its initial solitary condition. We discovered that at late times, the jerk parameter (j) approaches that of the ΛCDM model. We also go over the model's geometrical and physical characteristics.

II. REVIEW LITERATURE

Mishra, B., Esmeili, F.M. & Ray, S. (2021). In this article, anisotropic variable-parameter cosmological models are presented and analyzed. The field equations for a space-time with a $f(R,T)$ gravity have been expressed in the form of a Bianchi I metric. The functional form for the $f(R,T)$ gravity has been defined as $f\ (R,T) = R + 2\ f\ (T)$, where R and T are the Ricci scalar and the trace of the energy-momentum tensor, respectively. Two different models are constructed in terms of the scale factors, such as the hybrid scale factor and the power law scale factor. By representing the anisotropic parameter as a hyperbolic function, further information about the behavior of the equation of state parameter is revealed.

Tiwari, R. k.; Beesham, A. K.; Mishra Soma and Dubey Vipin, (2021). According to the available observational data, the universe is homogeneous and isotropic on a large enough scale. This does not, however, preclude the idea that there was some anisotropy in the early stages of the universe's development and that it was later reduced. This theory has led to the popularity of the homogeneous but anisotropic Bianchi models. Second, modified gravity has garnered a lot of attention due to problems with the conventional CDM model in general relativity. Thus, $f(R,T)$-modified gravity was used in this study to study the Bianchi type-I cosmological model. By assuming a certain form of the deceleration parameter, motivated by some cosmographic ideas, a model demonstrating a transition from early deceleration to late-time acceleration was produced.

Montani, G., (2024). In order to alleviate the Hubble strain, we examine a dynamic scenario for describing the late Universe evolution. We specifically examine the Jordan frame metric (R) gravity applied to the dynamics of a flat isotropic universe. Because of the cosmological gravitational field's time variation, this cosmological model includes a matter formation process. We use the isotropic Universe (i.e., a given fiducial volume) as an open thermodynamic system to explain particle production. Four unknowns are involved in the resulting dynamical model: the energy density of the matter component, the non-minimally coupled scalar field, its potential, and the Hubble parameter.

Shukla, B.K.; Sahlu, Shambel (2025). In the coupling of geometry with matter alternative theory $f\ (R,T)$ gravity, the accelerating expansion of the universe has been studied in the multi-components fluid. The gravitational Lagrangian is determined by an arbitrary function of the trace of the stress-energy tensor T and of the Ricci scalar R. We use the nonzero divergence of the energy-momentum tensor taken into consideration in the presence of a multi-component fluid to solve the Friedmann equations in order to address the late-time accelerating cosmos.

III. METHODOLOGY

(A) The Basic Equations

The f(R, T) theory of gravity is the generalization or modification of GR. The action for this theory is given by

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \quad (1)$$

where f (R, T) is an arbitrary function of the Ricci scalar (R), and the trace (T) of the stress-energy tensor of the matter Tμν. Lm is the matter Lagrangian density.

Where f (R,T) is any function that can be defined using the Ricci scalar R, and the trace T of the matter's stress-energy tensor, Tij. The Lagrangian density of matter is Lm.

The definition of the matter's stress-energy tensor is

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m}{\delta g^{ij}}, \quad (2)$$

and its trace by Tij $= g^{ij}$ Tij . Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives leads to

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \quad (3)$$

The gravitational field equations of f (R,T) gravity are produced as by changing the action S with regard to the metric tensor components gij.

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij} - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_r(R,T)T_{ij} - f_r(R,T)\theta_{ij} \quad (4)$$

∇_i and ∇_j denotes the covariant derivative. The contraction of eq. (7) yields

$$f_R(R,T)R + 3f_R(R,T) - 2f(R,T) = (8\pi - f_R(R,T)T - f_R(R,T)\theta, \quad (5)$$

We obtain The expansion tensor θ_{ij} is given by

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$$\theta_{ij} = -2\mathrm{Tij} + g_{ij}L_m - 2g^{\mu\nu}\frac{\partial^2 L_m}{\partial g^{\mu\nu} g^{ij}}, \quad (6)$$

The stress-energy tensor of the matter Lagrangian is given by under the assumption that matter is thought of as a perfect fluid

$$\mathrm{Tij} = (\rho + \mathrm{p})u_i u_j - \mathrm{p}g_{ij}, \quad (7)$$

where $u^i = (0,0,0,1)$ is the four velocity in the moving coordinates which satisfies the conditions $u^i u_i = 1$ and $u^i \nabla_j\, j\, u_i\, 0$. ρ and p are the energy density and pressure of the fluid respectively. With the use of Eq. (13), we obtain

$$\theta_{ij} = -2\mathrm{Tij} - \mathrm{p}g_{ij}, \quad (8)$$

Since the field equations in f (R, T) gravity also depend on the physical nature of the matter field (through the tensor θ_{ij}), for each choice of f , we obtain a different set of field equations. Three explicit specifications of the functional from of f (R, T) are usually considered

$$f(R,T) = R + 2f(T),$$

$$f(R,T) = f_1(R) + f_2(T)$$

$$f(R,T) = f_1(R) + f_2(R)f_3(T)$$

Harko et al. have constructed some FRW cosmological models by using different functional forms of f (R, T).

(B) The Metric and Field Equations

The Bianchi type-II metric is given by

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2, \quad (9)$$

Where the metric potentials A,B and C are functions of cosmic time t.

We use one of the most straightforward expressions for f (T), which is f (T) = λT, where λ is a constant. In essence, we are dealing with f (R) gravity with non-standard matter fields since we have thought about dividing f (R, T) into a f (R) component and a f (T) part. In actuality, the standard Einstein-Hilbert action describes the gravitational component when we select f (R) = R and the theory is identical to general relativity with a non-standard matter field in this instance. It has been demonstrated that this assumption results in a power-law shape for the scaling factor in the FRW situation.

Now assuming commoving coordinate system, the field Eqs. (7) for the metric (1) withthe help of (1) and (9) can be written as

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{4}\frac{A^2}{B^2C^2} = \rho\{8\pi - \lambda(3\omega + \delta - 3)\} + 2p\lambda, \quad (10)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3}{4}\frac{A^2}{B^2C^2} = -\rho\{8\pi(\omega + \delta) + \lambda(5\omega + 3\delta - 1)\} + 2p\lambda, \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{1}{4}\frac{A^2}{B^2C^2} = -\rho\{8\pi\omega + \lambda(5\omega + \delta - 1)\} + 2p\lambda, \quad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{A^2}{B^2C^2} = -\rho\{8\pi\omega + \lambda(5\omega + \delta - 1)\} + 2p\lambda, \quad (13)$$

where a derivative with respect to cosmic time t is represented as a dot in the sky. Determining certain physical parameters is a prerequisite to solving the field equations. The average scale factor R and the volume scale factor V are defined as

$$V = R^3 = \mathrm{ABC}, \quad (14)$$

The form of the generalized mean Hubble parameter H

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \quad (15)$$

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions.

of x, y and z axis respectively. Using Eqs. (14) and (15), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{R}}{R}, \quad (16)$$

The mean anisotropy parameter Δ is defined as

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$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{17}$$

The shear scalar σ^2 is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) \tag{18}$$

As a measure of the cosmic acceleration of the universe's expansion, the deceleration parameter q in cosmology is defined as

$$q = -\frac{\ddot{R}R}{\dot{R}^2}, \tag{19}$$

IV. RESULTS

Equations (11) through (13) include a set of three equations with four parameters: $A, B, p, and\ \rho$**.** We need one more physically feasible relation to get a complete solution. According to recent studies, the deceleration parameter has a value between -1 and 0, indicating that the cosmos is currently accelerating. The temporal dependence of the two parameters can be used to categorize universe models. Any model can be classified according to whether it accelerates or decelerates, expands or contracts:

(1) $H > 0, q > 0$: Expanding and decelerating.

(2) $H > 0, q < 0$: Expanding and accelerating.

(3) $H < 0, q > 0$: Contracting and decelerating.

(4) $H < 0, q < 0$: Contracting and accelerating.

(5) $H > 0, q = 0$: Expanding and zero decelerating.

(6) $H < 0, q = 0$: Contracting and zero decelerating.

(7) $H = 0, q = 0$: Static.

We assume that the deceleration parameter q is a simple linear function of the Hubble parameter H, i.e.

$$q = \alpha + (\beta)H\ , \tag{20}$$

where α, β are constant.

This assumption is made for the following reasons. We are aware that the universe was once decelerating but is now accelerating. As a result, the deceleration parameter q must exhibit signature flipping from positive in the past to negative in the present; it cannot remain constant during the universe's evolution. The deceleration parameter varies with time in the majority of models. As we demonstrate below, our selection results in the necessary behavior. A straightforward generalization of a constant q is a linear function of the Hubble parameter equation.

By taking α = -1, we can solve the Einstein field equations and obtain the deceleration parameter q, which is a linear function of Hubble's parameter. This allows us to create an initially decelerated and late time accelerated model. H i.e., $q = \alpha + (-\beta)\, H$, which gives scale factor

$a = k_1\left(e^{\beta t} - 1\right)^{\frac{1}{1+\alpha}}$ (where α, β, k_1 are constant and $\beta < 0$).

The spatial volume V, Hubble parameter H, expression scalar θ, shear scalar σ^2 , anisotropy parameter A_m, and declaration parameter q take the following forms, respectively:

$$V = k_1^3\left(e^{\beta t} - 1\right)^{\frac{1}{1+\alpha}}, \tag{21}$$

$$H = \frac{\beta e^{\beta t}}{(1+\alpha)(e^{\beta t}-1)}, \tag{22}$$

$$\theta = \frac{3\beta}{(1+\alpha)(1-e^{-\beta t})}, \tag{23}$$

$$\sigma^2 = \frac{k_1^2 + k_2^2 + k_1 k_2}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}, \tag{24}$$

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$$A_m = \frac{2(k_1^2+k_2^2+k_1k_2)(1+\alpha)^2}{9\beta^2k_1^6e^{2\beta t}(e^{\beta t}-1)^{\frac{4-2\alpha}{1+\alpha}}} \tag{25}$$

Equations (21) – (24) are determined essentially from (25), and are the kinematic quantities. The field Equation, on the other hand is basically used to determine the dynamical quantities, the energy density ρ, pressure p, and cosmological parameter Λ.

From Equations (17) – (19), we obtain energy density ρ and pressure p:

$$\rho = \frac{1}{(1+\omega)(1+2\lambda)}\left[\frac{2\beta^2e^{\beta t}}{(1+\alpha)(e^{-\beta t}-1)^2} - \frac{2(k_1^2+k_2^2+k_1k_2)}{3k_1^6(e^{\beta t}-1)^{\frac{6}{1+\alpha}}}\right], \tag{26}$$

$$p = \frac{\omega}{(1+\omega)(1+2\lambda)}\left[\frac{2\beta^2e^{\beta t}}{(1+\alpha)(e^{-\beta t}-1)^2} - \frac{2(k_1^2+k_2^2+k_1k_2)}{3k_1^6(e^{\beta t}-1)^{\frac{6}{1+\alpha}}}\right] \tag{27}$$

The cosmological parameter $\Lambda = \lambda(\rho - p)$ is given by:

$$\Lambda = \lambda\left[\frac{2(1-\omega)\beta^2e^{\beta t}}{(1+\omega)(1+\alpha)(1+2\lambda)(e^{-\beta t}-1)^2} - \frac{(1+\omega)}{(1+\omega)(1+2\lambda)}\frac{2(k_1^2+k_2^2+k_1k_2)}{3k_1^6(e^{\beta t}-1)^{\frac{6}{1+\alpha}}}\right] \tag{28}$$

For our Bianchi model (14), we observe that the spatial volume V is zero and expansion scalar θ are infinite at $t = 0$. Thus, the universe starts evolving with zero volume and an infinite rate of expansion at $t = 0$. Equations (26)–(27) and (28) show that the scale factors also vanish at $t = 0$, hence the model has a "point type" singularity at the initial approach. Initially, at $t = 0$ the Hubble parameter H and shear scalar σ^2 are infinite. The energy density ρ, pressure p and cosmological constant Λ are also infinite. As t tends to infinity, V becomes infinitely large, whereas σ^2 approaches zero. Later, the energy density ρ and pressure p converge to zero. The cosmological parameter Λ also approaches a constant later. The deceleration parameter q for the model is a constant α at $(t = 0)$, and as t increases—i.e., when it is $(1/\beta)log(1 + \alpha) - q$ is zero, which show that there will be a transition to acceleration. It is equal to1 when t tends to infinity, which shows thatthe model describes the accelerating phase of the universe. The anisotropy parameter A_m gives a measure of the anisotropy of the model, and is given by, which his large early on as $(t = 0)$ but decreases very rapidly.

As a matter of interest, the solution for (Λ= 0), which also means (λ= 0) from Equation (14), can now be easily given. All the kinematic quantities are the same as before, viz., Equations (26)–(28). The density and pressure are given by:

$$\rho = p = \frac{1}{(1+\omega)}\left[\frac{2\beta^2e^{\beta t}}{(1+\alpha)(e^{-\beta t}-1)^2} - \frac{2(k_1^2+k_2^2+k_1k_2)}{3k_1^6(e^{\beta t}-1)^{\frac{6}{1+\alpha}}}\right] \tag{29}$$

LRS Bianchi Type –I cosmological model with constant declaration parameter. Which shows that this model is an accelerating model of the universe. From eq. (29), we conclude that, the model has an initial singularity at $t = 0$ and expands with time.The line element with scale factors $A(t)$ and $B(t)$, given by equations (24) and (25) gives an exact stiff fluid solution of the universe. Scale factors A(t) vanishes while the other one $B(t)$ diverges at large time. From the above results we can discuss the physical behavior of the universe. From eq. (26)-(28), we observe that, at initial epoch energy density (ρ), the Hubble parameter (H), expansion scalar(θ), the share scalar (σ), the anisotropy parameter (Δ)all diverges and vanishes at large value of time. since, that $\frac{\sigma^2}{\theta}$ does not tends to zero at t → ∞, which indicates that this model of universe does not approaches isotropy at late times. positive constant value of declaration parameter shows that the model has a decelerating expansion ($q > 0$).

V. CONCLUSION

In this paper we cover some fundamental review as well as the fundamental formalism of the $f(R,T)$ gravity field equations from an action. Assuming that $f(R,T) = R + 2f(T)$ (general relativity with a non-standard matter field) and that the deceleration parameter (q) is a linear function of the Hubble parameter (H), the field equations in $f(R,T)$ gravity are solved. The cosmos begins in a unique condition and transitions over time from an early phase of deceleration to a late phase of acceleration. Assuming a constant jerk value($j = 1$), the essay examines significant cosmological parameters for the two different hypotheses. The exponential law comes after the power law. The first era is covered by

Model I, which does not achieve isotropy in later cosmic epochs. A positive constant value of the declaration parameter ($q > 0$) indicates decelerating expansion. Furthermore, model II is appropriate for late times and correlates with rapid expansion. The various kinematic and physical aspects have been discussed and evaluated.

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