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Diffraction of Non-Stationary Plane Pressure Waves on a Rigid Sphere in an Acoustic Half Space

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ABSTRACT: The paper considers the problem of diffraction of a non-stationary plane pressure wave on an absolutely rigid stationary sphere in an acoustic half-space. The integral Laplace transform with respect to dimensionless time was used to solve the problem. In the image space, the problem is reduced to an infinite system of linear algebraic equations, the solution of which is sought in the form of an infinite series of exponentials. Formulas for hydrodynamic pressure are obtained. The transition to the originals is carried out using the theory of residues. The results of numerical experiments are presented in the form of graphs.

KEY WORDS: Acoustic half-space, rigid sphere, integral Laplace transform, infinite system, non-stationary wave, acoustic medium, disturbance, pressure.

I. INTRODUCTION

The study of propagation and diffraction of non-stationary waves in continuous media, including acoustic media with obstacles of various kinds, is one of the urgent, fundamental, and applied problems in the dynamics of a deformable solid. Many publications are devoted to problems of propagation and diffraction of non-stationary waves in multiply connected regions of acoustic and elastic media. The interaction of a plane acoustic shock wave of pressure with a system of two rigid parallel circular cylinders of different radii is considered in the study [1]. The initial-boundary value problem is solved by the method proposed in [2]. A formula for the hydrodynamic pressure is obtained. In the monograph [3], an analytical method is presented for solving non-stationary problems of aerohydroelasticity and hydroelectroelasticity, allowing them to be reduced to Volterra integral equations with retarded arguments, and solutions are given to problems of diffraction of acoustic shock waves on hollow cylindrical or spherical shells located near a flat boundary. The influence of a rigid wall and a free boundary of a half-space on the stress-strain state of spherical and cylindrical shells is investigated. The validity of applying the reduction method to the obtained infinite system of integral equations is substantiated. A numerical solution to problems of non-stationary action of shock waves on thin-walled shells located near a rigid wall of an acoustic half-space is given in [4]. The fluid motion is described by a system of nonlinear equations of gas dynamics, and the dynamic behaviour of the shells is presented by equations of the geometric nonlinear theory of shells of the S. P. Timoshenko type. The article [5] is devoted to the study of an axisymmetric problem of propagation of non-stationary waves from a spherical cavity in an acoustic half-space. The problem is reduced to an infinite system of linear algebraic equations, the solution of which is constructed in the form of an infinite series in exponentials. Formulas for the characteristics of the acoustic medium are obtained. The paper [6] studies the problem of interaction between an oscillating spherical body and a thin elastic cylindrical shell filled with an ideal compressible fluid and immersed in an infinite ideal compressible medium with other parameters. The hydrodynamic characteristics of the fluid filling the cylindrical volume and surrounding it, as well as the deflections of the cylindrical shell, are determined. The paper [8] presents an approach to solving problems of scattering stationary acoustic waves by two non-parallel circular cylinders located in an infinite medium. As a result, an infinite system of integro-algebraic equations is obtained. The effect of semi-boundedness of the acoustic space when a plane wave is incident on a thin elastic spherical shell is studied in [11]. The sought pressure is represented in the form of expansions in series in spherical functions. An infinite system of algebraic equations is obtained using the method of imaginary sources and the addition theorem for spherical wave functions. The problem of diffraction of a plane wave by a system of two concentric spherical shells surrounded by acoustic media is investigated in the article [13]. In this case, the solution is represented as a superposition of elementary waves. In the work [14], a numerical solution is given to the problem of propagation of non-stationary waves from a spherical cavity in an acoustic half-space. The problem is solved by the finite element method. The results of numerical

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experiments are presented in the form of graphs. This article is devoted to the study of the problem of diffraction of a non-stationary plane wave by an absolutely rigid sphere in an acoustic half-space. The aim of the work is to develop an algorithm for solving the problem of diffraction of non-stationary plane waves by an absolutely rigid sphere in an acoustic half-space and to study wave processes in the vicinity of the rigid sphere.

II. STATEMENT OF THE PROBLEM

Let the acoustic half-space ($z \geq 0$) at a depth from the boundary plane (h) on the axis O_2z (the point lies on the flat boundary of the half-space) be the center of a fixed rigid ball of radius R ($R < h$).

At the moment $\tau = 0$ in time the front of a non-stationary plane pressure wave with a given potential touches φ_0 the frontal point of the surface of a rigid sphere.

$$\varphi_0 = f(\tau + r\cos\theta - 1)H(\tau + r\cos\theta - 1) \tag{1}$$

where $f(\tau)$ is an arbitrary function that defines the law of change of potential over time; $H(\tau)$ is the Heaviside unit function.

Then the potential of a plane φ_1 wave reflected from the boundary ($z = 0$) of a half-space without a ball has the following form:

$$\varphi_1 = \mp f(\tau - r\cos\theta - 1 - 2h)H(\tau - r\cos\theta - 1 - 2h), \tag{2}$$

where the upper sign corresponds to the free surface, and the lower sign to the rigid wall.

Taking the acoustic approximation model for the liquid and by virtue of the superposition principle (linear problem), we represent the particle velocity potential of the medium φ as the sum of three terms:

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2, \tag{3}$$

where φ_2 is potential for disturbances introduced by the sphere.

The pressure arising in the acoustic medium is determined by the formula

$$p = p_0 + p_1 + p_2, \; p_i = -\frac{\partial \varphi_i}{\partial \tau}, \, (i = \overline{0,2}). \tag{4}$$

Taking into account the axial symmetry of the problem, the motion of the acoustic medium relative to the velocity potential φ_2 is described by the wave equation

$$\gamma^2 \frac{\partial^2 \varphi_2}{\partial \tau^2} = \Delta \varphi_2, \tag{5}$$

where Δ is Laplace operator in the spherical coordinate system r, θ, ϑ .

The flat boundary of a half-space is either a free surface or a rigid wall. In the case of a free surface, the boundary condition on the flat boundary is given by the equality

$$p_2\big|_{z=0} = 0, \tag{6}$$

or in the case of a rigid wall, it is defined as follows:

$$V_2\big|_{z=0} = 0. \tag{7}$$

On the surface of the sphere, the boundary condition has the following form:

$$V_2\big|_{r=1} = -V_0\big|_{r=1} - V_1\big|_{r=1},\ V_i = \frac{\partial \varphi_i}{\partial r},\ (i=\overline{0,2}), \tag{8}$$

Initial conditions are homogeneous:

$$\varphi_2\big|_{\tau=0} = \dot{\varphi}_2\big|_{\tau=0} = 0 \tag{9}$$

and at infinity there is no disturbance:

$$\lim_{r\to\infty} \varphi_2 = 0\,. \tag{10}$$

The problem statement is given in the following dimensionless quantities (dimensional quantities are indicated by a prime):

$$\varphi = \frac{\varphi'}{c_* b},\ p = \frac{p'}{\rho_* c_*^2},\ v = \frac{v'}{c_*},\ r = \frac{r'}{b},\ \tau = \frac{c_* t}{b},\ h = \frac{h'}{b},\ \gamma = \frac{c}{c_*},$$

where b is some characteristic linear dimension; t is time; ρ_* and c_* are the density of the liquid and the speed of propagation of waves in the liquid.

III. SOLUTION METHOD

The initial boundary value problem (1) - (10) is solved by applying the integral Laplace transform with respect to time τ (the superscript L denotes the transform; s is the transform parameter).

Taking into account the absence of a wave at infinity (10) in the image space of the Laplace transform, the velocity potential is represented as

$$\varphi_2^L = \sum_{n=0}^{\infty} \frac{1}{\sqrt{r}} A_n^L(s) K_{n+1/2}(r\gamma s) P_n(\cos\theta) + \sum_{p=0}^{\infty} \frac{1}{\sqrt{r_1}} B_p^L(s) K_{p+1/2}(r_1\gamma s) P_p(\cos\theta_1) \tag{11}$$

where $K_{n+1/2}(x)$ are modified Bessel functions of the second kind [9]; $P_n(\cos\theta)$ are Legendre polynomials [9]; $A_n^L(s)$, $B_p^L(s)$ are unknown functions that are determined from the boundary conditions; r_1 and θ_1 are the coordinates of a spherical coordinate system centered at a point O_1 symmetrical to a point O relative to the boundary of the half-space.

Further, taking into account the relationship between the coordinates r, θ and r_1, θ_1 on the boundary of the half-space ($z = 0$)

$$r\big|_{z=0} = r_1\big|_{z=0}, \qquad \theta\big|_{z=0} + \theta_1\big|_{z=0} = \pi \tag{12}$$

and also the properties of Legendre polynomials [12]

$$(-1)^n P_n(\cos\theta)\big|_{z=0} = P_n(\cos\theta_1)\big|_{z=0}, \tag{13}$$

From the boundary conditions (6) and (7), we obtain the relationship between the functions $A_n^L(s)$ and $B_p^L(s)$:

$$B_n^L(s) = \pm(-1)^n A_n^L(s)\,,$$

where the upper sign corresponds to a rigid wall and the lower sign to a free surface.

Using addition theorems for Bessel functions $K_{n+1/2}(x)$ [7], we obtain the following expressions for the images of the velocity potential in a spherical coordinate system (r, θ, ϑ):

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$$\varphi_2^L(r,\theta,s)=\sum_{n=0}^{\infty}\varphi_{2n}^L(r,s)P_n(\cos\theta)\,, \tag{14}$$

$$\varphi_{2n}^L(r,s)=\frac{1}{r^{n+1}s^n}\left[R_{n0}(r\gamma s)A_n^L(s)e^{-r\gamma s}+G_{n0}(r\gamma s)\sum_{p=0}^{\infty}C_{np}(s)A_p^L(s)e^{-2h\gamma s}\right], \tag{15}$$

$$C_{np}(s)=\frac{\pm(-1)^p(2n+1)}{4h\gamma s}\sum_{\sigma=|p-n|}^{p+n}b_\sigma^{(n0p0)}\frac{R_{\sigma0}(2h\gamma s)}{(2h\gamma s)^\sigma}\,,\;G_{n0}(s)=R_{n0}(-s)e^s-R_{n0}(s)e^{-s}\,,$$

$$R_{n0}(s)=\sum_{k=0}^{n}\alpha_{nk}s^{n-k}\,,\;\alpha_{nk}=\frac{(n+k)!}{(n-k)!k!2^k}\,,\;\alpha_{nk}=0\,,\;k<0\,,\;k>n$$

Where $b_\sigma^{(n0p0)}$ are Clebsch-Gordon coefficients [7]; $I_{n+1/2}(x)$ are modified Bessel functions of the first kind [12].

By performing the Laplace transform in (1) and (2) and using addition theorems for Bessel functions [12], we find the following expressions for the images of the expansion coefficients of potentials, φ_0, φ_1 in series in Legendre polynomials:

$$\varphi_{0n}^L(r,s)=f^L(s)\frac{(-1)^n(2n+1)}{2(r\gamma s)^{n+1}}e^{(r-1)\gamma s}[R_{n0}(-r\gamma s)-e^{-2r\gamma s}R_{n0}(r\gamma s)]\,, \tag{16}$$

$$\varphi_{1n}^L(r,s)=\mp f^L(s)\frac{(2n+1)}{2(r\gamma s)^{n+1}}e^{-2h\gamma s}e^{(r-1)\gamma s}[R_{n0}(-r\gamma s)-e^{-2r\gamma s}R_{n0}(r\gamma s)]\,. \tag{17}$$

Next, we expand the pressure p^L and velocity v^L images into series using Legendre polynomials, and, using the relationships between the potential and the velocity of particles in the medium, we find the following expressions for the coefficients of the velocity series:

$$V_{0n}^L=-f^L(s)\frac{(-1)^n(2n+1)}{2r^{n+2}(\gamma s)^{n+1}}e^{(r-1)\gamma s}[R_{n1}(-r\gamma s)-e^{-2r\gamma s}R_{n1}(r\gamma s)]\,, \tag{18}$$

$$V_{1n}^L=\pm f^L(s)\frac{(2n+1)}{2r^{n+2}(\gamma s)^{n+1}}e^{-2h\gamma s}e^{(r-1)\gamma s}[R_{n1}(-r\gamma s)-e^{-2r\lambda s}R_{n1}(r\gamma s)] \tag{19}$$

$$V_{2n}^L=-\frac{1}{r^{n+2}(\gamma s)^n}\left[R_{n1}(r\gamma s)A_n^L(s)e^{-r\gamma s}+G_{n1}(r\gamma s)\sum_{p=0}^{\infty}C_{np}(s)A_p^L(s)e^{-2h\gamma s}\right], \tag{20}$$

$$G_{n1}(s)=R_{n1}(-s)e^s-R_{n1}(s)e^{-s}\,,\;R_{n1}(s)=\sum_{k=0}^{n+1}\beta_{nk}s^{n+1-k}\,,\;\beta_{nk}=\alpha_{nk}-k\alpha_{n,k-1}\,.$$

The boundary condition on the surface of the sphere with respect to the coefficients of the series has the following form:

$$V_{2n}\big|_{r=1}=-V_{0n}\big|_{r=1}-V_{1n}\big|_{r=1} \tag{21}$$

Substituting expressions (18), (19) and (20) into the boundary condition (21) on the surface of the sphere, we obtain an infinite system of linear algebraic equations with respect to unknown functions $A_n^L(s)$, which we write in the form of a matrix equation:

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$$\mathbf{MA}y^2+\mathbf{F}^{(1)}\mathbf{A}x-\mathbf{F}^{(2)}\mathbf{A}xy^2=\mathbf{p}^{(1)}y+\mathbf{p}^{(2)}xy+\mathbf{p}^{(3)}y^3+\mathbf{p}^{(4)}xy^3\,, \tag{22}$$

$$x=e^{-2h\gamma s}\,,\ y=e^{-\gamma s}\,,$$

where $\mathbf{M}(s)$ is an infinite diagonal matrix with elements $M_n(s)$; $\mathbf{F}^{(k)}(s)$ are infinite matrices with elements $F_{np}^{(k)}(s)$ ($k=1,2$); $\mathbf{p}^{(i)}(s)$ are given infinite vectors of elements $p_{in}^{L}(s)$ ($i=\overline{1,4}$); $\mathbf{A}(s)$ is an infinite unknown vector with elements $A_n^L(s)$ and these elements are defined by the following expressions:

$$M_n(s)=R_{n1}(s)\,,\ F_{np}^{(1)}(s)=C_{np}(s)R_{n1}(-s)\,,\ F_{np}^{(2)}(s)=C_{np}(s)R_{n1}(s)\,,$$

$$p_n^{(1)}(s)=-f^L(s)\frac{(-1)^n(2n+1)R_{n1}(-s)}{2s}\,,\ p_n^{(2)}(s)=\pm f^L(s)\frac{(2n+1)R_{n1}(-s)}{2s}\,,$$

$$p_n^{(3)}(s)=f^L(s)\frac{(-1)^n(2n+1)R_{n1}(s)}{2s}\,,\ p_n^{(4)}(s)=\mp f^L(s)\frac{(2n+1)R_{n1}(s)}{2s}\,.$$

The solution of the system is represented as an infinite exponential series.

$$\mathbf{A}=\sum_{i,j=0}^{\infty}\mathbf{a}_{ij}(s)x^i y^{-j+1}\,, \tag{23}$$

which allows us to construct a solution to an infinite system without using the reduction method. $\mathbf{a}_{ij}(s)$ are infinite unknown vectors with elements, $a_{ij}^{(p)}(s)$ ($p\geq 0$) respectively. Substituting this solution series into system (22) and equating the coefficients $a_{ij}^{(p)}(s)$ with the same powers, we obtain recurrence relations for the coefficients x and y the initial conditions for them.

$$a_{00}^{(n)}(s)=\frac{p_n^{(3)}(s)}{M_n(s)}\,,\ a_{02}^{(n)}(s)=\frac{p_n^{(1)}(s)}{M_n(s)}\,,\ a_{0j}^{(n)}(s)=0\,,\ j\geq 1\,,\ j\neq 2\,,$$

$$a_{10}^{(n)}(s)=\frac{p_n^{(4)}(s)}{M_n(s)}+\sum_{p=0}^{\infty}C_{np}(s)a_{00}^{(p)}(s)\,,\ a_{1j}^{(n)}(s)=0\,,\ j\geq 1\,,\ j\neq 2\,,$$

$$a_{12}^{(n)}(s)=\frac{p_n^{(2)}(s)}{M_n(s)}+\sum_{p=0}^{\infty}C_{np}(s)a_{02}^{(p)}(s)-\frac{M_n(-s)}{M_n(s)}\sum_{p=0}^{\infty}C_{np}(s)a_{00}^{(p)}(s)\,, \tag{24}$$

$$a_{ij}^{(n)}(s)=\sum_{p=0}^{\infty}C_{np}(s)a_{i-1,j}^{(p)}(s)-\frac{M_n(-s)}{M_n(s)}\sum_{p=0}^{\infty}C_{np}(s)a_{i-1,j-2}^{(p)}(s)\,,\ i\geq 1\,,\ j\geq 2\,.$$

Recurrence relations allow us to define all elements $a_{ij}^{(p)}(s)$, ($p\geq 0$) of the corresponding columns $\mathbf{a}_{ij}(s)$ in the form of rational functions, which makes it possible to calculate their originals and, therefore, to find the originals of the coefficients of the pressure p^L and velocity V^L series using residue theory.

For the coefficients of the series, initial conditions and recurrence relations are obtained that do not require the use of the reduction method. The coefficients of the series of the sought functions are determined in the form of rational functions, which makes it possible to find the originals using residue theory.

In this case, the expansion coefficients of the hydrodynamic pressure and velocity are determined by the formulas

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$$p_{2n}^L(r,s) = -\frac{1}{r^{n+1}(\gamma s)^n}\sum_{i,j=0}^{\infty}\left[R_{n0}(r\gamma s)a_{ij}^{(n)}(s)y^r + +G_{n0}(r\gamma s)\sum_{p=0}^{\infty}C_{np}(s)a_{ij}^{(p)}(s)x\right]x^i y^{-j+1}, \quad (25)$$

$$V_{2n}^L(r,s) = -\frac{1}{r^{n+2}(\gamma s)^n}\sum_{i,j=0}^{\infty}\left[R_{n1}(r\gamma s)a_{ij}^{(n)}(s)y^r + +G_{n1}(r\gamma s)\sum_{p=0}^{\infty}C_{np}(s)a_{ij}^{(p)}(s)x\right]x^i y^{-j+1}.$$

IV. NUMERICAL RESULTS

As an example, we consider the diffraction of non-stationary plane pressure waves on a stationary rigid ball in a half-space of water, in which $c_* = 1500 m/s$, $\rho_* = 1000\, kg/m^3$. $b = R$. Dimensionless quantities are equal $h = 1.5$, $R = 1$, $\gamma = 1$. The function chosen as the law of change of the incident wave over time was $f(s) = \frac{1}{s}$.

The hydrodynamic pressure at points $\theta = 0$ (curve 1), $\theta = \pi/2$ (curve 2), and $\theta = \pi$ (curve 3) of the sphere surface is calculated. The numerical results are obtained taking into account five terms of the Legendre polynomial series. The graphs in Fig. 1 are obtained for a free surface, and the graphs in Fig. 2 are obtained for a rigid wall. At $\tau > 3$, the flat boundary, there is a significant effect on the change in the distribution of hydrodynamic pressure. In the case of a rigid wall (Fig. 2), at moments of time $\tau = 3$ and 4 at point $\theta = \pi$ (curve 3); $\tau = 4$ at point $\theta = \pi/2$ (curve 2); $\theta = 5$ at point $\theta = 0$ (curve 1), the pressure value increases significantly, which is caused by the influence of waves reflected from the flat boundary.

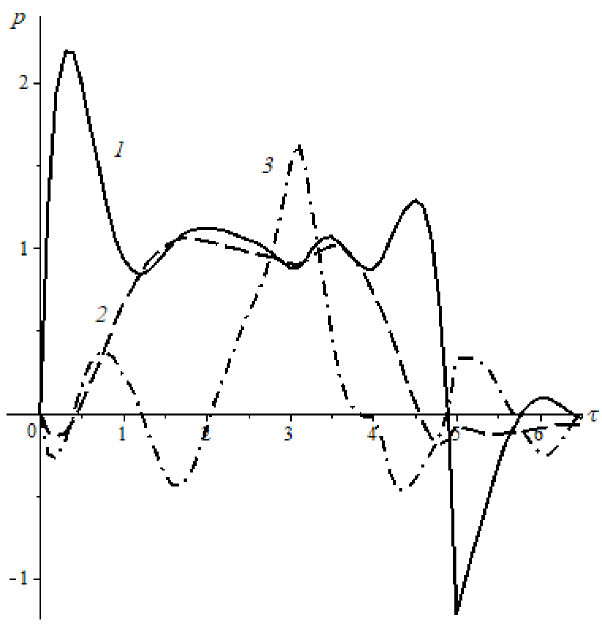

Fig. 1. Change in pressure in the case of a free surface.

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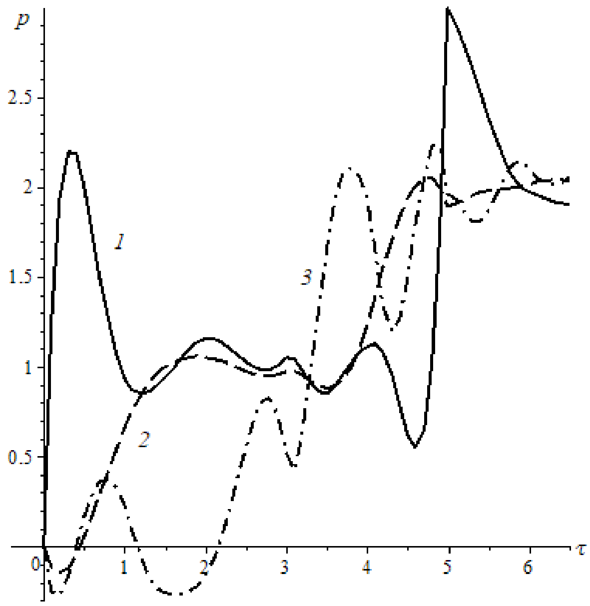

Fig. 2. Pressure change in the case of a rigid wall.

V. CONCLUSION

An algorithm for solving the problem of diffraction of non-stationary plane waves on an absolutely rigid sphere in an acoustic half-space has been developed. In the Laplace image space, an infinite system of linear algebraic equations has been obtained, the solution of which is constructed as an infinite series in exponentials, which made it possible to obtain a solution to the infinite system without using the reduction method. The graphs of the pressure dependence on time show that the waves reflected from the plane boundary affect the hydrodynamic state of the medium.

REFERENCES

[1] Babaev A.E. Problem of interaction of a non-stationary pressure wave with a system of two cylinders in an acoustic medium // Mat. Isled. "Approximate and numerical methods for solving boundary value problems." - Chisinau, 1988. - Issue. 101, pp. 3-9.
[2] Babaev A.E. Problems of diffraction of non-stationary waves on shells located near a flat boundary // Applied mechanics. - 1989. - Vol. 25, No. 1, pp. 71-83.
[3] Babaev A.E. Non-stationary waves in continuous media with a system of reflecting surfaces. - Kyiv: Nauk. Dumka, 1990. - 176 p.
[4] Babaev A.E., Zhirnov M.V., Kubenko V.D. Interaction of underwater shock waves with shells located near a rigid sphere // Abstract of the report of the All-Union Conf. "Numerical implementation of physical and mathematical problems of strength." - Gorky, 1987. - pp. 18-19.
[5] Gorshkov A.G., Tarlakovskiy D.V., Shukurov A.M. Propagation of waves from a spherical cavity in an acoustic half-space // PMM. - 1991. Vol. 55. No. 1, pp. 172-174.
[6] Dzyuba V. D., Kubenko V. D. Internal axisymmetric problem of interaction of a thin elastic cylindrical shell filled with a compressible fluid and immersed in an infinite compressible fluid with an oscillating sphere, Acoustic Bulletin, 2002. Vol. 5, N 2, pp. 33–42.
[7] Ivanov E. A. Diffraction of electromagnetic waves by two bodies. – Minsk: Science and Technology, 1968. – 584 p.
[8] Kubenko, V. D. Problem of diffraction of acoustic waves by two non-parallel circular cylinders. Appl. Mech. –1989. – Vol. 25, N 1. – Pp. 43–51.
[9] Kuznetsov D. S. Special functions. – Moscow: Higher School, 1985. – 423 p.
[10] Lavrentiev M.A., Shabat B.V. Methods of the Theory of Functions of a Complex Variable. – Moscow: Nauka, 1987. – 688 p.
[11] Ryabukha Yu. N. On the Solution of the Three-Dimensional Stationary Problem of Plane Wave Diffraction by a Spherical Shell in an Acoustic Half-Space // Prikl. Mekh. – 1985. Vol. 21, No. 8. – Pp. 40-45.
[12] Handbook of Special Functions with Formulas, Graphs, and Tables // Ed. M. Abramovits, I. Stigan. – Moscow: Nauka, 1979. – 832 p.
[13] Huang H., Transient Response of Two Fluid-Coupled Spherical Elastic Shells to an Incident Pressure Pulse, J. Acoust. Soc. Amer., Vol. 65, N 4, 1979, pp. 881-887.
[14] Lee, M. Finite element modeling of the acoustic wave propagation from a cavity near a free surface // Journal of Fluids Engineering, March 1999, Vol. 121, pp. 205-207.