



A Crisp and Fuzzy Inventory Model for a Two-Storage Management System Under a Linear Trend in Demand: A Structural Review

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ABSTRACT: The present paper deals with the development of an inventory model under crisp (deterministic) and Fuzzy (uncertainty) environment by considering the facilities of a two-storage named Owned Storage (OS) and Rented Storage (RS). A linear trend in demand is incorporated in the present model assuming constant deteriorating items in both OS and RS. The owned storage stands with finite capacity and the rented storage has large capacity of items. The total inventory costs for both crisp model and fuzzy model are derived. Signed Distance Method (SDM) is used to defuzzify the total cost function. Two illustrations are given for both the models separately. A sensitivity analysis of the optimal solution towards the changes of the system parameters is furnished. A pictorial presentation of the inventory cost with time is given for both the models. Lastly a structural review is demonstrated on the basis of the observation of the outcomes of the solution and pictorial scenario.

KEY WORDS: Inventory, crisp, fuzzy, deterioration, linear trend, owned storage, rented storage and defuzzify.

I. INTRODUCTION

Over the years, many inventory models for deteriorating items like food products, vegetables, chemicals, pharmaceuticals, blood, drugs, radioactive substances etc have attracted much attention in inventory analysis because these products deteriorate over time. Whitin [18] has considered first the fashion goods deteriorating during the storage period. After that many researchers Ghare & Schrader [8], Shah & Jaiswal [14] etc. developed several inventory models assuming different types deteriorating items.

Normally, the inventory models are mostly developed with single storage i.e. single warehouse space. But it is very difficult for big shops or showrooms located in main market places of city or town having another storage space due to inaccessibility of space. If it is managed to get such storage, they have to pay very high rents. Moreover, when a firm vast amount of goods for the future demand, they have a need for owned storage (OS) in existing market place, and the excess goods are stocked in rented storage (RS) with large capacity. This type of storage based inventory model was first developed by Hartley [9] in 1976. Thereafter, several publications were made under two-storage based inventory models. Researchers Bhunia et al [1], Mandal & Islam [11], Sheikh & Patel [15], Malik & Garg [12], Biswaranjan [2] are mentioned a few.

We know, the inventory management system has weakness on its applications under the uncertain or unrealistic behaviour of the different system parameters. Even many parameters have not clearly defined, their values are approximated base on subjective beliefs. So in various diverse circumstances, a fuzzy set theory plays a key role regarding development of inventory model under fuzzy sense. The main cost parameters are considered as fuzzy parameters to describe their uncertainties in a well manner. Even the fuzzy theory has the potential to provide a better result in comparison to a crisp model. Signed Distance Method(SDM) is used to defuzzify the total cost function. Zadeh [20] first discussed the new set theory named fuzzy set theory. Later many researchers like P K De & Rawat [6], Shekarian et al [16], Nayek et al [13], Yadav et al [19], Biswaranjan [3] etc have developed several fuzzy inventory models under various uncertainty constraints.

In inventory model, demand has an important role in real life. It is observed that demand has different types like constant demand, time-dependent demand, price-dependent demand, Weibul distributed demand, ramp type demand, quadratic demand, cubic demand and many more. In the literature, several researchers have developed many inventory models assuming different types of demand pattern. In our present model, efforts have been made to focus a crisp model and fuzzy model assuming linear trended in demand. On this view, we mentioned few research papers developed by Donaldson [7], Tripathy et al [17], Biswaranjan [4], Halim et al [10], Chowdhury & Ghosh [5] and many more.

Under the consideration of the above scenario, the present paper deals with an inventory model under both crisp and fuzzy senses describing a two-storage facilities with a linear trend in demand. The deterioration effects have been assumed in both the goods kept in owned storage (OS) and rented storage (RS). The Signed Distance Method (SDM) is used for defuzzification of the fuzzy inventory model. Shortages are not allowed here. Two illustrations are given for both the models separately. A sensitivity analysis of the optimal solution towards the changes of the system parameters is furnished. A pictorial presentation of the inventory cost with time is given for both the models. Lastly a structural review is demonstrated on the basis of the observation of the outcomes of the solution and pictorial scenario.

II. DEFINITIONS AND PRELIMINARIES

We have stated the following definitions for development of the fuzzy inventory model.

a) A fuzzy set X on the given universal set is a set of order pairs and defined by

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}, \text{ where } \lambda_{\tilde{A}} : X \rightarrow [0,1] \text{ is called membership function.}$$

b) A fuzzy number \tilde{A} is a fuzzy set on the real number R , if its membership function $\lambda_{\tilde{A}}$ has the following properties

- (i). $\lambda_{\tilde{A}}(x)$ is upper semi continuous.
- (ii). $\lambda_{\tilde{A}}(x) = 0$, outside some interval $[a_1, a_4]$

Then \exists real numbers a_2 and a_3 , $a_1 \leq a_2 \leq a_3 \leq a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$ and decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x) = 1$ for each $x \in [a_2, a_3]$.

c) A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is represented with membership function $\lambda_{\tilde{A}}$ as

$$\lambda_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2; \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3; \\ 0, & \text{otherwise} \end{cases}$$

d) Let \tilde{A} be a fuzzy set defined on R , then the signed distance of \tilde{A} is defined as

$$d(\tilde{A}, 0) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$$

Where $A_\alpha = [A_L(\alpha) + A_R(\alpha)] = [a + (b - a)\alpha, d - (d - c)\alpha]$, $\alpha \in [0, 1]$ is an α cut of a fuzzy set A .

e) Let $A = (a_1, a_2, a_3)$ is a triangular fuzzy number, then the Signed Distance Method of A is defined as

$$d(A, 0) = \frac{a_1 + 2a_2 + a_3}{4}$$

III. ASSUMPTIONS AND NOTATIONS

Notations:

- (i) $R(t)$: Demand rate.
- (ii) OS : Owned storage with finite capacity.
- (iii) RS : Rented storage with infinite capacity.
- (iv) $n_r(t)$: Stock amount in RS at time t.
- (v) $n_o(t)$: Stock amount in OS at time t.
- (vi) W_o : Storage capacity of OS
- (vii) x : The deterioration rate in RS where $0 \leq x < 1$
- (viii) y : The deterioration rate in OS where $0 \leq y < 1$
- (ix) T : The fixed length of each production cycle.
- (x) d_o : Ordering cost per order
- (xi) d_c : Deterioration cost per unit item in RS/OS.
- (xii) h_r : The storage cost per unit item in RS.
- (xiii) h_o : The storage cost per unit item in OS.
- (xiv) TAC : The total average cost of the system per unit time in the crisp model.
- (xv) d_c : The fuzzy deterioration cost per unit item in RS/OS.
- (xvi) h_r : The fuzzy storage cost per unit item in RS.
- (xvii) h_o : The fuzzy storage cost per unit item in OW.
- (xviii) TAC : The fuzzy total average cost of the system per unit time in the fuzzy model.

Assumptions:

- (i) Lead time is zero.
- (ii) Replenishment rate is infinite but size is finite.
- (iii) The time horizon is finite.
- (iv) There is no repair of deteriorated items occurring during the cycle.
- (v) The demand rate is a linear trended given by

$R(t) = a + bt$, where a and b are positive constants, a being initial demand rate and b being positive trend in demand.

- (vi) The storage cost per unit in RS is more than that of OS.
- (vii) Items are kept in OS first.
- (viii) The priority has been given to RS for first consumption.

IV. MATHEMATICAL FORMULATION

The proposed model deals with a two-storage inventory model. For RS, the inventory level $n_r(t)$ reaches at zero level at time $t = t_1$. During period $(0, t_1)$, the demand of the customer fulfils from RS, and in between some items deteriorates in OS in same period having inventory level $n_{o1}(t)$. After RS empty, the customers' demand fulfils by OS during the period (t_1, T) having inventory level $n_{o2}(t)$. The initial inventory for OS is W_o . A pictorial presentation of the proposed two-storage inventory model is given in Fig. 1.

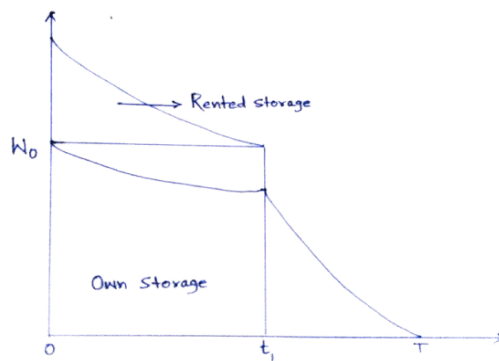


Figure 1: Proposed two-storage based inventory model

4.1: Crisp Model

$$\frac{dn_r(t)}{dt} + xn_r(t) = -(a + bt), 0 \leq t \leq t_1 \tag{4.1.1}$$

$$\frac{dn_{o1}(t)}{dt} + yn_{o1}(t) = 0, 0 \leq t \leq t_1 \tag{4.1.2}$$

and $\frac{dn_{o2}(t)}{dt} + yn_{o2}(t) = -(a + bt), t_1 \leq t \leq T \tag{4.1.3}$

Boundary Conditions $n_r(t_1) = 0, n_{o1}(0) = W_o$ and $n_{o2}(T) = 0 \tag{4.1.4}$

Solutions of the equations (4.1.1), (4.1.2) and (4.1.3) using (4.1.4) are the following:

$$n_r(t) = \left(\frac{a}{x} - \frac{b}{x^2}\right)[\exp\{x(t_1 - t)\} - 1] + \frac{bt_1}{x} \exp\{x(t_1 - t)\} - \frac{bt}{x}, 0 \leq t \leq t_1 \tag{4.1.5}$$

$$n_{o1}(t) = W_o \exp(-yt), 0 \leq t \leq t_1 \tag{4.1.6}$$

And $n_{o2}(t) = \left(\frac{a}{y} - \frac{b}{y^2}\right)[\exp\{y(T - t)\} - 1] + \frac{bT}{y} \exp\{y(T - t)\} - \frac{bt}{y}, t_1 \leq t \leq T \tag{4.1.7}$

Again $n_{o1}(t_1) = n_{o2}(t_1)$ gives

$$W_o = \left(\frac{a}{y} - \frac{b}{y^2}\right)\{\exp(yT) - \exp(yt_1)\} + \frac{b}{y}\{T \exp(yT) - t_1 \exp(yt_1)\} \quad (4.1.8)$$

Cost Components:

The total average cost (TAC) has the following components

1. Inventory Ordering Cost (IOC) = d_o
2. Inventory Storage Cost (ISC) in RS

$$\begin{aligned} ISC_R &= h_r \int_0^{t_1} n_r(t) dt = h_r \int_0^{t_1} \left[\left(\frac{a}{x} - \frac{b}{x^2}\right)\{\exp\{x(t_1 - 1)\} - 1\} + \frac{bt_1}{x} \exp\{x(t_1 - 1)\} - \frac{bt}{x}\right] dt \\ &= h_r \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2}\right)\{\exp(xt_1) - 1\} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x}\right] \end{aligned}$$

3. Inventory Storage Cost (ISC) in OS

$$\begin{aligned} ISC_O &= h_o \left[\int_0^{t_1} n_{o1}(t) dt + \int_{t_1}^T n_{o2}(t) dt\right] \\ &= h_o \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2}\right)\{\exp(yT) - \exp(yt_1)\} + \frac{b}{y^2}\{T \exp(yT) - t_1 \exp(yt_1)\} - \frac{a}{y}(T - t_1) - \frac{b}{2y}(T^2 - t_1^2)\right] \end{aligned}$$

4. Inventory Deteriorating Cost (IDC) in RS

$$\begin{aligned} IDC_R &= d_c \int_0^{t_1} x n_r(t) dt \\ &= d_c x \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2}\right)\{\exp(xt_1) - 1\} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x}\right] \end{aligned}$$

5. Inventory Deteriorating Cost (IDC) in OS

$$\begin{aligned} IDC_O &= d_c y \left[\int_0^{t_1} n_{o1}(t) dt + \int_{t_1}^T n_{o2}(t) dt\right] \\ &= d_c y \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2}\right)\{\exp(yT) - \exp(yt_1)\} + \frac{b}{y^2}\{T \exp(yT) - t_1 \exp(yt_1)\} - \frac{a}{y}(T - t_1) - \frac{b}{2y}(T^2 - t_1^2)\right] \end{aligned}$$

Therefore the total average cost per unit time is given by

$$TAC(t_1) = \frac{1}{T} [IOC + ISC_R + ISC_O + IDC_R + IDC_O]$$

$$= \frac{d_o}{T} + \frac{h_r + d_c x}{T} \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2} \right) \{ \exp(xt_1) - 1 \} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x} \right] + \frac{h_o + d_c y}{T} \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2} \right) \{ \exp(yT) - \exp(yt_1) \} + \frac{b}{y^2} \{ T \exp(yT) - t_1 \exp(yt_1) \} - \frac{a}{y} (T - t_1) - \frac{b}{2y} (T^2 - t_1^2) \right] \tag{4.1.9}$$

For minimum, the necessary condition is $\frac{dTAC(t_1)}{dt_1} = 0$

$$\text{Or, } \frac{h_r + d_c x}{x} [(a + bt_1) \{ \exp(xt_1) - 1 \}] - \frac{h_o + d_c y}{y} [(a + bt_1) \{ \exp(yt_1) - 1 \}] = 0 \tag{4.1.10}$$

Which is the equation for optimum solution.

Solving the equation (4.1.10), we get the optimum value of $t_1 = t_1^*$.

The optimum value of storage capacity (W_o) and the optimum total average cost of TAC (t_1) are obtained from the expressions (4.1.8) and (4.1.9) by putting $t_1 = t_1^*$.

4.2 Fuzzy Model :

We discussed here a fuzzy inventory model through the signed distance method (SDM). Due to global market scenario, the price of items fluctuating with storage costs and deterioration cost in RS and OS are seen in nature. These triangular fuzzy numbers

$$\square_{d_c} = (d_{c1}, d_{c2}, d_{c3}), \square_{h_r} = (h_{r1}, h_{r2}, h_{r3}) \text{ and } \square_{h_o} = (h_{o1}, h_{o2}, h_{o3})$$

Using signed distance method (SDM), the fuzzy total average cost is given by

$$\square_{TAC_{SDM}}(t_1) = \frac{1}{4T} \left\langle \begin{array}{c} \square_{TAC_{SDM1}}(t_1) \\ \square_{TAC_{SDM2}}(t_1) \\ \square_{TAC_{SDM3}}(t_1) \end{array} \right\rangle \tag{4.2.1}$$

Where

$$\square_{TAC_{SDM1}}(t_1) = \frac{d_o}{T} + \frac{h_{r1} + d_{c1} x}{T} \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2} \right) \{ \exp(xt_1) - 1 \} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x} \right] + \frac{h_{o1} + d_{c1} y}{T} \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2} \right) \{ \exp(yT) - \exp(yt_1) \} + \frac{b}{y^2} \{ T \exp(yT) - t_1 \exp(yt_1) \} - \frac{a}{y} (T - t_1) - \frac{b}{2y} (T^2 - t_1^2) \right]$$

$$\square_{TAC_{SDM2}}(t_1) = \frac{d_o}{T} + \frac{h_{r2} + d_{c2} x}{T} \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2} \right) \{ \exp(xt_1) - 1 \} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x} \right] + \frac{h_{o2} + d_{c2} y}{T} \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2} \right) \{ \exp(yT) - \exp(yt_1) \} + \frac{b}{y^2} \{ T \exp(yT) - t_1 \exp(yt_1) \} - \frac{a}{y} (T - t_1) - \frac{b}{2y} (T^2 - t_1^2) \right]$$

$$\text{And } \square_{TAC_{SDM3}}(t_1) = \frac{d_o}{T} + \frac{h_{r3} + d_{c3} x}{T} \left[\frac{1}{x} \left(\frac{a}{x} - \frac{b}{x^2} \right) \{ \exp(xt_1) - 1 \} - \frac{at_1}{x} + \frac{bt_1}{x^2} \exp(xt_1) - \frac{bt_1^2}{2x} \right] + \frac{h_{o3} + d_{c3} y}{T} \left[\frac{1}{y} \left(\frac{a}{y} - \frac{b}{y^2} \right) \{ \exp(yT) - \exp(yt_1) \} + \frac{b}{y^2} \{ T \exp(yT) - t_1 \exp(yt_1) \} - \frac{a}{y} (T - t_1) - \frac{b}{2y} (T^2 - t_1^2) \right]$$

Here

$$TAC_{SDM}(t_1) = \frac{1}{4T} [TAC_{SDM1}(t_1) + 2TAC_{SDM2}(t_1) + TAC_{SDM3}(t_1)] \quad (4.2.2)$$

The necessary condition for the minimization of the average cost $TAC_{SDM}(t_1)$ is

$$\frac{dTAC_{SDM}(t_1)}{dt_1} = 0$$

Or,
$$\frac{(h_{r1} + 2h_{r2} + h_{r3}) + x(d_{c1} + 2d_{c2} + d_{c3})}{x} [(a + bt_1)\{\exp(xt_1) - 1\}] - \frac{(h_{o1} + 2h_{o2} + h_{o3}) + y(d_{c1} + 2d_{c2} + d_{c3})}{y} [(a + bt_1)\{\exp(yt_1) - 1\}] = 0 \quad (4.2.3)$$

which gives the optimum values of t_1 .

$TAC_{SDM}(t_1)$ is minimum only if $\frac{d^2 TAC_{SDM}(t_1)}{dt_1^2} > 0$ would be satisfied for $t_1 > 0$.

The optimal fuzzy total average cost $TAC^*_{SDM}(t_1)$ is obtained by putting the optimal value $t_1 = t_1^*$ in the equation (4.2.2).

V. NUMERICAL ILLUSTRATION

The following two numerical illustrations are given for both the inventory models namely crisp model and fuzzy model.

Illustration 1: (Crisp Model):

The values of the parameters be as follows

$d_o = 500$ per order; $h_r = \$ 0.75$ per unit ; $h_o = \$ 0.4$ per unit ; $d_c = \$ 0.5$ per unit; $x = 0.06$; $y = 0.07$; $a = 200$; $b = 10$; $T = 1$ year.

Solving the equation (4.1.10) with the help of computer using the above parameter values, we find the following optimum outputs

$$t_1^* = 0.018 \text{ year; } W_o^* = 208.90 \text{ units and } TAC^* = \$ 546.032$$

It is also checked that this solution satisfies the sufficient condition for optimality.

Illustration 2: (Fuzzy Model):

Consider the fuzzy parameters are

$d_o = 500$ per order; $h_r = (0.5, 0.7, 0.9)$; $h_o = (0.2, 0.4, 0.6)$; $d_c = (0.4, 0.5, 0.6)$; $x = 0.06$; $y = 0.07$; $a = 200$; $b = 10$; $T = 1$ year.

Solving the equations (4.2.3) with the help of computer using the above values of fuzzy parameters, we find the following optimum outputs

$$t_1^* = 0.017 \text{ year; } W_o^* = 208.95 \text{ units and } TAC^* = \$ 546.031$$

VI. SENSITIVITY ANALYSIS AND PICTORIAL PRESENTATION.

The table indicates the comparative study of the inventory models for the storage capacity and the optimal total average costs. Also a pictorial presentation is furnished on the basis of following data. The results are shown in the following tables.

Table A: Effect of changes in the crisp and Fuzzy parameters on the model.

Changing parameter	% change in the system parameter	% change in Crisp Model		% change in Fuzzy Model	
		W_o^*	TAC^*	W_o^*	TAC^*
d_o	-50	-0.0004	-45.78	-1.02	-45.78
	-20	-0.0003	-18.31	-0.79	-18.31
	+20	0.0003	18.31	0.54	18.31
	+50	0.0004	45.78	1.03	45.78
h_r or h_r	-50	0.25	-0.003	0.22	-0.002
	-20	0.11	-0.001	0.06	-0.001
	+20	-0.04	0.001	-0.05	0.001
	+50	-0.08	0.003	0.19	0.002
h_o or h_o	-50	-0.09	-3.87	-0.11	-3.87
	-20	-0.05	-1.55	-0.06	-1.55
	+20	0.08	1.55	0.11	1.55
	+50	0.36	3.87	0.61	3.87
d_c or d_c	-50	-0.009	-0.34	0.004	-0.34
	-20	-0.0004	-0.14	0.0003	-0.14
	+20	0.0004	0.14	-0.0002	0.29
	+50	0.009	0.34	-0.002	0.54
x	-50	0.815	-0.001	0.91	-0.001
	-20	0.316	-0.0007	0.35	-0.0006
	+20	-0.307	0.0008	-0.33	0.0007
	+50	-0.739	0.0022	-0.81	0.0021
y	-50	-2.26	-0.43	-2.33	-0.43
	-20	-0.92	-0.17	-0.95	-0.19
	+20	0.95	0.18	0.98	0.18
	+50	2.42	0.44	2.54	0.44
a	-50	-49.25	-4.08	-49.25	-4.08
	-20	-19.71	-1.63	-19.70	-1.63
	+20	19.71	1.63	19.71	1.63
	+50	49.29	4.08	49.28	4.08
b	-50	-0.71	-0.14	-0.71	-0.14
	-20	-0.29	-0.05	-0.28	-0.05
	+20	0.29	0.06	0.29	0.06
	+50	0.73	0.14	0.73	0.14

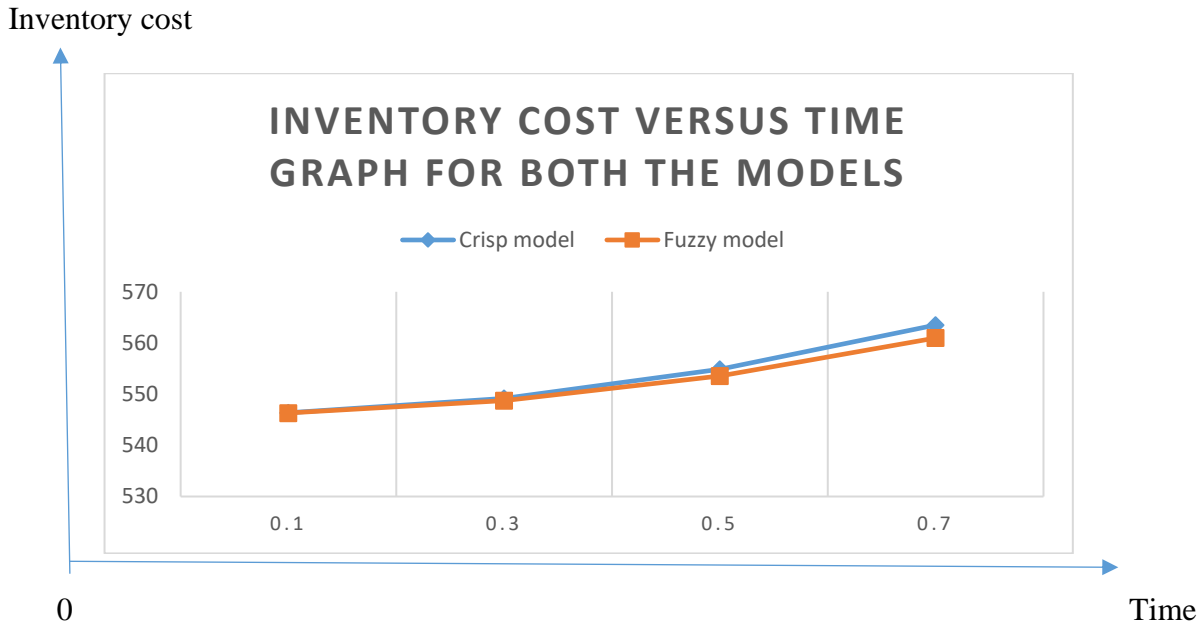


Figure-2

From Table A and Figure-2, the sensitivity analysis is performed by changing all parameters -50%, -20%, +20% and +50% taking one parameter at a time and keeping the other parameters unchanged. The observations may be made

- (i). The optimum value of storage capacity (W_o) increase or decrease with the increase or decrease in the values of the parameters d_o , h_o or h_r , d_c or d_r , y , a and b for both crisp model and fuzzy model. On the other hand, it increases or decreases with the decrease or increase of the system parameters h_r or h_r and x . The results obtained show that the optimum values of storage capacity for crisp model and fuzzy model are almost insensitive towards changes of the parameters d_o , h_r or h_r , h_o or h_o , d_c or d_c , x , y and b ; whereas these are highly sensitive towards changes of the initial demand rate a .
- (ii). The optimum value of total average cost (TAC) increase or decrease with the increase or decrease in the values of the parameters d_o , h_r or h_r , h_o or h_o , d_c or d_c , x , y , a and b for both crisp model and fuzzy model. The results obtained show that the optimum values of total average cost (TAC) are almost insensitive towards changes of the parameters h_r or h_r , h_o or h_o , d_c or d_c , x , y , a and b ; whereas these are highly sensitive towards changes of the inventory ordering cost parameter d_o .
- (iii) The pictorial presentation (Figure 2) on inventory cost versus time shows that the value of the optimal fuzzy total average cost is less than the value of the optimum total average cost. So the fuzzy model is more optimum than the crisp model.

**VII. CONCLUDING REMARKS**

The present article is a two-storage based inventory model for deteriorating items with a linear trend in demand under crisp and fuzzy environment. The owned storage stands with finite capacity and the rented storage has large capacity of items. The total inventory costs for both crisp model and fuzzy model are derived. Signed Distance Method (SDM) is used to defuzzify the total cost function. Shortages are not allowed in the proposed paper. On the outcomes of the solution from two illustrations and a sensitivity analysis of the optimal solution with pictorial presentation shows that the initial demand rate a and the inventory ordering cost parameter d_o are highly sensitive on the optimum value of storage capacity (W_o) and the optimum value of total average cost (TAC) respectively. Moreover, it is also reviewed from the graphical presentation that the total average cost in fuzzy model is smaller as compared to crisp model. This research work can be extended further to an inflationary inventory model assuming shortages with trade credit policy.

REFERENCES

- (1) Bhunia A K, Jaggi C K, Sharma A and Sharma R (2014) : "A two warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging", Applied Mathematics and Computation, 232(1), pp. 1125-1137.
- (2) Biswaranjan Mandal (2023) : "A two-warehouse fuzzy inventory model of deteriorating items under exponentially increasing demand", Int. J. Sc. & Res., 12(9), pp. 977-983.
- (3) Biswaranjan Mandal (2023) : "Optimization of fuzzy inventory model for deteriorating items under stock dependent linear trended demand with variable holding cost function", Strad research, 17(5), pp. 250-261.
- (4) Biswaranjan Mandal (2020) : "An inventory management system for deteriorating and ameliorating demand with linear trended in demand and partial backlogging", Global Journal of Pure and Applied Mathematics, 16(6), pp. 759-770.
- (5) Chowdhury R R and Ghosh S K (2022) : "A production inventory model for perishable items with demand dependent demand rate and a variable holding cost". Int. J. Prod. Mgmt., 15, pp. 424-446.
- (6) De P K and Rawat A (2011) : "A fuzzy inventory model without shortages using triangular fuzzy number", Fuzzy information and engineering, 3, pp. 59-68.
- (7) Donaldson W A (1977): "Inventory replenishment policy for a linear trend in demand – an analytical solution". Operational Research Quarterly, 28, pp. 663 -670.
- (8) Ghare P M and Schrader G F (1963) : "A model for an exponentially decaying inventory", Journal of Indus Engr, 14, pp. 238-243.
- (9) Hertley V Ronald (1976) : "On the EOQ model two levels of storage", Opsearch, 13, pp. 190-196.
- (10) Halim A., Paul M., Mahmoud M., Alshahrani B., Alazzawi A Y M and Ismail G M (2021): "An overtime production inventory model for deteriorating items with non-linear price and stock dependent demand", Alexandria Eng. Journal, 60, pp. 2779-2786.
- (11) Mandal W A and Islam S (2015) : "A fuzzy two-warehouse inventory model for weibull deteriorating items with constant demand, shortages and fully backlogged", Int. J of Sc. & Res, 4(7), pp. 1621-1624.
- (12) Malik A K and Garg H (2021) : "An improved fuzzy inventory model under two warehouses", J. of Artificial Intelligence and Systems, 3, pp. 115-129.
- (13) Nayek D K, Routray S S, Paikray S K and Dutta H (2021) : "A fuzzy inventory model for Weibull deteriorating items under completely backlogged shortages", Discrete and Continuous Dynamical Systems, 14, pp. 2435-2453.
- (14) Shah Y K and Jaiswal M C (1977) : "An order level inventory model for a system with constant rate of deterioration", Opsearch, 14, pp. 174-184.
- (15) Sheikh S R and Patel R (2017) : Two warehouse inventory model with different deterioration rated under time dependent demand and shortages", Global J of Pure and Applied Math, 13(8), pp. 3951 -3960.
- (16) Shekarian E, Kazemi N, Abdul Rashid S H and Olugu E U (2017) : "Fuzzy inventory models : A comprehensive review", Applied soft computing, 55, pp. 588-621.
- (17) Tripathy R. P., Pareek S. and Kaur M. (2017): "Inventory model with exponential time dependent demand rate, variable deterioration, shortages and production cost", Int. J of Appl. and Comp. Mathematics, 3, pp. 1407-1419.
- (18) Whittin T M (1957) : Theory of inventory management", Princeton University Press, Princeton, NJ, pp. 62-72.
- (19) Yadav V, Chaturvedi B K and Malik A K (2022) : "Development of fuzzy inventory model under decreasing demand and increasing deterioration rate", Int. J of Future Revolution in Comp. Sc. & Communication Eng., 8(4), pp. 1-7.
- (20) Zadeh L A (1965) : Fuzzy set. Information control, 8, pp. 338-353.