



A Two -Warehouse Inventory Management System for Deteriorating Items with Time-Dependent Quadratic Demand Rate

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ABSTRACT: My present paper deals with an inventory management system for deteriorating items with time-dependent quadratic demand rate. The movement of stock under the warehouse management has been considered in the proposed model. As the warehousing plays an important role in a firm's logistics system, we tried to incorporate two warehouses : the first is the owned warehouse (OW) with fixed capacity of W_o and the other one is rented warehouse (RW) with unlimited capacity. The nature of deterioration occurs in both the warehouses. The constant rate of deterioration is considered here. It is also noted to fact that firstly the demand is fulfilled from the inventory of RW and then it is used in OW. Normally retailer wants to sell the inventory in RW first, inventory of RW reaches zero level due to deterioration and demand, while that of OW is depleted due to deterioration only. A numerical example is given to illustrate the model. Some special cases are discussed along with its pictorial presentation furnishing the behaviour of the optimal cost in the proposed model and discussing the observation of the outcomes of the system solution.

KEY WORDS: Inventory, deteriorating, owned warehouse, rented warehouse, quadratic demand.

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I. INTRODUCTION

In today's era of globalization, poor inventory management affects sales, customer services and revenue, which have negative impact on any business section. Maintaining accurate records of inventory improves customer service by providing knowledge of customer's demand. So inventory management is absolutely concerned with the development of policies in any branch of business management. These policies are focused with: planning and programming, purchasing, storage and care, disposal of surplus stores, loss due to deterioration of products etc. Maintaining all these factors, optimization of loss or profit of the system becomes fulfilment.

However, we know, the inventory management and warehouse management are two facets of managing stock. Warehousing is a set of activities that are involved in receiving, storing goods and preparing them for the next destination. Again warehouse reduces lead time, logistic costs and increase competitive advantage. It is also very important for "storage of raw materials, WIP and finished goods seasonal and in transit, efficient distribution of goods during adverse time, price stabilization during time of scarcity, presentation of deteriorating items like fruits, meats, drugs, pharmaceutical products, radioactive substance etc. So apart from single warehouse system, the retailers use two storage facilities : one generally in the market named as owned warehouse (OW) with finite capacity, other is placed nearby it with large capacity named rented warehouse (RW). In this context, I must mention many researchers like Pakkala T P [1], Bhunia and Maiti [2], Lee C C [3], Kumar B. et al [4], Deepa Khurana [5], Sheikh S R and Patel R[6], Lee S. S. and Kim Y. G[7] etc. developing several inventory models on two-warehouse management system.

The assumption of a constant demand rate may not be always appropriate for many inventory items. For example milk, vegetables etc., the age of inventory has a negative impact on demand due to loss of consumer confidence on the quality of such products. So many researchers like Donaldson [8], Silver [9], Goswami et al [10], Biswaranjan [11] etc. are names to only a few to develop linear trended time varying demand models. In the competitive market, the demand of some product may increase due to the consumer's preference on some eye-catching product. Therefore, the demand

of the product at the time of its growth and the phase of declination may be approached by continuous-time-dependent function. These continuous-time-dependent functions may be a function of exponential or ramp type, linear type, quadratic, cubic in nature. Ritchie [12] discussed the solution of a linear increasing time-dependent demand, which is obtained by Silver and Meal [13] developed a model for deterministic time-varying demand, which also gives an approximate solution procedure termed as Silver-Meal Heuristic. Ramp type demand pattern is considered by Biswaranjan [14], Exponential demand has been developed by M. Dhivya Lakshmi [15] and Quadratic Time-Dependent Demand by many researchers like Shukla et al [16] etc.

For these sort of solutions, efforts have been made to develop a two-warehouse inventory model in presence of deteriorating items under the time-dependent quadratic demand rate over a fixed time horizon. Shortages are not allowed here. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a pictorial presentation furnishing the behaviour of the optimal cost in the proposed model and discussing the observation on the outcomes of the system solution.

II. NOTATIONS AND ASSUMPTIONS

The present inventory model is developed under the following notations and assumptions:

Notations:

- (i) $R(t)$: Demand rate.
- (ii) OW : Owned warehouse
- (iii) RW : Rented warehouse
- (iv) $N_r(t)$: Stock amount in RW at time t.
- (v) $N_o(t)$: Stock amount in OW at time t.
- (vi) Q : The initial ordered quantity.
- (vii) W_0 : Storage capacity of OW.
- (viii) x : The deterioration rate in RW where $0 \leq x < 1$
- (ix) y : The deterioration rate in OW where $0 \leq y < 1$
- (x) T : The fixed length of each production cycle.
- (xi) d_o : Ordering cost per order
- (xii) d_c : Deterioration cost per unit item in RW/OW.
- (xiii) h_r : The storage cost per unit item in RW.
- (xiv) h_o : The storage cost per unit item in OW.
- (xv) TC: The average total cost in the system.

Assumptions:

- (i). Lead time is zero.
- (ii). Replenishment rate is infinite but size is finite.
- (iii). The time horizon is finite.
- (iv). The products considered in this model are deteriorating in nature.
- (v). The demand rate is a time-dependent quadratic demand function

$R(t) = a + bt + ct^2$, $a, b, c \geq 0$ where a is the initial demand rate, b is the initial rate of change of demand and c is the rate at which the demand rate increases.

- (vi). The storage cost per unit in RW is more than that of OW.
- (vii). Items are kept in OW first.

(viii). The priority has given to RW for first consumption.

III. MATHEMATICAL FORMULATION

The proposed model deals with a two-warehouse inventory model. For RW, the inventory level $N_r(t)$ reaches at zero level at time $t = t_1$. During period $(0, t_1)$, the demand of the customer fulfils from RW, and in between some items deteriorates in OW in same period having inventory level $N_{o1}(t)$. After RW empty, the customers' demand fulfils by OW during the period (t_1, T) having inventory level $N_{o2}(t)$. The initial inventory for OW is W_o . A pictorial presentation of the proposed two-warehouse inventory model is given in Fig. 1.

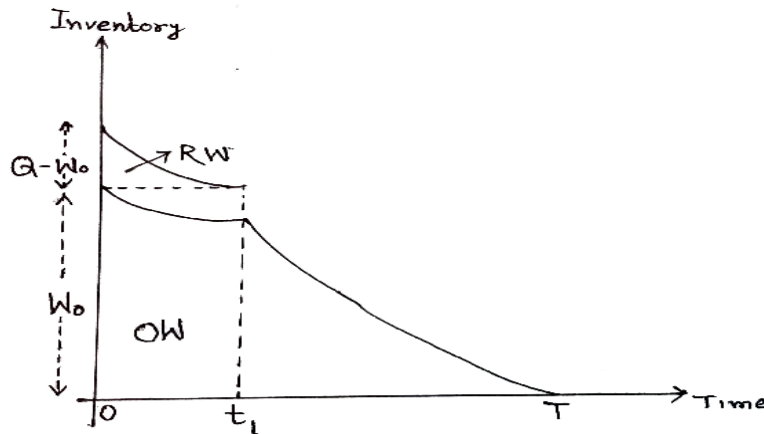


Fig. 1: The proposed two-warehouse inventory model

The differential equations of the proposed model are governed by the following

$$\frac{dN_r(t)}{dt} + xN_r(t) = -(a + bt + ct^2), 0 \leq t \leq t_1 \tag{3.1}$$

$$\frac{dN_{o1}(t)}{dt} + yN_{o1}(t) = 0, 0 \leq t \leq t_1 \tag{3.2}$$

And $\frac{dN_{o2}(t)}{dt} + yN_{o1}(t) = -(a + bt + ct^2), t_1 \leq t \leq T$ (3.3)

The boundary conditions are $N_r(t_1) = 0, N_r(0) = Q - W_o, N_{o1}(0) = W_o, N_{o2}(T) = 0$ (3.4)

The solutions of the equations (3.1), (3.2) and (3.3) using (3.4) are given by the following (neglecting second and higher order powers of x and y as $0 < x, y < 1$)

$$N_r(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{ax}{2}(t_1^2 - 2tt_1 + t^2) + \frac{bx}{6}(2t_1^3 - 3tt_1^2 + t^3) + \frac{cx}{12}(3t_1^4 - 4tt_1^3 + t^4), 0 \leq t \leq t_1 \tag{3.5}$$

$$N_{o1}(t) = W_o e^{-yt} = W_o(1 - yt), 0 \leq t \leq t_1 \tag{3.6}$$

And $N_{o2}(t) = a(T - t) + \frac{b}{2}(T^2 - t^2) + \frac{c}{3}(T^3 - t^3) + \frac{ay}{2}(T^2 - 2tT + t^2) + \frac{by}{6}(2T^3 - 3tT^2 + t^3) + \frac{cy}{12}(3T^4 - 4tT^3 + t^4)$, $t_1 \leq t \leq T$ (3.7)

Since $N_{o1}(t_1) = N_{o2}(t_1)$, we get the following expression of storage capacity of OW using the equations (3.6) and (3.7)

$$W_o = a(T - t_1) + \frac{b + ay}{2}(T^2 - t_1^2) + \frac{c + by}{3}(T^3 - t_1^3) + \frac{cy}{4}(T^4 - t_1^4) \quad (3.8)$$

Again $N_r(0) = Q - W_o$ gives

$$Q = W_o + at_1 + \frac{b + ax}{2}t_1^2 + \frac{c + bx}{3}t_1^3 + \frac{cx}{4}t_1^4 \quad (3.9)$$

Now, TC (The average total cost in the system) consists of the following cost components:

1. The inventory ordering cost (IOC) = d_o (3.10)
2. The inventory storage cost (ISC) in RW is

$$\begin{aligned} ISC_R &= h_r \int_0^{t_1} N_r(t) dt \\ &= h_r \int_0^{t_1} \left\{ a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{ax}{2}(t_1^2 - 2tt_1 + t^2) + \frac{bx}{6}(2t_1^3 - 3tt_1^2 + t^3) + \frac{cx}{12}(3t_1^4 - 4tt_1^3 + t^4) \right\} dt \\ &= h_r \left\{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 + \frac{cx}{10}t_1^5 \right\} \end{aligned} \quad (3.11)$$

3. The inventory storage cost (ISC) in OW is

$$ISC_o = h_o \left[\int_0^{t_1} N_{o1}(t) dt + \int_{t_1}^T N_{o2}(t) dt \right]$$

Integrating and then putting the value of W_o given in (3.8), we get the following

$$ISC_o = h_o \left\{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{c}{4}(T^4 - t_1^4) + \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) + \frac{cy}{10}(T^5 - t_1^5) \right\} \quad (3.12)$$

4. The inventory deterioration cost in RW is given by

$$IDC_R = d_c \int_0^{t_1} xN_r(t) dt = d_c x \left\{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 + \frac{cx}{10}t_1^5 \right\} \quad (3.13)$$

5. The inventory deterioration cost in OW is given by

$$\begin{aligned} IDC_o &= d_c \left[\int_0^{t_1} yN_{o1}(t) dt + \int_{t_1}^T yN_{o2}(t) dt \right] \\ &= d_c y \left\{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{c}{4}(T^4 - t_1^4) + \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) + \frac{cy}{10}(T^5 - t_1^5) \right\} \end{aligned} \quad (3.14)$$

Therefore, the average total cost per unit time is

$$TC(t_1) = \frac{1}{T} [IOC + ISC_R + ISC_o + IDC_R + IDC_o]$$

$$\begin{aligned}
 &= \frac{1}{T} [d_o + h_r \{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 + \frac{cx}{10}t_1^5 \} + h_0 \{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{c}{4}(T^4 - t_1^4) \\
 &+ \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) + \frac{cy}{10}(T^5 - t_1^5) \} + d_c x \{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 + \frac{cx}{10}t_1^5 \} + \\
 &d_c y \{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{c}{4}(T^4 - t_1^4) + \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) + \frac{cy}{10}(T^5 - t_1^5) \}] \quad (3.15)
 \end{aligned}$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

$$\begin{aligned}
 \text{Or, } (h_r + d_c x) \{ at_1 + (b + \frac{ax}{2})t_1^2 + (c + \frac{bx}{2})t_1^3 + \frac{cx}{2}t_1^4 \} \\
 - (h_o + d_c y) \{ at_1 + (b + \frac{ay}{2})t_1^2 + (c + \frac{by}{2})t_1^3 + \frac{cy}{2}t_1^4 \} = 0 \quad (3.16)
 \end{aligned}$$

For minimum, the sufficient condition $\frac{d^2 TC(t_1)}{dt_1^2} > 0$ would be satisfied.

Solving the equation (3.16), we get the optimal value of $t_1 = t_1^*$.

The optimal values of the storage capacity (W_0), the initial ordered quantity (Q) and the average total cost (TC) are obtained by putting $t_1 = t_1^*$ from the expressions (3.8), (3.9) and (3.15).

IV. SOME SPECIAL CASES

(a). Absence of deterioration:

If the deterioration of items is ignored in both the warehouses i.e. $x = 0, y = 0$, then the expressions (3.8), (3.9) and (3.15) of the storage capacity (W_0) and initial ordered quantity (Q) and average total cost per unit time (TC (t_1)) during the period [0,T] become

$$W_o = a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3}(T^3 - t_1^3) \quad (4.1)$$

$$Q = W_o + at_1 + \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3 \quad (4.2)$$

And

$$TC(t_1) = \frac{1}{T} [d_o + h_r \{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 \} + h_0 \{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{c}{4}(T^4 - t_1^4) \}] \quad (4.3)$$

The equation (3.16) becomes

$$h_r (at_1 + bt_1^2 + ct_1^3) - h_o (at_1 + bt_1^2 + ct_1^3) = 0 \quad (4.4)$$

This gives the optimum value of t_1 .

(b). If the demand rate is linear trended function of time then $c = 0$

From (3.8) and (3.9), the storage capacity (W_0) and initial ordered quantity (Q) become

$$W_o = a(T - t_1) + \frac{b + ay}{2}(T^2 - t_1^2) + \frac{by}{3}(T^3 - t_1^3) \tag{4.5}$$

$$Q = W_o + at_1 + \frac{b + ax}{2}t_1^2 + \frac{bx}{3}t_1^3 \tag{4.6}$$

From (3.15), the average total cost per unit time of the system during the cycle [0,T] becomes

$$\begin{aligned} TC(t_1) = & \frac{1}{T} [d_o + h_r \{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 \} + h_o \{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) \\ & + \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) \} + d_c x \{ \frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{ax}{6}t_1^3 + \frac{bx}{8}t_1^4 \} + \\ & d_c y \{ \frac{a}{2}(T^2 - t_1^2) + \frac{b}{3}(T^3 - t_1^3) + \frac{ay}{6}(T^3 - t_1^3) + \frac{by}{8}(T^4 - t_1^4) \}] \end{aligned} \tag{4.7}$$

The equation (3.16) becomes

$$(h_r + d_c x) \{ at_1 + (b + \frac{ax}{2})t_1^2 + \frac{bx}{2}t_1^3 \} - (h_o + d_c y) \{ at_1 + (b + \frac{ay}{2})t_1^2 + \frac{by}{2}t_1^3 \} = 0 \tag{4.8}$$

which is the equation to find out the optimum value of t_1 .

(c). If the demand rate is constant function of time then $b = 0$ and $c = 0$

From (3.8) and (3.9), the storage capacity (W_o) and initial ordered quantity (Q) become

$$W_o = a(T - t_1) + \frac{ay}{2}(T^2 - t_1^2) \tag{4.9}$$

$$Q = W_o + at_1 + \frac{ax}{2}t_1^2 \tag{4.10}$$

From (3.15), the average total cost per unit time of the system during the cycle [0,T] becomes

$$\begin{aligned} TC(t_1) = & \frac{1}{T} [d_o + h_r (\frac{a}{2}t_1^2 + \frac{ax}{6}t_1^3) + h_o \{ \frac{a}{2}(T^2 - t_1^2) + \frac{ay}{6}(T^3 - t_1^3) \} + d_c x (\frac{a}{2}t_1^2 + \frac{ax}{6}t_1^3) + \\ & d_c y \{ \frac{a}{2}(T^2 - t_1^2) + \frac{ay}{6}(T^3 - t_1^3) \}] \end{aligned} \tag{4.11}$$

The equation (3.16) becomes

$$(h_r + d_c x)(at_1 + \frac{ax}{2}t_1^2) - (h_o + d_c y)(at_1 + \frac{ay}{2}t_1^2) = 0 \tag{4.12}$$

This gives the optimum value of t_1 .

V. NUMERICAL EXAMPLES:

To explain the outcomes of offered model, we used a numerical example.

The following are the parametric values associated with the model:

$d_o = 500$ per order ; $a=30$; $b=20$; $c = 10$; $h_r = \$10$ per unit ; $h_o = \$8$ per unit ; $d_c = \$5$ per unit ; $x = 0.06$; $y = 0.07$; $T = 1$ year.

Solving the equation (3.16) using computer based optimization technique, we find the following optimum outcomes

$$t_1^* = 0.212 \text{ years, } W_o^* = 38.15 \text{ units, } Q^* = 45.02 \text{ units and } TC^* = \$ 708.20$$

It is checked that this solution satisfies the sufficient condition for optimality.

VI. COMPARISON STUDY AND PICTORIAL PRESENTATION.

The comparative stud is furnished here to illustrate the different aspects of the inventory model

Table A:

Inventory model	Optimum values of			
	t_1	W_o	Q	TC
Absence of deterioration	0.199	36.93	43.33	694.64
Linear Trended in Demand	0.201	35.01	41.51	686.60
Constant rate of demand	0.169	25.95	31.05	629.01



Fig 2: Pictorial presentation of different special cases of inventory models.

VII. OBSERVATIONS

Analyzing the results of table, A and pictorial presentations (Fig 2), the following observations may be made:

- (i) The optimum value of storage capacity in OW and initial ordered quantity (Q) change very less sensitively in the presence or in the absence of deterioration of the items proposed in the inventory model. On the other hand, the optimum value of average total cost changes moderately for the same.
- (ii) Similarly the optimum value of storage capacity in OW and initial ordered quantity (Q) change very less sensitively when the demand rate is time dependent function with quadratic, linear and constant in nature. On the other hand, the optimum value of average total cost changes moderately for the same.



- (iii) It is also observed that the optimum value of average total cost of the inventory system is minimum when the demand rate is constant in nature. So the demand parameters b and c play an important role on the estimation of optimum cost of the present model and we need adequate attention to estimate these two demand parameters.

VIII. CONCLUDING REMARKS

Many researchers ignored the assumption of deterioration of two-warehouse inventory model. The present paper deals with a two-warehouse inventory model where the optimal inventory cost function is obtained using demand function assumed as a time dependent quadratic function with constant rate of deteriorating items. Shortages are not considered in the present model. Numerical illustration is performed along with some special cases and a pictorial presentation furnished to observe the nature of variation of optimal TC (average total cost) for different demand functions like quadratic, linear and constant. Also observation is made when deterioration of items are ignored. A future work will be further incorporated in the present model by considering fuzzy, inflation and time discounting, trade credit policy or profit based inventory model under this imprecise environment with shortages having fully or partially backlogged.

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