ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 10, Issue 3, March 2023

Mathematical models for modeling two-dimensional unsteady water movement at water facilites

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ABSTRACT: The article develops mathematical models and numerical methods for modeling two-dimensional unsteady water movement at water management facilities. And also, the models can be classified according to the solution methods used. The existing methods for solving the Saint-Venant equations are conditionally divided into three groups. The first category includes solutions obtained as a result of attempts to find the general integral of the Saint-Venant equations using rigorous mathematical analysis, when the method of differential characteristics is applied, followed by the use of equations in finite differences.

KEYWORDS: Mathematical models, numerical methods, hydraulic methods, solutions, Saint-Venant equations, Convection-diffuse model.

I. INTRODUCTION

Currently, approximate methods are very common for solving one-dimensional equations of unsteady motion of water, and they are very widely used in practical calculations. Here it is necessary to note two directions- the use of modified equations and the use of complete systems of Saint-Venant equations.
In the one-dimensional Saint-Venant case, the equation has the form

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=q$$
$$\frac{1}{g\omega}\left(\frac{\partial Q}{\partial t}+2v\frac{\partial Q}{\partial x}\right)+\left[1-\left(\frac{v}{c}\right)^2\right]\frac{\partial z}{\partial x}=\left[i+\frac{1}{B}\left(\frac{\partial \omega}{\partial x}\right)_{h=const}\right]\left(\frac{v}{c}\right)^2-\frac{Q|Q|}{K^2}, \qquad (1)$$

where:
$Q=Q(x,t)$ – water consumption; $z=z(x,\ t)$ is the coordinate of the free surface; g is the gravitational constant; i –bottom slope; $B=B(z)$ is the width of the flow over the surface of the free section; $\omega=\ \omega(z)$ is the free area of the flow; $c=c(z)$ is the propagation velocity of small waves; $K=K(z)$ – consumption module.
A significant advantage of hydraulic models is their versatility. They are applicable both in the design and operation of sections of rivers and canals. The disadvantages of hydraulic models are mainly related to processes in the riverbeds, where the emergence of so-called non-transit zones is observed - bushy or other sections of the river, where water hardly moves. Non-transit zones play the role of storage tanks; therefore, such zones should not be taken into account in the free flow section. Methods for identifying transition zones have not yet been developed, as a result of which they are not taken

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 10, Issue 3, March 2023

into account in the commonly used one-dimensional equations of water motion. In channels with proper maintenance, the appearance of non-transit zones is almost not observed, as a result of which the indicated disadvantages of hydraulic models are insignificant.
Thus, it is hydraulic models that are of the greatest interest for the study of dynamic processes in water management objects and systems.

II. MATERIAL AND METHODS

The presented models can be classified according to the solution methods used. The existing methods for solving the Saint-Venant equations are conditionally divided into three groups. The first includes solutions obtained as a result of attempts to find the general integral of the Saint-Venant equations using rigorous mathematical analysis, when the method of differential characteristics is applied, followed by the use of equations in finite differences.
The second group consists of solutions found with the help of mathematical analysis involving the theory of small amplitude waves.
The third group includes solutions obtained as a result of approximate integration of the Saint-Venant equations with their preliminary replacement by equations in finite differences.
Models based on solving modified one-dimensional Saint-Venant equations [1-3]. The convection-diffuse model is based on the neglect of the inertial terms of the equations and has the form

$$\frac{\partial Q}{\partial t}+\left(\frac{Q}{K}\frac{\partial K}{\partial h}\right)\frac{\partial Q}{\partial x}-\frac{K^2}{2b|Q|}\frac{\partial^2 Q}{\partial x^2}=0, \quad (2)$$

where K is the flow modulus.
In the cace of neglecting the slope of free surface, we obtain the kinematic wave equation

$$\frac{\partial \omega}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (3)$$
$$Q=\omega c\sqrt{Ri}$$

Models of the theory of small amplitude waves [4] suggest that all changes in hydraulic elements due to wave motion are, in fact, small quantities, so that the squares of these quantities, as well as their products, can be neglected. By linearizing the Saint-Venant equation about the steady motion, it is reduced to linear equations of hyperbolic type with constant coefficients, the values of which are determined for the initial uniform regime.
The advantage of the above models is the use of a small number of generally accepted and repeatedly tested initial positions, a clear and rigorous mathematical formulation of the problems that arise.
In many cases, based on the hydrodynamic theory, it is possible to perform detailed calculations of the course of the corresponding physical phenomena in a multidimensional spatial region and in time. An example of such a successful application of the theory is the calculation of the movement of water in the form of long waves. Among the long-wave motions subdivided according to dynamic features, two-dimensional processes in wide river channels, lakes, canals, and reservoirs are practically the most significant.
Multidimensional hydrodynamic processes characterized by long-wave disturbances find analogy in various areas of mechanics and geophysics, acoustics, gas dynamics, hydraulics, meteorology, seismology and other areas of science.
The theory of long waves belongs to the classical branches of hydrodynamics. The starting position of the theory is the hydrostatic law for pressure [5]

$$p(x,y)=\rho g(\xi(x,y)-z(x,y))+p_0(x,y), \quad (4)$$

where x, y are horizontal coordinates, the XOY coordinate plane coincides with the undisturbed surface of the liquid, the vertical z axis is directed upwards; ξ - excess of the water level above the equilibrium position, ρ - water density,

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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g - acceleration of gravity. So how ρ it is constant everywhere, which makes it possible to exclude internal waves from consideration.

The assumption of pressure hydrostaticity in the case of an ideal fluid has the consequence of independence from the z horizontal accelerations of the fluid particle (and hence the horizontal components of the velocity, if the motion begins from a state of rest). Neglecting vertical acceleration leads to the law of hydrostatics. This makes it possible to reduce the dimension of the space in which the process is studied and to consider motion in the two-dimensional XOY plane.

III. SIMULATION AND RESULTS:

The motion of a long wave (Figure. 1) is described by a differential equation (Stoker, 1959) [6]

$$\frac{dU}{dt} = F - g\nabla\xi \tag{5}$$

Where $U = \{u(x,y,t),\ v(x,y,t)\}$ and $F = \{Fx(x,y,t),\ Fy\ (x,y,t),\}$ is the velocity of an external force per unit mass, and the vector of external forces that do not depend on the vertical coordinate, g – acceleration of gravity, ξ – excess of the liquid level above its equilibrium position.

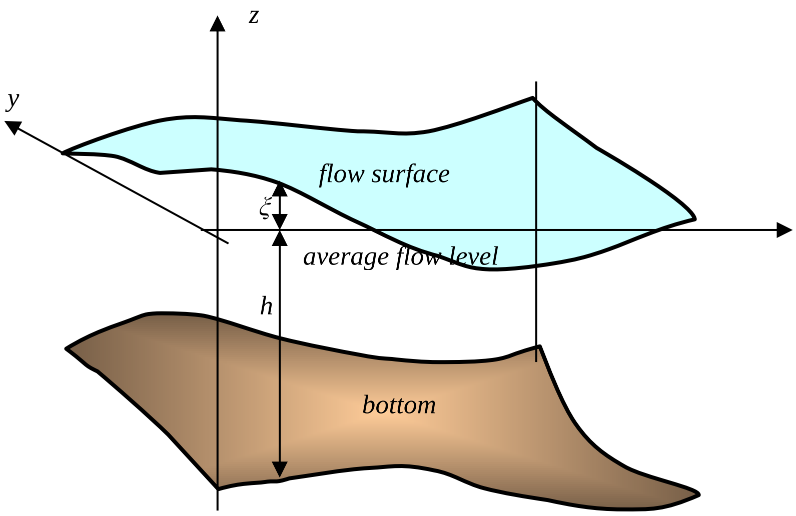

Figure.1. Diagram of the long wave motion in the XYZ coordinate axes..

Equation (1.3) expresses the law of conservation of the amount of motion. It is obtained from the basic equation of continuum mechanics [4]

$$\frac{dV}{dt} = f + \frac{1}{\rho} divT \tag{6}$$

where $\rho = \rho(x, y\ t)$ – density, $V = \{u, v, \omega\}$ – particle velocity, f – external force.

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International Journal of Advanced Research in Science, Engineering and Technology
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Assuming the absence of tangential stresses for the stress tensor T, which characterizes the reaction of the medium to internal forces; the stress system at any point of the liquid is reduced to uniform pressure (compression) and equation (5) is obtained if we take the hydrostatic law of change for the magnitude of this pressure p.

$$p = \rho g\left(\xi - z\right) + pa \qquad (7)$$

(pa – atmospheric pressure on the free surface) and put p =const.
The external forces for the tasks under consideration are, in addition to gravity, the friction force of wind on the water surface, the friction of water on the bottom, shores and atmospheric pressure. These forces are given as functions of spatial coordinates and time, they should be included in the expression for the vector of external forces F on the right side of equation (3). Let's attribute the acceleration of a liquid particle in this equation to a reference frame that is motionlessly connected to the Earth, replacing the left side of the equation with $\frac{dU}{dt} + 2\omega x U$ (ω is the vector of the angular velocity of the Earth's rotation). The resulting equation, the Euler equation, describes the motion of a long wave in an ideal incompressible fluid in a hydrostatic approximation, taking into account the Coriolis force. Unknown functions $U = \{u, v\}$ and ξ are determined under certain initial and conditions from equation (3) and the continuity equation expressing the law of conservation of mass in a prismatic column of fluid between infinitely close vertical planes [7]

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} Hu + \frac{\partial}{\partial y} Hv = 0 \qquad (8)$$

where $H\left(x, y, t\right) = h + \xi,\ h\ \left(x, y\right)$undisturbed depth of the liquid.
Using the formula

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + (U\nabla)U\,, \qquad (9)$$

let's write down the projections of equation (3) on the axis coordinates:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial H}{\partial x} = \Phi_X\,, \qquad (10)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial H}{\partial y} = \Phi_y\,, \qquad (11)$$

where is the vector

$$\Phi = \{\Phi x\ ,\ \Phi y\ \} \equiv F - 2\omega\ x\ U + gh. \qquad (12)$$

In the future, we will often use a convenient notation of the system of equations (1.4), (1.5) in the form of a single vector equation [8]

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial t} + B\frac{\partial U}{\partial y} = \Phi. \qquad (13)$$

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International Journal of Advanced Research in Science, Engineering and Technology
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Here

$$U = \begin{pmatrix} u \\ v \\ H \end{pmatrix} \quad A = \begin{pmatrix} u & 0 & g \\ 0 & u & 0 \\ H & 0 & u \end{pmatrix} \quad B = \begin{pmatrix} v & 0 & 0 \\ 0 & v & g \\ 0 & H & v \end{pmatrix} \tag{14}$$

If A, B do not depend on U, and the vector Φ depends on U non-linearly, the system of equations (13) is called almost linear. In general, when are matrices A and B depend on the components of the vector U, (13) represents a quasi-linear system of hyperbolic equations.
Two-dimensional Saint-Venant equations describing the unsteady flow of water in open channels [9]

$$\begin{aligned} &\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} + i = 0, \\ &\frac{\partial (uh)}{\partial t} + \frac{\partial (u^2 h)}{\partial x} + \frac{\partial (uvh)}{\partial y} + g\frac{\partial (h^2/2)}{\partial x} = gh(S_{ax} - S_{fx}), \\ &\frac{\partial (vh)}{\partial t} + \frac{\partial (v^2 h)}{\partial y} + \frac{\partial (uvh)}{\partial x} + g\frac{\partial (h^2/2)}{\partial y} = gh(S_{ay} - S_{fy}). \end{aligned} \tag{15}$$

Here x – the coordinate of the axis along the length; y – the coordinate of the axis in width; t – time; $h = h(x, y, t)$ – depth of the water surface; $u = u(x, y, t)$ – the longitudinal component of the water flow velocity; $v = v(x, y, t)$ – the transverse component of the water flow velocity; S_{ax} – slope of the bottom along the axis x, S_{ay} – slope of the bottom along the axis y, S_{fx} – slope of the free surface of the water along the axis x, S_{fy} – slope of the free surface of the water along the axis y; g – acceleration of gravity; $i(x, y, t)$ – intensity of water intake.
The ordinate of the channel bottom is given by the function ?, then the bottom slopes according to the corresponding coordinates are determined by [10]

$$S_{ax} = \frac{\partial z_0}{\partial x}, \quad S_{ay} = \frac{\partial z_0}{\partial y}, \tag{16}$$

Using the Manning formula, we obtain the slopes of free surfaces in ordinates [11].

$$\begin{aligned} &S_{fx} = \frac{n^2 u (u^2 + v^2)^{1/2}}{h^{4/3}} \\ &S_{fy} = \frac{n^2 v (u^2 + v^2)^{1/2}}{h^{4/3}}, \end{aligned} \tag{17}$$

Equation (15) refers to two-dimensional equations, quasi-linear equations of hyperbolic type.
Let's introduce the replacement of variables $p = uh$, $q = vh$ [12]. Then equation (15) has the form

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International Journal of Advanced Research in Science, Engineering and Technology
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$$\frac{\partial h}{\partial t}+\frac{\partial p}{\partial x}+\frac{\partial q}{\partial y}+i=0,$$

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}(\frac{p^2}{h}+\frac{gh^2}{2})+\frac{\partial}{\partial y}(\frac{pq}{h})+gh\frac{\partial z_0}{\partial x}+gn^2\frac{p(p^2+q^2)^{\frac{1}{2}}}{h^{\frac{7}{3}}}=0,$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial y}(\frac{q^2}{h}+\frac{gh^2}{2})+\frac{\partial}{\partial x}(\frac{pq}{h})+gh\frac{\partial z_0}{\partial y}+gn^2\frac{p(p^2+q^2)^{\frac{1}{2}}}{h^{\frac{7}{3}}}=0. \qquad (18)$$

Writing these equations in vector form, we get

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}+\mathbf{D}=0, \qquad (19)$$

where $\mathbf{U}$, F, G and D are vectors of the function

$$\mathbf{U}=\begin{pmatrix} h \\ p \\ q \end{pmatrix}, \qquad \mathbf{F}=\begin{pmatrix} p \\ \frac{p^2}{h}+\frac{gh^2}{2} \\ \frac{pq}{h} \end{pmatrix}, \qquad \mathbf{G}=\begin{pmatrix} p \\ \frac{pq}{h} \\ \frac{q^2}{h}+\frac{gh^2}{2} \end{pmatrix}, \qquad (20)$$

$$\mathbf{D}=\begin{pmatrix} i \\ gh\frac{\partial z_0}{\partial x}+gn^2\frac{p(p^2+q^2)^{\frac{1}{2}}}{h^{\frac{7}{3}}} \\ gh\frac{\partial z_0}{\partial y}+gn^2\frac{q(p^2+q^2)^{\frac{1}{2}}}{h^{\frac{7}{3}}} \end{pmatrix}, \qquad (21)$$

Since the functions $F(U)$ and $G(U)$ depend on the function U, equation (21) is written in the following form [13]

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial \mathbf{U}}\frac{\partial \mathbf{U}}{\partial x}+\frac{\partial \mathbf{G}}{\partial \mathbf{U}}\frac{\partial \mathbf{U}}{\partial y}+\mathbf{D}=0. \qquad (22)$$

Finally, we write equation (22) in vector-matrix form

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$$\frac{\partial \mathbf{U}}{\partial t}+\mathbf{A}\frac{\partial \mathbf{U}}{\partial x}+\mathbf{B}\frac{\partial \mathbf{U}}{\partial y}+\mathbf{D}=0, \tag{23}$$

where

$$\mathbf{A}=\frac{\partial \mathbf{F}}{\partial \mathbf{U}}=\begin{pmatrix} 0 & 1 & 0 \\ -\frac{p^2}{h^2}+gh & \frac{2p}{h} & 0 \\ -\frac{pq}{h^2} & \frac{q}{h} & \frac{q}{h} \end{pmatrix},\quad \mathbf{B}=\frac{\partial \mathbf{G}}{\partial \mathbf{U}}=\begin{pmatrix} 0 & 0 & 1 \\ -\frac{pq}{h} & \frac{q}{h} & 0 \\ -\frac{q^2}{h^2}+gh & 0 & \frac{2q}{h} \end{pmatrix}. \tag{24}$$

Without taking into account the inertial terms, equation (1.16) has the form [14]

$$\begin{aligned} &\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}+i=0, \\ &\frac{\partial h}{\partial x}=S_{ax}-S_{fx}, \\ &\frac{\partial h}{\partial y}=S_{ay}-S_{fy}. \end{aligned} \tag{25}$$

This equation refers to two-dimensional equations of the parabolic type
Thus, the two-dimensional Saint-Venant equation describing unsteady water flows in open channels in vector-matrix form has the form [15]

$$\frac{\partial \mathbf{U}}{\partial t}+\mathbf{A}\frac{\partial \mathbf{U}}{\partial x}+\mathbf{B}\frac{\partial \mathbf{U}}{\partial y}+\mathbf{D}=0, (x,\, y)\ \epsilon\ \Omega \tag{26}$$

where $U=\{h,\ p,\ q\}$.

$$\mathbf{A}=\begin{pmatrix} 0 & 1 & 0 \\ -\frac{p^2}{h^2}+gh & \frac{2p}{h} & 0 \\ -\frac{pq}{h^2} & \frac{q}{h} & \frac{p}{h} \end{pmatrix},\quad \mathbf{B}=\begin{pmatrix} 0 & 0 & 1 \\ -\frac{pq}{h} & \frac{q}{h} & \frac{p}{h} \\ -\frac{q^2}{h^2}+gh & 0 & \frac{2q}{h} \end{pmatrix},$$

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International Journal of Advanced Research in Science, Engineering and Technology
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$$\mathbf{D} = \begin{pmatrix} i \\ gh\frac{\partial z_0}{\partial x} + gn^2 \frac{p(p^2+q^2)^{1/2}}{h^{7/3}} \\ gh\frac{\partial z_0}{\partial y} + gn^2 \frac{q(p^2+q^2)^{1/2}}{h^{7/3}} \end{pmatrix}.$$

Here x – the coordinate of the axis along the length; y – the coordinate of the axis in width; t – time; $h = h(x, y, t)$ – the depth of the water flow; $p = uh = p(x, y, t)$ – the longitudinal component of the water flow rate; $q = vh = q(x, y, t)$ – the transverse component of the water flow rate; $u = u(x, y, t)$ – the longitudinal component of the water velocity of the water flow; $v = v(x, y, t)$ – the transverse component of the water flow velocity; $\partial z_0 / \partial x$ - slope of the bottom along the axis x, $\partial z_0 / \partial y$ – slope of the bottom along the axis y, n – roughness coefficient, g – acceleration of gravity; $i(x, y, t)$ – intensity of water intake.

IV. DISCUSSIONS

This article was written based on the material of the applied project QXA-7-031 “Development of a system for mathematical modeling of processes occurring in water management facilities, taking into account their multidimensional spatial distribution”, which was carried out under a contract with the Uzbek Scientific and Production Center for Agriculture of the Ministry of Agriculture and Water Resources of the Republic of Uzbekistan in the laboratory “Modern Information technologies in the water sector and water resources management "of the Research Institute of Irrigation and Water Problems 2012-2014, according to the priority areas for the development of the national economy of the Republic PFI - 5" Biology, biotechnology, soil science, water problems, genetics, plant and animal breeding ”, section “Creation of the theory of water flow with discrete distribution”.

V. CONCLUSION

The article develops mathematical models and numerical methods for modeling two-dimensional unsteady movement of water at water management facilities. And also, the created models can be classified according to the solution methods used. The existing methods for solving the Saint-Venant equations are conditionally divided into three groups. The first category includes solutions obtained as a result of attempts to find the general integral of the Saint-Venant equations using rigorous mathematical analysis, when the method of differential characteristics is applied, followed by the use of equations in finite differences.

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ISSN: 2350-0328

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