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Several Forms of $h(hv)-$Torsion Tensor C_{jkh} in Generalized $\mathcal{B}P$ $-$Birecurrent Space

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ABSTRACT: The aim of this paper is to extend the generalized $\mathcal{B}P-$birecurrent space by using the properties of $C2-$like space, $C-$reducible space, semi$-C-$reducible space and $C3-$like to get new spaces that are called $C2-\text{like}-G(\mathcal{B}P)-BRF_n$, $C-\text{reducible}-G(\mathcal{B}P)-BRF_n$, $semi-C-\text{reducible}-G(\mathcal{B}P)-BRF_n$ and $C3-\text{like}-G(\mathcal{B}P)-BRF_n$, respectively.

KEY WORDS: $C2-$like space, $C-$reducible space, semi$-C-$reducible space, $C3-$like space, generalized $\mathcal{B}P-$birecurrent space.

I. INTRODUCTION

Four forms of the $h(hv)-$torsion tensor C_{jkh} are called $C2-$like space, $C-$reducible space, semi$-C-$reducible space and $C3-$like space have been studied by the Finslerian geometers. Matsumoto and Numata [10] and Aveesh et al. [17] introduced definition for $C2-$like space. Singh and Gupta [18] discussed some properties for $C2-$like space.

Saxena [12] studied $C-$reducible Finsler space with Douglas tensor and gave the condition for Finsler space to be $C-$reducible Finsler space. Dwivedi [11] obtained every $C-$reducible Finsler space is $P-$reducible and converse is not necessarily true.

Tiwari et al. [7] and Heydari [3] introduced a definition for semi$-C-$reducible space and studied its properties. Also, Chethana and Narasimhamurthy [5] showed that every semi$-C-$reducible manifold with $C-$reducible metric reduces to a Landsberg manifold.

Tayebi and Peyghan [4], Tripathi and Pandey [6] and Numata [16] introduced a definition for $C3-$like space and discussed its relationship with other spaces in Finsler space. In addition, Gangopadhyay and Tiwari [13] obtained that $C3-$like Finsler metric may be considered as a generalization of $C-$reducible, semi$-C-$reducible and $C2-$like Finsler metrics.

Beizavi [14] introduced a definition for semi$-C-$reducible space and $C3-$like space. Also, he studied the relationship between $C3-$like metric with $C-$reducible metric, semi$-C-$reducible metric and $C2-$like metric. In this paper, special forms of the $h(hv)-$torsion tensor C_{ikh} in the generalized $\mathcal{B}P-$birecurrent space have been studied.

II. PRELIMINARIES

In this section, some conditions and definitions will be given for the purpose of this paper. An $n-$dimensional space X_n equipped with a function $F(x,y)$ which denoted by $F_n = (X_n, F(x,y))$ called a Finsler space if the function $F(x,y)$

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satisfying the request conditions [15, 20]. Matsumoto [9] introduced the $(h)hv$ −torsion tensor C_{ijk} that is positively homogeneous of degree -1 in y^i and symmetric in all its indices which is defined by

$$C_{ijk} = \frac{1}{2}\dot{\partial}_i\, g_{jk} = \frac{1}{4}\dot{\partial}_i\dot{\partial}_j\dot{\partial}_k\, F^2.$$

The above tensor satisfies the following

(2.1) $C^h_{ik} = g^{hj}C_{ijk}$,

where C^i_{jk} is called associate tensor of the tensor C_{ijk} .

Berwald's covariant derivative $\mathcal{B}_k T^i_j$ of an arbitrary tensor field T^i_j with respect to x^k is given by [8]

$$\mathcal{B}_k T^i_j = \partial_k T^i_j - (\dot{\partial}_r T^i_j)G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}.$$

Let Berwald's covariant derivative of second order for the $(h)hv$ −torsion tensor C_{ijk} and its associative C^i_{jk} which satisfy [19]

(2.2) $\mathcal{B}_l\mathcal{B}_m C^i_{kh} = a_{lm}C^i_{kh} + b_{lm}(\delta^i_k y_h - \delta^i_h y_k)$

and

(2.3) $\mathcal{B}_l\mathcal{B}_m C_{jkh} = a_{lm}C_{jkh} + b_{lm}(g_{jk}y_h - g_{jh}y_k)$,

where a_{lm} and b_{lm} are non - zero covariant tensors field.

Definition 2.1. A Finsler space $F_n (n \geq 2)$ with $C^2 = C_j C^j \neq 0$, it is called a $C2-$ *like space* if the $(h)hv$ −torsion tensor C_{jkh} can be written in the form [14, 18]

(2.4) $C_{jkh} = C_j C_k C_h / C^2$,

where $C_j = g^{kh}C_{jkh}$.

Definition 2.2. A Finsler space F_n is called a C −*reducible space* if the $(h)hv$ −torsion tensor C_{jkh} is characterized by the condition [12, 17]

(2.5) $C_{jkh} = \dfrac{1}{(n+1)}\left(h_{jk}C_h + h_{kh}C_j + h_{hj}C_k\right)$,

where $h_{jk} = g_{jk} - l_j l_k$ is an angular metric tensor.

Definition 2.3. A Finsler metric F_n is called *a semi−C −reducible* if the $(h)hv$ −torsion tensor C_{jkh} is given by [3, 5]

(2.6) $C_{jkh} = \left[\dfrac{p}{1+n}\left(h_{jk}I_h + h_{kh}I_j + h_{hj}I_k\right) + \dfrac{q}{\|\mathbf{I}\|^2} I_j I_k I_h\right]$,

where $p = p(x,y)$ and $q = q(x,y)$ are scalar function on F_n and $\|\mathbf{I}\|^2 = I^i I_i$.

Definition 2.4. A Finsler metric F_n is called a $C3$ −*like space* if the $(h)hv$ −torsion tensor C_{jkh} is given by [4, 16]

(2.7) $C_{jkh} = \left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]$,

where $A_i = A_i(x,y)$ and $B_i(x,y)$ are y −homogeneous scalar functions on F_n of degree -1 and 1, respectively.

Alaa et al. [1, 2] introduced the generalized $\mathcal{B}P-$birecurrent space which Cartan's second curvature tensor P^i_{jkh} satisfies the condition

(2.8) $\mathcal{B}_l\mathcal{B}_m P^i_{jkh} = a_{lm}P^i_{jkh} + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t\mu_m \mathcal{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}), \qquad P^i_{jkh} \neq 0$.

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This space is denoted by $G(\mathcal{B}P)-BRF_n$.

III. A $C2$ –LIKE– GENERALIZED $\mathcal{B}P$ – BIRECURRENT SPACE

In this section, we extened the generalized $\mathcal{B}P$ –birecurrent space i.e. characterized by the condition (2.8), by using the properties of $C2$ –like space to obtain new space contain the same properties of the main space.

Definition 3.1. *The generalized $\mathcal{B}P$ –birecurrent space which is $C2$ –like space i.e, satisfies the condition (2.4), will be called a $C2$ –like generalized $\mathcal{B}P$ –birecurrent space and will be denoted briefly by* $C2-\text{like}-G(\mathcal{B}P)-BRF_n$.

Let us consider a $C2-\text{like}-G(\mathcal{B}P)-BRF_n$.

Taking $\mathcal{B}$ –covariant derivative for the condition (2.4) twice with respect to x^m and x^l, respectively, using eq. (2.3) we get

$$\mathcal{B}_l\mathcal{B}_m\left(C_jC_kC_h/C^2\right)=a_{lm}C_{jkh}+b_{lm}\left(g_{jk}y_h-g_{jh}y_k\right).$$

Using the condition (2.4) in above equation, we get

$$\mathcal{B}_l\mathcal{B}_m\left(C_jC_kC_h/C^2\right)=a_{lm}\left(C_jC_kC_h/C^2\right)+b_{lm}\left(g_{jk}y_h-g_{jh}y_k\right). \tag{3.1}$$

Transvecting the condition (2.4) by g^{ij} using (2.1), we get

$$C^i_{kh}=C^iC_jC_k/C^2, \tag{3.2}$$

where $C^i=g^{ij}C_j$.

Taking $\mathcal{B}$ –covariant derivative for eq. (3.2) twice with respect to x^m and x^l, respectively, using eq. (2.2) we get

$$\mathcal{B}_l\mathcal{B}_m\left(C^iC_jC_k/C^2\right)=a_{lm}C^i_{kh}+b_{lm}\left(\delta^i_ky_h-\delta^i_hy_k\right).$$

Using eq. (3.2) in above equation, we get

$$\mathcal{B}_l\mathcal{B}_m\left(C^iC_jC_k/C^2\right)=a_{lm}\left(C^iC_jC_k/C^2\right)+b_{lm}\left(\delta^i_ky_h-\delta^i_hy_k\right). \tag{3.3}$$

From eqs. (3.1) and (3.3), we conclude the following theorem:

Theorem 3.1. *In* $C2-\text{like}-G(\mathcal{B}P)-BRF_n$, *Berwald's covariant derivative of second order for the tensors* $\left(C_jC_kC_h/C^2\right)$ *and* $\left(C^iC_jC_k/C^2\right)$ *are given by eqs. (3.1) and (3.3), respectively.*

IV. A C –REDUCIBLE– GENERALIZED $\mathcal{B}P$ – BIRECURRENT SPACE

In this section, we extened the generalized $\mathcal{B}P$ –birecurrent space i.e. characterized by the condition (2.8) by using the properties of C –reducible space to obtain new space contain the same properties of the main space.

Definition 4.1. *The generalized $\mathcal{B}P$ –birecurrent space which is C –reducible space i.e, satisfies the condition (2.5), will be called a C – reducible generalized $\mathcal{B}P$ – birecurrent space and will be denoted briefly by* $C-\text{reducible}-G(\mathcal{B}P)-BRF_n$.

Let us consider a $C-\text{reducible}-G(\mathcal{B}P)-BRF_n$.

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Taking $\mathcal{B}$ – covariant derivative for the condition (2.5) twice with respect to x^m and x^l, respectively, using eq. (2.3) we get

$$\mathcal{B}_l\mathcal{B}_m[\frac{1}{n+1}(h_{jk}C_h+h_{kh}C_j+h_{hj}C_k)]=a_{lm}C_{jkh}+b_{lm}(g_{jk}y_h-g_{jh}y_k).$$

Using the condition (2.5) in above equation, we get

(4.1) $$\mathcal{B}_l\mathcal{B}_m[\frac{1}{n+1}(h_{jk}C_h+h_{kh}C_j+h_{hj}C_k)]=a_{lm}[\frac{1}{n+1}(h_{jk}C_h+h_{kh}C_j+h_{hj}C_k)] +b_{lm}(g_{jk}y_h-g_{jh}y_k).$$

Transvecting the condition (2.5) by g^{ij}, using (2.1), we get

(4.2) $$C^i_{kh}=\frac{1}{n+1}(h^i_kC_h+h_{kh}C^i+h^i_hC_k),$$

where $C^i=g^{ij}C_j$ and $h^i_k=g^{ij}h_{jk}$.

Taking $\mathcal{B}$ – covariant derivative for eq. (4.2) twice with respect to x^m and x^l, respectively, using eq. (2.2) we get

$$\mathcal{B}_l\mathcal{B}_m[\frac{1}{n+1}(h^i_kC_h+h_{kh}C^i+h^i_hC_k)]=a_{lm}C^i_{kh}+b_{lm}(\delta^i_ky_h-\delta^i_hy_k).$$

Using eq. (4.2) in above equation, we get

(4.3) $$\mathcal{B}_l\mathcal{B}_m[\frac{1}{n+1}(h^i_kC_h+h_{kh}C^i+h^i_hC_k)]=a_{lm}[\frac{1}{n+1}(h^i_kC_h+h_{kh}C^i+h^i_hC_k)] +b_{lm}(\delta^i_ky_h-\delta^i_hy_k).$$

From eqs. (4.1) and (4.3), we conclude the following theorem:

Theorem 4.1. *In C – reducible – $G(\mathcal{B}P)$ – BRF_n, Berwald's covariant derivative of first order for the tensors $[\frac{1}{n+1}(h_{jk}C_h+h_{kh}C_j+h_{hj}C_k)]$ and $[\frac{1}{n+1}(h^i_kC_h+h_{kh}C^i+h^i_hC_k)]$ are given by eqs. . (4.1) and (4.3), respectively.*

V. A SEMI–C –REDUCIBLE–GENERALIZED $\mathcal{B}P$ – BIRECURRENT SPACE

In this section, we extened the generalized $\mathcal{B}P$ – birecurrent space i.e. characterized by the condition (2.8), by using the properties of semi–C –reducible space to obtain new space contain the same properties of the main space.

Definition 5.1. *The generalized $\mathcal{B}P$ – recurrent space which is semi–C –reducible space i.e, satisfies the condition (2.6), will be called a semi –C – reducible generalized $\mathcal{B}P$ – birecurrent space and will be denoted briefly by semi – C – reducible – $G(\mathcal{B}P)$ – BRF_n.*

Let us consider a *semi* – C – reducible – $G(\mathcal{B}P)$ – BRF_n.

Taking $\mathcal{B}$ – covariant derivative for the condition (2.6) twice with respect to x^m and x^l, respectively, using eq. (2.3) we get

$$\mathcal{B}_l\mathcal{B}_m[\frac{p}{1+n}(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h]=a_{lm}C_{jkh}+b_{lm}(g_{jk}y_h-g_{jh}y_k).$$

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Using the condition (2.6) in above equation, we get

(5.1) $$\mathcal{B}_l\mathcal{B}_m\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big]=a_{lm}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big]+b_{lm}\big(g_{jk}y_h-g_{jh}y_k\big).$$

Transvecting the condition (2.6) by g^{ij} , using eq. (2.1), we get

(5.2) $$C^i_{kh}=g^{ij}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big].$$

Taking $\mathcal{B}$ – covariant derivative for eq. (5.2) twice with respect to x^m and x^l, respectively, using eq. (2.2), we get

$$\mathcal{B}_l\mathcal{B}_m\Big(g^{ij}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big]\Big)=a_{lm}C^i_{kh}+b_{lm}\big(\delta^i_ky_h-\delta^i_hy_k\big).$$

Using eq. (5.2) in above equation, we get

(5.3) $$\mathcal{B}_l\mathcal{B}_m\Big(g^{ij}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big]\Big)$$
$$=a_{lm}\Big(g^{ij}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathbf{I}\|^2}I_jI_kI_h\Big]\Big)+b_{lm}\big(\delta^i_ky_h-\delta^i_hy_k\big).$$

From eqs. (5.1) and (5.3), we conclude the following theorem:

Theorem 5.1. *In* $semi-C-\text{reducible}-G(\mathcal{B}P)-BRF_n$, *Berwald's covariant derivative of second order for the tensors* $\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathrm{I}\|^2}I_jI_kI_h\Big]$ *and* $\Big(g^{ij}\Big[\frac{p}{1+n}\big(h_{jk}I_h+h_{kh}I_j+h_{hj}I_k\big)+\frac{q}{\|\mathrm{I}\|^2}I_jI_kI_h\Big]\Big)$ *are given by eqs. (5.1) and (5.3), respectively.*

VI. A $C3$ –LIKE–GENERALIZED $\mathcal{B}P$– BIRECURRENT SPACE

In this section, we extened the generalized $\mathcal{B}P$–biecurrent space i.e. characterized by the condition (2.8), by using the properties of $C3$ –like space to obtain new space contain the same properties of the main space.

Definition 6.1. *The generalized* $\mathcal{B}P$–*birecurrent space which is* $C3$ –*like space i.e, satisfies the condition (2.7), will be called a* $C3$ –*like generalized* $\mathcal{B}P$–*birecurrent space and will be denoted briefly by* $C3-\text{like}-G(\mathcal{B}P)-BRF_n$.

Let us consider a $C3-\text{like}-G(\mathcal{B}P)-BRF_n$.

Taking $\mathcal{B}$ – covariant derivative for the condition (2.7) twice with respect to x^m and x^l, respectively, using eq. (2.3), we get

$$\mathcal{B}_l\mathcal{B}_m\big[\big(A_jh_{kh}+A_kh_{hj}+A_hh_{jk}\big)+\big(B_jI_kI_h+I_jB_kI_h+I_jI_kB_h\big)\big]=a_{lm}C_{jkh}+b_{lm}\big(g_{jk}y_h-g_{jh}y_k\big).$$

Using the condition (2.7) in above equation, we get

(6.1) $$\mathcal{B}_l\mathcal{B}_m\big[\big(A_jh_{kh}+A_kh_{hj}+A_hh_{jk}\big)+\big(B_jI_kI_h+I_jB_kI_h+I_jI_kB_h\big)\big]$$
$$=a_{lm}\big[\big(A_jh_{kh}+A_kh_{hj}+A_hh_{jk}\big)+\big(B_jI_kI_h+I_jB_kI_h+I_jI_kB_h\big)\big]+b_{lm}\big(g_{jk}y_h-g_{jh}y_k\big).$$

Transvecting the condition (2.7) by g^{ij}, using eq. (2.1), we get

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(6.2) $C^i_{kh} = \left(g^{ij}\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]\right).$

Taking $\mathcal{B}$ – covariant derivative for eq. (6.2) twice with respect to x^m and x^l, respectively, using eq. (2.2), we get

$$\mathcal{B}_l \mathcal{B}_m \left(g^{ij}\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]\right) = a_{lm} C^i_{kh} + b_{lm}\left(\delta^i_k y_h - \delta^i_h y_k\right).$$

Using eq. (6.2) in above equation, we get

(6.3) $\mathcal{B}_l \mathcal{B}_m \left(g^{ij}\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]\right)$

$$= a_{lm}\left(g^{ij}\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]\right) + b_{lm}\left(\delta^i_k y_h - \delta^i_h y_k\right).$$

From eqs. (6.1) and (6.3), we conclude the following theorem:

Theorem 6.1. *In* $C3-\text{like}-G(\mathcal{B}P)-BRF_n$ *, Berwald's covariant derivative of second order for the tensors* $\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]$ *and* $\left(g^{ij}\left[\left(A_j h_{kh} + A_k h_{hj} + A_h h_{jk}\right) + \left(B_j I_k I_h + I_j B_k I_h + I_j I_k B_h\right)\right]\right)$ *are given by eqs. (6.1) and (6.3), respectively.*

VII. CONCLUSION

We discussed some special spaces in the generalized $\mathcal{B}P$ – birecurrent space to get new spaces related to it. Certain identities belong to these spaces have been obtained.

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