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Calculation of the contact interaction of gear teeth from composite materials

V.V. Mozharovsky, D.S. Kuzmenkov, S.V. Kirhintsava

Doctor of Technical Sciences, professor of computational mathematics and programming department,
Francisk Skorina Gomel State University, Gomel, Belarus

Candidate of Physical and Mathematical Sciences, Head of computational mathematics and programming department,
Francisk Skorina Gomel State University, Gomel, Belarus

Assistant of computational mathematics and programming department,
Francisk Skorina Gomel State University, Gomel, Belarus

ABSTRACT: This paper presents a method for determining and calculating the contact zone, pressure in the contact zone, contact stresses and displacements during the interaction of gear teeth made of various materials - isotropic and anisotropic (composite). The calculation is based on the Hertz theory [1] and the anisotropic theory of elasticity [2]. The Hertz's theory reflects the calculation of the contact interaction of isotropic cylinders. The problem of interaction of cylinders made of isotropic and fibrous composite materials is considered. When determining the contact parameters of the cylinders, the characteristics of the materials are taken into account, i.e. the volumetric content of fiber in the matrix, considering the materials isotropic. The values of the modulus of elasticity and Poisson's ratios, the ultimate strengths are different depending on the direction of the fibers. The comparison of the developed technique and the Hertz theory for isotropic cylinders is given.

I. INTRODUCTION

The use of gears is widely applicable in many industries, such as aircraft, automotive, shipbuilding, etc. The work considers spur gears. The specified type of wheels is easy to manufacture and is applicable for transferring power from one shaft to another shaft, changing the speed and torque, for example, in watches, gearboxes and other mechanisms. The service life of the gear teeth is related to the ability of the teeth to withstand contact loads. Contact stresses can cause many factors: pitting corrosion in the contact zone, material wear, which, in turn, can lead to failure of the gear teeth. The methods of increasing the service life of the teeth of gear wheels include the use of composite materials for their manufacture, as composites provide improved mechanical properties (strength-to-weight ratio, hardness), lower operating costs, and are also characterized by reduced levels of noise and corrosion, wear of the tooth material. These problems attract a lot of attention from researchers. The research results are reflected, for example, in works [3-6] and in numerous other articles and scientific publications. Increasing the strength of gear wheels is an urgent task, since a sudden failure of the gear teeth poses potential hazards to the operating elements of machines, and their replacement is necessary to restore the operation of systems using gears, which in turn is an expensive factor and entails the loss of working time.

II. FORMULATION OF THE PROBLEM

Let us consider the problem of contact interaction of gear teeth, assuming that their contact can be modeled by mating two cylinders with radii R_1 and R_2 . The materials of the gears in the gear train are different: anisotropic (composite) and isotropic.

To simulate the interaction of teeth and calculate them for contact strength, we replace them with cylinders pressed against each other by force P . It is necessary to determine the contact zone, contact stress and movement of the cylinders, as well as the pressure arising in the contact zone during their interaction. The contact diagram of two teeth of a gear train is shown in Fig. 1.

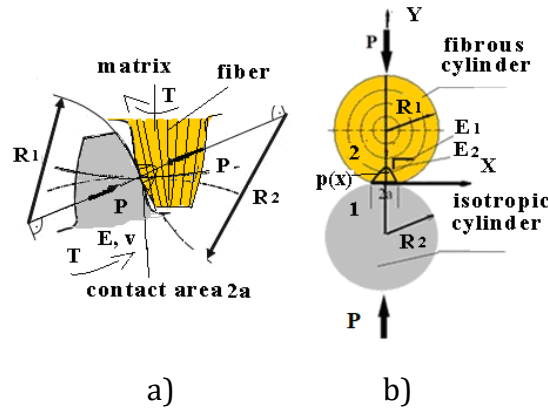


Fig. 1 Contact scheme of two teeth of a gear train made of isotropic and composite materials (a); mathematical model of contact, modulus of elasticity E_1 in the direction of the ox axis, E_2 - in the direction of the oy axis (b).

III. DESIGN OF GEAR

The geometrical characteristics of the pinion and cogwheel are presented in Table 1.

Table 1. Geometric parameters of gear and cogwheel

Parameter	Value
Pressure angle ϕ (degrees)	20
Number of teeth	17
Module m (mm)	2
Tooth thickness b_3 (mm)	12
Pitch circle diameter d_p (mm)	34
Rotational speed n_p (rpm)	1500
Power P_m (KW)	1

The physical and mechanical characteristics of the gear and gear materials are presented in Table 2. Elastic modulus, Poisson's ratio and shear modulus with subscripts m characterize the material Unreinforced Nylon 66, with subscripts f - Glass fiber [6].

Table 2. Physical and mechanical characteristics of gear and gear materials

Property \ Element	Gear	Toothed wheel
Elastic modulus (MPa)	$E_m = 3500; E_f = 85000$	$E = 207000$
Poisson's ratio	$\nu_m = 0,35; \nu_f = 0,2$	$\nu = 0,3$
Shear modulus (MPa)	$G_m = 1300; G_f = 35420$	$G = 79615$
Fiber content V in the material matrix (%)	20	—

IV. SOLUTION METHODS

Earl Buckingham (1926) used the Hertz's theory to calculate the contact stress between a rotating pair of teeth [3, 7]. According to the Hertz theory [1], when two isotropic cylinders are pressed against each other, the contact stress is determined by the expression:

$$\sigma_c = \frac{2P}{\pi a L}, \quad a = \sqrt{\frac{2P \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}{\pi L \left(\frac{1}{d_1} + \frac{1}{d_2} \right)}}$$

where σ_c is the maximum value of the contact stress (N/mm^2); P is the pressure force of the two cylinders (N); a - half-width of the contact zone (mm); L is the axial length of the cylinders (mm); d_1 and d_2 - diameters of two cylinders (mm); E_1 and E_2 - moduli of elasticity of materials of two cylinders (N/mm^2); ν_1 and ν_2 are the Poisson's ratios of the materials of the two cylinders.

When a pair of teeth is in contact, the diameters d_1 and d_2 are represented by the diameters of curvature at the point of contact [1]: where ϕ is the pressure angle (in degrees); d_{pp} and d_{pg} are the diameters of the pitch circle of the pinion and toothed wheel (mm).

Pressure force of two cylinders [3, 7]

$$P = \frac{P_t}{\cos \phi} = \frac{2T}{d_p \cos \phi},$$

where P_t is the tangential load (N); d_p - the diameter of the pitch circle of the gear and toothed wheel (mm); T - torque ($N \cdot mm$), determined from the formula [1]

$$P_m = \frac{2 \pi n_p T}{60 \times 10^6},$$

where n_p is the speed of rotation of the gear and gear wheel (revolutions per minute); P_m - transmitted power (kW).

The axial length of the cylinders L is equal to the tooth thickness b_3 (mm). The diameters of the pitch circle of the pinion and the cogwheel d_p can be calculated by the formula $d_p = mz$, where m is the modulus (mm); z is the number of teeth of a gear and a gear wheel.

The relationship between stresses and strains, using the generalized Hooke's law [6, 8], can be written in the form:

$$\{\sigma\} = [D] \{\varepsilon^{el}\} \quad \text{or} \quad \{\varepsilon^{el}\} = [D]^{-1} \{\sigma\},$$

where σ , ε^{el} - components of stress and strain tensors; in what follows, for an orthotropic cylinder material, we assume that

$$E_x = E_1, \quad E_y = E_2, \quad \nu_{xy} = \nu_{12}, \quad \nu_{yz} = \nu_{23};$$

here E_j , G_{12} , ν_{ij} , $i, j = \overline{1,3}$ - technical material constants, $[D]^{-1}$ - system stiffness matrix.

To determine the contact stress during the interaction of orthotropic cylinders, the size of the contact zone (half-width) a (mm) is determined by the formula [2]:

$$a = \sqrt{\frac{2 P R_1 R_2}{\pi M L (R_1 + R_2)}},$$

where P is the pressure force of two cylinders (N); L is the axial length of the cylinders (cylinder thickness) (mm); R_1 and R_2 - radii of two interacting cylinders (mm); parameter $M = \left[((\beta_1 + \beta_2) S_{22})^{(1)} + ((\beta_1 + \beta_2) S_{22})^{(2)} \right]^{-1}$, indices (1) and (2) - characterize the materials of two cylinders, the values $\beta_{1,2}$ for each cylinder are calculated by the formulas [2]

$$\beta_{1,2} = \left(\frac{S_{66} + 2S_{12} \pm \sqrt{(S_{66} + 2S_{12})^2 - 4S_{11}S_{22}}}{2S_{11}} \right)^{-1},$$

where the constant S_{ij} at plane deformation is determined from [2] as follows:

$$S_{11} = (1 - \nu_{13}\nu_{31})/E_1, S_{12} = -(\nu_{12} + \nu_{13}\nu_{31})/E_1, S_{22} = (1 - \nu_{32}\nu_{23})/E_2, S_{66} = 1/G_{12},$$

indices 1,2,3 designate the technical constants of the cylinder materials (modulus of elasticity E (MPa), Poisson's ratio ν and shear modulus G (MPa)) characterize different directions. Elastic constants are calculated according to the rule of mixtures:

$$E_1 = VE_f + (1-V)E_m, E_2 = \frac{E_m(1+\eta V)}{1-\eta V}, \eta = (E_f - E_m)/(E_f + E_m); G_{12} = G_m \frac{G_f(1+V) + G_m(1-V)}{G_f(1-V) + G_m(1+V)},$$

$$\nu_{12} = \nu_{13} = V\nu_f + (1-V)\nu_m, \nu_{21} = \nu_{31} = E_2/E_1 \cdot \nu_{12}, \nu_{23} = \nu_{32} = 1 - \nu_{21} - E_2/(3K),$$

$$K = K_f K_m / (VK_m + (1-V)K_f), K_f = E_f / (3 - 6\nu_f), K_m = E_m / (3 - 6\nu_m);$$

the subscripts f and m denote fiber and matrix, respectively; V is the volumetric content of fiber in the material matrix; K_f, K_m - bulk modulus of elasticity of fiber and matrix.

The equation describing the pressure $p(x)$ in the contact zone a has the form [2]:

$$p(x) = M \frac{R_1 + R_2}{R_1 R_2} \sqrt{a^2 - x^2}, -a \leq x \leq a.$$

The maximum contact stresses are determined by the formula $\sigma_c = \frac{2P}{\pi a L}$.

It should be noted that the calculated contact voltages must not exceed the permissible voltages $\sigma_c < [\sigma_w]$, which are calculated by the approximate dependence

$$[\sigma_w] = [\sigma_{wf}] \cdot V + (1 - V) \cdot [\sigma_{wm}],$$

here $[\sigma_{wf}]$ и $[\sigma_{wm}]$ - allowable stresses for the materials of the fiber and the matrix of the composite, respectively, and are determined experimentally.

One of the main parameters characterizing the contact interaction of the teeth of the gear wheels is the contact approach or displacement. If the material of the teeth of the wheels has orthotropy properties, then the contact displacement v (mm) is determined by the relationship [2]

$$v = -\frac{P}{\pi} \left\{ S_{22}(\beta_1 + \beta_2) \left[\ln \frac{a}{2q} - \frac{1}{2} \right] + \frac{S_{22}}{\beta_1 - \beta_2} (\beta_1^2 \ln \beta_1 - \beta_2^2 \ln \beta_2) + \frac{S_{12}}{\beta_1 - \beta_2} \ln \frac{\beta_2}{\beta_1} \right\},$$

where q is a measure of the contact approach of the distance (mm), equal, for example, to $20a$ (we assume that at this distance from the contact zone, the deformation is attenuated).

This dependence can be easily converted into a formula for determining the contact approach of an isotropic cylinder at $\beta_1 = \beta_2 = 1$ and has the form [2, 9]

$$v_1 = -\frac{2(1-\nu^2)}{\pi E} P \left[\ln \frac{a}{2q} + \frac{\nu}{2(1-\nu)} \right].$$

The convergence of two contacting teeth δ (mm) is the sum of the contact movements of each tooth.

V. NUMERICAL CALCULATION

For an example of calculating the contact interaction of the teeth of gears according to the developed method, let us take the case when $d_{pg} = d_{pp} = d_p$. The geometrical, physical and mechanical characteristics of the gear and cogwheel materials correspond to the data in Table 1 and Table 2.

$$E_1 = 0.2 \times 85000 + (1 - 0.2) \times 3500 = 19800 \text{ MPa},$$

$$\eta = (85000 - 3500) / (85000 + 3500) = 0.92;$$

$$E_2 = \frac{3500 \times (1 + 0.92 \times 0.2)}{1 - 0.92 \times 0.2} = 5080.33 \text{ MPa},$$

$$G_{12} = 1300 \times \frac{35420 \times (1 + 0.2) + 1300 \times (1 - 0.2)}{35420 \times (1 - 0.2) + 1300 \times (1 + 0.2)} = 1893.47 \text{ MPa},$$

$$\nu_{12} = \nu_{13} = 0.2 \times 0.2 + (1 - 0.2) \times 0.35 = 0.32,$$

$$\nu_{21} = \nu_{31} = 5080.33 / 19800 \times 0.32 = 0.08,$$

$$K_f = 85000 / (3 - 6 \times 0.2) = 47222.22 \text{ MPa},$$

$$K_m = 3500 / (3 - 6 \times 0.35) = 3888.89 \text{ MPa},$$

$$K = \frac{47222.22 \times 3888.89}{0.2 \times 3888.89 + (1 - 0.2) \times 47222.22} = 4763.05 \text{ MPa},$$

$$\nu_{23} = \nu_{32} = 1 - 0.08 - 5080.33 / (3 \times 4763.05) = 0.56.$$

The obtained moduli of elasticity of the composite material of one of the cylinders E_1 and E_2 in the direction of the ox and oy axes, respectively, characterize the material 20% Glass fiber reinforced nylon66 [6].

Odds S_{ij} for a composite material will be equal to:

$$S_{11} = (1 - 0.32 \times 0.08) / 19800 = 4.92 \times 10^{-5} \text{ MPa}^{-1},$$

$$S_{12} = -(0.32 + 0.32 \times 0.08) / 19800 = -1.75 \times 10^{-5} \text{ MPa}^{-1},$$

$$S_{22} = (1 - 0.56 \times 0.56) / 5080.33 = 13 \times 10^{-5} \text{ MPa}^{-1},$$

$$S_{66} = 1 / 1893.47 = 53 \times 10^{-5} \text{ MPa}^{-1},$$

$$\beta_1 = 0.32, \beta_2 = 1.89.$$

Odds S_{ij} for an isotropic material will be equal to:

$$S_{11} = (1 - 0.3 \times 0.3) / 207000 = 4.40 \times 10^{-6} \text{ MPa}^{-1},$$

$$S_{12} = -(0.3 + 0.3 \times 0.3) / 207000 = -1.88 \times 10^{-6} \text{ MPa}^{-1},$$

$$S_{22} = (1 - 0.3 \times 0.3) / 207000 = 4.40 \times 10^{-6} \text{ MPa}^{-1},$$

$$S_{66} = 1 / 79615 = 1.26 \times 10^{-5} \text{ MPa}^{-1},$$

$$\beta_1 = 0.998, \beta_2 = 1.002.$$

Parameter M

$$M = \left[((0.32 + 1.89) \times 13 \times 10^{-5})^{(1)} + ((0.998 + 1.002) \times 4.40 \times 10^{-6})^{(2)} \right]^{-1} = 3269.06 \text{ MPa},$$

$$T = \frac{1 \times 60 \times 10^6}{2 \times 3.14 \times 1500} = 6369.43 \text{ N} \cdot \text{mm},$$

$$d_p = 17 \times 2 = 34 \text{ mm},$$

$$P_t = \frac{2 \times 6369.43}{34} = 374.67 \text{ N},$$

$$r_p = 34/2 = 17 \text{ mm}.$$

The contact area a will be equal to

$$a = \sqrt{\frac{2 \times 374.67 \times 17 \times \sin 20^\circ \times 17 \times \sin 20^\circ}{3.14 \times \cos 20^\circ \times 3269.06 \times 12 \times (17 \times \sin 20^\circ + 17 \times \sin 20^\circ)}} = 0.14 \text{ mm}.$$

Then the contact stress σ_c will take the value

$$\sigma_c = \frac{2 \times 374.67}{3.14 \times \cos 20^\circ \times 0.14 \times 12} = 154.27 \text{ MPa}.$$

Let us assume for our calculation that the permissible stresses for the fiber material Glass fiber $[\sigma_{wf}] = 2300 \text{ MPa}$, for matrix material Unreinforced Nylon 66 when compressed $[\sigma_{wm}] = 130 \text{ MPa}$ [10].

$$\text{Then } [\sigma_w] = 2300 \cdot 0.2 + 0.8 \cdot 130 = 564 \text{ MPa}.$$

Therefore, the condition $\sigma_c < [\sigma_w]$ is satisfied.

VI. COMPARISON OF THE DEVELOPED METHOD WITH THE HERTZ'S THEORY

Let us consider several options for the task, depending on the percentage of fibers in the matrix of the composite material (0% and 100%): this means that the second material of the cylinder will also be isotropic, which means that the Hertz's theory is applicable, and we will compare the size of the contact zone and the contact stress, obtained according to the developed method and the Hertz theory.

According to the Hertz's theory, at zero fiber content, the half-width of the contact zone and the contact stress will be equal:

$$a = \sqrt{\frac{2 \times 374.67 \times \left(\frac{1-0.3^2}{207000} + \frac{1-0.35^2}{3500} \right)}{3.14 \times \cos 20^\circ \times 12 \times \left(\frac{1}{2 \times 17 \times \sin 20^\circ} + \frac{1}{2 \times 17 \times \sin 20^\circ} \right)}} = 0.18 \text{ mm},$$

$$\sigma_c = \frac{2 \times 374.67}{3.14 \times \cos 20^\circ \times 0.18 \times 12} = 120.80 \text{ MPa};$$

at 100% fiber content, the half-width of the contact zone and the contact stress will be:

$$a = \sqrt{\frac{2 \times 374.67 \times \left(\frac{1-0.3^2}{207000} + \frac{1-0.2^2}{85000} \right)}{3.14 \times \cos 20^\circ \times 12 \times \left(\frac{1}{2 \times 17 \times \sin 20^\circ} + \frac{1}{2 \times 17 \times \sin 20^\circ} \right)}} = 0.04 \text{ mm},$$

$$\sigma_c = \frac{2 \times 374.67}{3.14 \times \cos 20^\circ \times 0.04 \times 12} = 481.49 \text{ MPa}.$$

The difference in the size of the contact zone and contact stresses for cylinders made of steel and composite material with a fiber content of 0% and 100% in the matrix of a fibrous material when calculating according to the Hertz theory and the developed method is shown in Table 3. As can be seen from Table 3, the differences between the results obtained according to the Hertz theory and the developed technique are insignificant for isotropic materials.

At zero fiber content in the composite matrix, the permissible stress is $[\sigma_w] = [\sigma_{wm}] = 130 \text{ MPa}$, and at 100% fiber content $[\sigma_w] = [\sigma_{wf}] = 2300 \text{ MPa}$.

Table 3. Differences in the calculation according to the Hertz theory and the developed method

	Steel+composite (0% fibers)		Steel+composite (100% fibers)	
	a, mm	σ_c , MPa	a, mm	σ_c , MPa
Hertz's theory	0.1752	120.80	0.0440	481.50
The developed technique	0.1767	119.80	0.0441	479.70
% difference	0,9	0,8	0,2	0,4

As can be seen from the data in Table 3, the condition $\sigma_c < [\sigma_w]$ is satisfied.

VII.COMPUTER MODELLING

On the basis of the described technique, an algorithm was developed and a program for calculating the contact zone of two interacting cylinders made of anisotropic (composite) and isotropic materials, pressure in the contact zone, as well as contact stresses and displacements of the cylinders was developed.

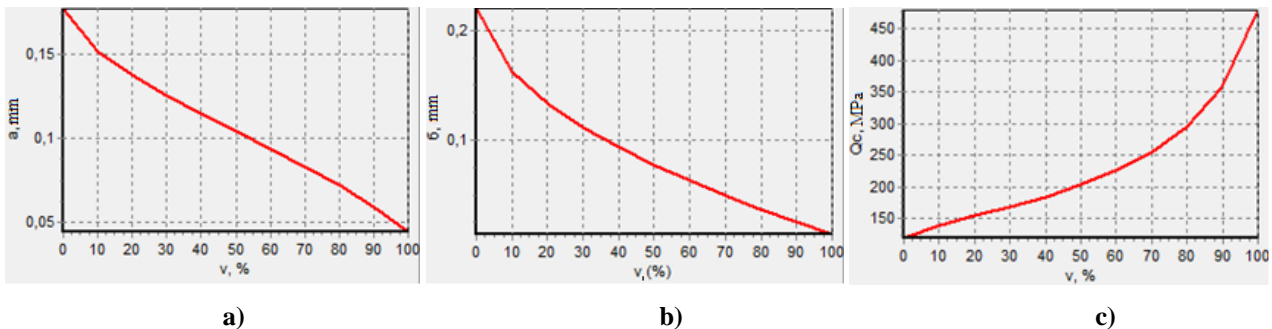
Using the method of computer simulation and the developed program, the influence of the rotation speed n_p on the half-width of the contact zone a , contact displacements v_1 and v_2 , the convergence of the cylinders δ and the contact stress σ_c at the interaction of two cylinders were analyzed. The input data correspond to the data in Table 1 and Table 2. The results are reflected in Table 4 and Fig.2.

Table 4. Computer simulation results

n_p (rpm)	a (mm)	v_1 (mm)	v_2 (mm)	δ (mm)	σ_c (N/mm ²)
1300	0,147	0,150	0,004	0,154	165,70
1500	0,137	0,130	0,004	0,134	154,30
1700	0,129	0,114	0,003	0,117	144,90

The results of computer simulation showed a significant effect of the rotation speed on the size of the contact zone of two cylinders made of composite and isotropic materials, as well as on contact stresses, contact displacements and the maximum pressure arising from their contact.

It should be noted that the condition $\sigma_c < [\sigma_w]$ is satisfied.



**Fig.2 a) The plot of the dependence of the half-width of the contact zone a on the percentage of fibers;
b) A graph of the dependence of the convergence of the teeth δ on the percentage of fibers;
c) A graph of the dependence of the contact stress σ_c on the percentage of fibers.**

VIII. CONCLUSIONS




A methodology and a program for calculating and plotting diagrams of contact stresses and displacements, pressure and contact zones during the interaction of teeth of gear wheels made of anisotropic and isotropic materials have been developed. In this case, it seems possible to choose the optimal characteristics (geometric, physical and mechanical) of the materials for the teeth of the gear wheels. The technique showed good agreement with the Hertz's theory for the case of isotropy of materials of two contacting cylinders.

The presented technique shows a general calculation of teeth for contact strength, and for a more accurate calculation, it is necessary to add coefficients characterizing the dynamic and other operational properties for a real gear transmission.

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AUTHOR’S BIOGRAPHY

	<p>Mozharovsky Valentin Vasiljvich, Doctor of Technical Sciences, Professor, was born April 4, 1948 year in Ukraine. Studies at the Faculty of Mechanics and Mathematics, Lviv State University. Ivan Franko, postgraduate studies at the Moscow Institute of Electronic Engineering (MIEM, Moscow). Associate Professor of the Departments of Mathematical Problems of Control and Computing Machines and Programming on a full-time basis, Francisk Skorina Gomel State University. Head of the Department of Computational Mathematics. Member of the Council at "Belarusian State University of Transport" in the specialty mechanics of a deformable solid. Member of the international scientific society EUROMECH. State University on the terms of full-time part-time job. Has more than 400 published scientific works in the form of articles, journals, theses and tutorials. Currently works at the department of “Computational mathematics and programming department” in Francisk Skorina Gomel State University, Gomel, Belarus. Badge "Excellence in Education"</p>
	<p>Kuzmenkov Dmitry Sergeevich, Head of Department, Candidate of Physical and Mathematical Sciences, was born February 3, 1983 in Gomel, Belarus. Has more than 140 published scientific works in the form of articles, journals, theses and tutorials. Currently works at the department of “Computational mathematics and programming department” in Francisk Skorina Gomel State University, Gomel, Belarus.</p>
	<p>Kirhintsava Sviatlana Viktorovna, was born April 30, 1989 year in Mogilev region, Belarus. Has more than 30 published scientific works in the form of articles, theses. Currently works at the department of “Computational mathematics and programming department” in Francisk Skorina Gomel State University, Gomel, Belarus.</p>