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Models of Pipeline Deformation Taking into Account Damage During Cyclic Loading

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ABSTRACT: The article presents the formulation of the problem and develops mathematical models of deformation of underground pipelines under repeated-variable loading. Based on the theory of small elastic-plastic deformations and the Hamilton-Ostrogradsky variational principle, a system of differential equations of motion (equilibrium) under repeated-variable loading with appropriate boundary and initial conditions was obtained for the first time. The boundary value problem is solved by the method of finite differences and matrix run. For a particular case, the calculation results are given.

KEYWORDS: repeated, variable loading, underground pipeline, deformation models, finite difference method.

I. INTRODUCTION

The modeling of deformation processes and the analysis of the stress-strain state of shell structures - pipelines have recently been given particular attention. This is due to the increasing role of pipeline transport in supplying the national economy with oil and gas, a significant increase in the length and diameters of pipelines, increased requirements for environmental protection.

The theory of small elastoplastic deformations of A.A. Ilyushin - V.V. Moskvitin is used in the calculation of pipelines beyond elasticity under repeated-variable loads. Moskvitin [1,2]. In the works of V.V. Moskvitin, the damageability function and the criteria for assessing the strength of elastoplastic, viscoplastic materials under alternating loads are introduced.

In calculation of main pipelines quite widespread methods of finite elements, finite differences and method of boundary elements were used. The paper by V. Korolev [3] describes a method for calculating plates and shells under elastic and elastoplastic deformations. He considered reinforced plates, bilayered and multilayered shells, and flat three-layered shells with elastic fillers. In [4], finite element methods are considered, with the main attention being paid to the reliability of calculation results. Many computational examples illustrate the importance of theoretical conclusions for practical calculations. The authors of [5] present modern continuum mechanics and mathematical foundations for studying the physical behavior of shells. Some modern finite element methods are presented for linear and nonlinear analyses. The publications [6,7] contain methods for solving nonlinear problems used in the calculation of strength, stability and vibrations of structures. Calculations are performed taking into account physical and geometric nonlinearities using the variational method. On the basis of the developed algorithm, a software product has been created.

In [8], an improved finite-element model for the linear analysis of anisotropic and layered double-curved composite shells is considered. The results are compared with the available analytical and numerical solutions.

In [9] variation finite-difference methods for solving linear and nonlinear problems for thin-walled shells and plates of homogeneous isotropic and orthotropic materials are considered. A numerical analysis of the stress-strain state of shells of complex geometry is given. Methods of successive approximations, the finite difference method and the numerical discretization algorithm are used.

The publication by A. F. Revuzhenko [10] deals with an elastic-plastic model and a numerical solution of the problem. A finite-element algorithm and a computer program allowing one to investigate the continuous media mode of deformation are developed.

The works by T. Buriev [11,12] consider the safety of structures of complex objects. Numerical methods for solving boundary value problems are developed, the issues of computer implementation, construction of an algorithmic system for calculating load-bearing elements of structures within and beyond elasticity under alternating loads are presented.

The book [13] shows the realization of some damage models with finite elements. The damage models are based on the principles of continuous damage mechanics and the concept of effective stress. The developed program can be used to

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analyze the behavior of an elastoplastic material, considering the damage consequences within the framework of damage mechanics.
In [14] geometrically nonlinear formulation of the problem of stress-strain state of pipelines laid in complex engineering and geological conditions is formulated. A method for calculating pipeline strength has been developed, in which the latter is modeled by a rod system consisting of curvilinear and straight rods of tubular cross-section, interconnected in nodal elements. Work [15] contains a description of methods for numerical modeling of trunk pipeline systems. The proposed concepts and methods are basic elements of theoretical foundation of modern computer tools for effective solution of a wide range of technical and technological problems of designing structures and objects of pipeline transport.
In [16] the parameters characterizing nonlinear elastic, elastoplastic and viscoelastic properties of interaction of underground pipeline with ground are determined.
Work [17] investigates single and alternating bending of layered plates and shells taking into account the viscoelastic-plastic properties. The work [18] is devoted to the construction of defining relations on the basis of experimental research, the areas of physical reliability of plasticity theory as applied to the processes of complex loading are determined.
The development of models and calculation methods for thin-walled pipelines under cyclic loading is an actual problem, since the reliability of calculations and the reliability of design solutions for specific classes of problems directly depend on the adequacy of the selected mathematical models, solution methods and boundary conditions.

II. METHODS

This paper formulates the problem statement and modeling of deformation processes of trunk pipelines beyond elasticity under repeated-variable loading. The theory of small elastic-plastic deformations of A.A. Ilyushin - V.V. Moskvitin is used in modeling calculations.
Following [2], we introduce the differences

$$\overline{U}_i^{(n)} = (-1)^n (U_i^{(n-1)} - U_i^{(n)}),\ \overline{e}_{ij}^{(n)} = (-1)^n (e_{ij}^{(n-1)} - e_{ij}^{(n)}), \overline{\sigma}_{ij}^{(n)} = (-1)^n (\sigma_{ij}^{(n-1)} - \sigma_{ij}^{(n)}) \quad (1)$$

To determine the components of displacements $\overline{U}_i^{(n)}$ under n-loading, we have the following relations [23]:

$$\overline{U}_\alpha^{(n)} = \overline{U}^{(n)} - \frac{\gamma}{A} \cdot \frac{\partial \overline{W}^{(n)}}{\partial \alpha}, \quad \overline{U}_\beta^{(n)} = (1 + k_2 \gamma) \overline{V}^{(n)} - \frac{\gamma}{B} \cdot \frac{\partial \overline{W}^{(n)}}{\partial \beta}, \quad \overline{U}_\gamma^{(n)} = \overline{W}^{(n)}(\alpha, \beta) \cdot \quad (2)$$

The deformations are determined by the refined formulas:

$$\overline{e}_{\alpha\alpha}^{(n)} = \frac{1}{R} \frac{\partial \overline{U}^{(n)}}{\partial \alpha} - \frac{\gamma}{R^2} \frac{\partial^2 \overline{W}^{(n)}}{\partial \alpha^2}, \quad \overline{e}_{\beta\beta}^{(n)} = \frac{\partial \overline{V}^{(n)}}{R \partial \beta} - (\gamma - k_2 \gamma^2) \frac{\partial^2 \overline{W}^{(n)}}{R^2 \partial \beta^2} + (1 - k_2 \gamma + k_2^2 \gamma^2) k_2 \overline{W}^{(n)},$$

$$\overline{e}_{\alpha\beta}^{(n)} = (1 - k_2 \gamma + k_2^2 \gamma^2) \frac{\partial \overline{U}^{(n)}}{R \partial \beta} - (\gamma - k_2 \gamma^2) \frac{\partial^2 \overline{W}^{(n)}}{R^2 \partial \alpha \partial \beta} + (1 + k_2 \gamma) \frac{\partial \overline{V}^{(n)}}{R \partial \alpha} - \frac{\gamma}{R^2} \frac{\partial^2 \overline{W}^{(n)}}{\partial \alpha \partial \beta}. \quad (3)$$

Under alternating loading, the stress and strain components are related as follows [12]:

$$\sigma_{\alpha\alpha}^{(n)} = G_1 \left\{ \left(e_{\alpha\alpha}^{(n)} + \mu e_{\beta\beta}^{(n)} \right) - \left[\omega^{(n)} \left(\overline{e}_{\alpha\alpha}^{(n)} + \mu \overline{e}_{\beta\beta}^{(n)} \right) + \sum_{m=1}^{k-1} \omega^{0(n-m)} \left(\overline{e}_{\alpha\alpha}^{0(n-m)} + \mu \overline{e}_{\beta\beta}^{0(n-m-1)} \right) \right] \right\},$$

$$\sigma_{\beta\beta}^{(n)} = G_1 \left\{ \left(e_{\beta\beta}^{(n)} + \mu e_{\alpha\alpha}^{(n)} \right) - \left[\omega^{(n)} \left(\overline{e}_{\beta\beta}^{(n)} + \mu \overline{e}_{\alpha\alpha}^{(n)} \right) + \sum_{m=1}^{k-1} \omega^{0(n-m)} \left(\overline{e}_{\beta\beta}^{0(n-m)} + \mu \overline{e}_{\alpha\alpha}^{0(n-m-1)} \right) \right] \right\},$$

$$\sigma_{\alpha\beta}^{(n)} = G_1 \left\{ e_{\alpha\beta}^{(n)} - \omega^{(n)} \overline{e}_{\alpha\beta}^{(n)} + \sum_{m=1}^{k-1} \omega^{0(n-m)} \overline{e}_{\alpha\beta}^{0(n-m)} \right\}, \quad (4)$$

To obtain the equation of motion of the main pipeline under variable loading, we will use the Hamilton-Ostrogradsky variational principle [12]:

$$\int_t (\delta T^{(n)} - \delta \Pi^{(n)} + \delta A^{(n)}) dt = 0 \quad (5)$$

First, we determine the variation of the kinetic energy by the formula:

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$$\int_t \delta T^{(n)} dt = \int_t \int_v \left(\rho \frac{\partial U_\alpha^{(n)}}{\partial t} \delta \frac{\partial U_\alpha^{(n)}}{\partial t} + \rho \frac{\partial U_\beta^{(n)}}{\partial t} \delta \frac{\partial U_\beta^{(n)}}{\partial t} + \rho \frac{\partial U_\gamma^{(n)}}{\partial t} \delta \frac{\partial U_\gamma^{(n)}}{\partial t} \right) dVdt \qquad (6)$$

We integrate relations (6) in parts:

$$\int_t \delta T^{(n)} dt = \int_v \left(\rho \frac{\partial U_\alpha^{(n)}}{\partial t} \delta \frac{\partial U_\alpha^{(n)}}{\partial t} + \rho \frac{\partial U_\beta^{(n)}}{\partial t} \delta \frac{\partial U_\beta^{(n)}}{\partial t} + \rho \frac{\partial U_\gamma^{(n)}}{\partial t} \delta \frac{\partial U_\gamma^{(n)}}{\partial t} \right) \Bigg|_t - \qquad (7)$$
$$- \int_t \int_v \left(\rho \frac{\partial^2 U_\alpha^{(n)}}{\partial t^2} \delta U_\alpha^{(n)} + \rho \frac{\partial^2 U_\beta^{(n)}}{\partial t^2} \delta U_\beta^{(n)} + \rho \frac{\partial^2 U_\gamma^{(n)}}{\partial t^2} \delta U_\gamma^{(n)} \right) dvdt$$

Here we substitute the expressions of displacements by (2) under the signs of variation

$$\int_t \delta T^{(n)} dt = \int_v \rho \frac{\partial U_\alpha^{(n)}}{\partial t} \delta \left(U^{(n)} - \frac{\gamma}{R} \frac{\partial W^{(n)}}{\partial \alpha} \right) + \rho \frac{\partial U_\beta^{(n)}}{\partial t} \delta \left(1 + (k_2 \gamma) V^{(n)} - \frac{\gamma}{R} \frac{\partial W^{(n)}}{\partial \beta} + \right.$$
$$\left. \left. + \rho \frac{\partial U_\gamma^{(n)}}{\partial t} \delta W^{(n)} \right) \right] dV^{(n)} \Bigg|_t - \int_t \int_v \rho \frac{\partial^2 U_\alpha^{(n)}}{\partial t^2} \delta \left(U^{(n)} - \frac{\gamma}{R} \frac{\partial W^{(n)}}{\partial \alpha} \right) + \rho \frac{\partial^2 U_\beta^{(n)}}{\partial t^2} \delta \left((1 + k_2 \gamma) V^{(n)} - \right.$$
$$\left. + \frac{\gamma}{R} \frac{\partial W^{(n)}}{\partial \beta} \right) + \rho \frac{\partial^2 U_\gamma^{(n)}}{\partial t^2} \delta W^{(n)} \Bigg] dV^{(n)} dt. \qquad (8)$$

Here we substitute the displacement expressions in (2) under the variation signs

$$\int_t \delta T^{(n)} dt = \int_v \left(\rho \frac{\partial U_\alpha^{(n)}}{\partial t} \delta U^{(n)} - \rho \gamma \frac{\partial U_\alpha^{(n)}}{\partial t} \delta \frac{\partial W^{(n)}}{R \partial \alpha} + \rho (1 + k_2 \gamma) \frac{\partial U_\beta^{(n)}}{\partial t} \delta V^{(n)} - \right.$$
$$\left. - \rho \gamma \frac{\partial U_\beta^{(n)}}{\partial t} \delta \frac{\partial W^{(n)}}{R \partial \beta} + \rho \frac{\partial U_\gamma^{(n)}}{\partial t} \delta W^{(n)} \right) dV^{(n)} \Bigg|_t - \int_t \int_v \left[\rho \frac{\partial^2 U_\alpha^{(n)}}{\partial t^2} \delta U^{(n)} - \rho \gamma \frac{\partial^2 U_\alpha^{(n)}}{\partial t^2} \delta \frac{\partial W^{(n)}}{R \partial \alpha} + \right.$$
$$\left. + \rho (1 + k_2 \gamma) \frac{\partial^2 U_\beta^{(n)}}{\partial t^2} \delta V^{(n)} - \rho \gamma \frac{\partial^2 U_\beta^{(n)}}{\partial t^2} \delta \frac{\partial W^{(n)}}{R \partial \beta} + \rho \frac{\partial^2 U_\gamma^{(n)}}{\partial t^2} \delta W^{(n)} \right] dV^{(n)} dt. \qquad (9)$$

Performing integration operations on the pipeline thickness, we obtain:

$$\int_t \delta T^{(n)} dt = \int_\beta \int_\alpha \left\{ \rho h \frac{\partial U^{(n)}}{\partial t} \delta U^{(n)} + \left[\rho \left(h + k_2^2 \frac{h^3}{12} \right) \frac{\partial V^{(n)}}{\partial t} - \rho k_2 \frac{h^3}{12R} \frac{\partial^2 W^{(n)}}{\partial t \partial \beta} \right] \delta V^{(n)} + \left[\rho h \frac{\partial W^{(n)}}{\partial t} - \rho \frac{h^3}{12R^2} \frac{\partial^3 W^{(n)}}{\partial t \partial \alpha^2} + \right. \right.$$
$$\left. \left. + \rho k_2 \frac{h^3}{12R} \frac{\partial^2 V^{(n)}}{\partial t \partial \beta} - \rho \frac{h^3}{12R^2} \frac{\partial^3 W^{(n)}}{\partial t \partial \beta^2} \right] \delta W^{(n)} \right\} R^2 d\alpha d\beta \Bigg|_t + \int_\beta \rho \frac{h^3}{12R} \frac{\partial^2 W^{(n)}}{\partial t \partial \alpha} \delta W^{(n)} R d\beta \Bigg|_t \Bigg|_\alpha - \int_\alpha \left(\rho k_2 \frac{h^3}{12} \frac{\partial V^{(n)}}{\partial t} - \right.$$
$$\left. - \rho \frac{h^3}{12R} \frac{\partial^2 W^{(n)}}{\partial t^2 \partial \beta} \right) \delta W^{(n)} R d\alpha \Bigg|_t \Bigg|_\beta - \int_t \int_\beta \int_\alpha \left[\rho h \frac{\partial^2 U^{(n)}}{\partial t^2} \delta U^{(n)} + \left(\rho \left(h + k_2^2 \frac{h^3}{12} \right) \frac{\partial^2 V^{(n)}}{\partial t^2} - \rho k_2 \frac{h^3}{12R} \frac{\partial^3 W^{(n)}}{\partial t^2 \partial \beta} \right) \delta V^{(n)} - \right.$$
$$\left. + \left[\rho h \frac{\partial^2 W^{(n)}}{\partial t^2} - \rho \frac{h^3}{12R^2} \frac{\partial^4 W^{(n)}}{\partial t^2 \partial \alpha^2} + \rho k_2 \frac{h^3}{12R} \frac{\partial^3 V^{(n)}}{\partial t^2 \partial \beta} - \rho \frac{h^3}{12R^2} \frac{\partial^4 W^{(n)}}{\partial t^2 \partial \beta^2} \right] \delta W^{(n)} \right] R^2 d\alpha d\beta dt -$$

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$$-\int_t\int_\beta \rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\alpha}\delta W^{(n)}Rd\beta dt\Big|_\alpha+\int_t\int_\alpha\left(\rho k_2\frac{h^3}{12}\frac{\partial^2 V^{(n)}}{\partial t^2}-\rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)}Rd\alpha dt\Big|_\beta . \quad (10)$$

The variations of the work of external forces are determined using expressions (2):

$$\int_t\delta A^{(n)}dt=\int_t\int_V\left[P_1^{(n)}\delta\left(U^{(n)}-\frac{\gamma}{R}\frac{\partial W^{(n)}}{\partial\alpha}\right)+P_2^{(n)}\delta(1+k_2\gamma)V^{(n)}-\gamma\frac{\partial W^{(n)}}{R\partial\beta}+P_3^{(n)}\delta W^{(n)}\right]dV^{(n)}dt+$$
$$+\int_t\int_\beta\int_\alpha\left[q_1^{(n)}\delta\left(U^{(n)}-\left(\pm\frac{h}{2}\right)\frac{\partial W^{(n)}}{R\partial\alpha}\right)+q_2^{(n)}\delta\left(1+k_2\left(\pm\frac{h}{2}\right)\right)V^{(n)}-\left(\pm\frac{h}{2}\right)\frac{\partial W^{(n)}}{R\partial\beta}+q_3^{(n)}\delta W^{(n)}\right]R^2d\alpha d\beta dt+$$
$$+\int_t\int_\beta\int_\alpha\left[\varphi_1^{(n)}\delta\left(U^{(n)}-\gamma\frac{\partial W^{(n)}}{R\partial\alpha}\right)+\varphi_2^{(n)}\delta\left[(1+k_2\gamma)V^{(n)}-\gamma\frac{\partial W^{(n)}}{R\partial\beta}\right]+\varphi_3^{(n)}\delta W^{(n)}\right]Rd\beta dt\Big|_\alpha+$$
$$+\int_t\int_\alpha\int_\beta\left[f_1^{(n)}\delta\left(U^{(n)}-\gamma\frac{\partial W^{(n)}}{R\partial\alpha}\right)+f_2^{(n)}\delta\left[(1+k_2\gamma)V^{(n)}-\gamma\frac{\partial W^{(n)}}{R\partial\beta}\right]+f_3^{(n)}\delta W^{(n)}\right]Rd\alpha d\gamma dt\Big|_\beta . \quad (11)$$

Introducing some notations, let us rewrite the variations of the work of external forces in the following form:

$$\int_t\delta A^{(n)}dt=\int_t\int_\beta\int_\alpha\Big\{\left[N^{(n)}\left(P_1^{(n)}\right)+N^{(n)}\left(q_1^{(n)}\right)\right]\delta U^{(n)}+\left[N^{(n)}\left(P_2^{(n)}\right)+N^{(n)}\left(q_2^{(n)}\right)\right]\delta V^{(n)}+\Big[\frac{\partial}{R\partial\alpha}\left(M^{(n)}\left(P_1^{(n)}\right)+M^{(n)}\left(q_1^{(n)}\right)\right)+$$
$$+\frac{\partial}{R\partial\beta}\left(M^{(n)}\left(P_2^{(n)}\right)+M^{(n)}\left(q_2^{(n)}\right)\right)+Q^{(n)}\left(P_3^{(n)}\right)+Q^{(n)}\left(q_3^{(n)}\right)\Big]\delta W^{(n)}\Big\}R^2d\alpha d\beta dt+\int_t\int_\beta\Big\{N^{(n)}\left(\varphi_1^{(n)}\right)\delta U^{(n)}+$$
$$+N^{(n)}\left(\varphi_2^{(n)}\right)\delta V^{(n)}+\left[q^{(n)}\left(\varphi_3^{(n)}\right)+\frac{\partial M^{(n)}\left(\varphi_2^{(n)}\right)}{R\partial\beta}\right]\delta W^{(n)}-M^{(n)}\left(\varphi_1^{(n)}\right)\delta\frac{\partial W^{(n)}}{R\partial\alpha}\Big\}Rd\beta dt\Big|_\alpha-$$
$$\int_t M^{(n)}\left(\varphi_2^{(n)}\right)\delta W^{(n)}dt\Big|_\alpha\Big|_\beta+\int_t\int_\alpha\Big\{N^{(n)}\left(f_1^{(n)}\right)\delta U^{(n)}+N^{(n)}\left(f_2^{(n)}\right)\delta V^{(n)}+\Big[Q^{(n)}\left(f_3^{(n)}\right)+$$
$$+\frac{\partial M^{(n)}\left(f_1^{(n)}\right)}{R\partial\alpha}\Big]\delta W^{(n)}-M^{(n)}\left(f_2^{(n)}\right)\delta\frac{\partial W^{(n)}}{R\partial\beta}\Big\}Rd\alpha dt\Big|_\beta-\int_t M^{(n)}\left(f_1^{(n)}\right)\delta W^{(n)}dt\Big|_\beta\Big|_\alpha \quad (12)$$

We turn to the definition of the variation of potential energy in this formulation:

$$\int_t\delta\Pi^{(n)}dt=\int_t\int_v\left(\sigma_{\alpha\alpha}^{(n)}\delta l_{\alpha\alpha}^{(n)}+\sigma_{\beta\beta}^{(n)}\delta l_{\beta\beta}^{(n)}+\sigma_{\alpha\beta}^{(n)}\delta l_{\alpha\beta}^{(n)}\right)dVdt \quad (13)$$

Substitute in (13) formulas (3), as a result, the variation of potential energy has the following form:

$$\int_t\delta\Pi^{(n)}dt=\int_t\int_v\Big[\sigma_\alpha\delta\left(\frac{1}{R}\frac{\partial U^{(n)}}{\partial\alpha}-\gamma\frac{1}{R^2}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}\right)+\sigma_\beta\delta\left(\frac{\partial V^{(n)}}{R\partial\beta}-\left(\gamma-k_2\gamma^2\right)\frac{\partial^2 W^{(n)}}{R^2\partial\beta^2}+\left(1-k_2\gamma+k_2^2\gamma^2\right)k_2W^{(n)}\right)+$$
$$+\sigma_{\alpha\beta}\delta\left(\left(1-k_2\gamma+k_2^2\gamma^2\right)\frac{\partial U^{(n)}}{R\partial\beta}-\left(\gamma-k_2\gamma^2\right)\frac{\partial^2 W^{(n)}}{R^2\partial\alpha\partial\beta}+\left(1+k_2\gamma\right)\frac{\partial V^{(n)}}{R\partial\beta}-\frac{\gamma}{R^2}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}\right)\Big]dv^{(n)}dt \quad (14)$$

Let us open the brackets under the variation signs. Then the variation of the potential energy (14) will be

$$\int_t\delta\Pi^{(n)}dt=\int_t\int_v\Big\{\sigma_\alpha\delta\frac{\partial U^{(n)}}{R\partial\alpha}-\gamma\sigma_\alpha\delta\frac{\partial^2 W^{(n)}}{R^2\partial\alpha^2}+\sigma_\beta\delta\frac{\partial V^{(n)}}{R\partial\beta}-\left(\gamma-k_2\gamma^2\right)\cdot\sigma_\beta\delta\frac{\partial^2 W^{(n)}}{R^2\partial\beta^2}+$$

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$$+\left(1-k_2\gamma+k_2^2\gamma^2\right)\cdot k_2\sigma_\beta\delta W^{(n)}+\left(1-k_2\gamma+k_2^2\gamma^2\right)\cdot\sigma_{\alpha\beta}\frac{\partial U^{(n)}}{R\partial\beta}-$$
$$-\left(\gamma-k_2\gamma^2\right)\sigma_{\alpha\beta}\frac{\partial^2 W^{(n)}}{R^2\partial\alpha\partial\beta}+\left(1+k_2\gamma\right)\sigma_{\alpha\beta}\delta\frac{\partial V^{(n)}}{R\partial\alpha}-\gamma\sigma_{\alpha\beta}\delta\frac{\partial^2 W^{(n)}}{R^2\partial\alpha\partial\beta}\Bigg\}dVdt. \quad (15)$$

Performing integration operations on parts in (15), after some transformations for the variation of potential energy under n-loading we have the following:

$$\int_t \delta\Pi dt=\int_t\int_\beta\left(N_\alpha^{(n)}\delta U^{(n)}+N_{\alpha\beta}^{(n)}\delta V^{(n)}+\left(\frac{\partial M_\alpha^{(n)}}{R\partial\alpha}+2\frac{\partial M_{\alpha\beta}^{(n)}}{R\partial\beta}\right)\delta W^{(n)}-M_\alpha^{(n)}\delta\frac{\partial W^{(n)}}{R\partial\alpha}\right)Rd\beta dt\Big|_\alpha-$$
$$-\int_t M_{\alpha\beta}^{(n)}\delta W^{(n)}\Big|_\alpha\Big|_\beta dt+\int_t\int_\alpha\left(\left(1+k_2^2\frac{h^2}{12}\right)N_{\alpha\beta}^{(n)}\delta U^{(n)}+N_\beta^{(n)}\delta V_\alpha^{(n)}+\left(\frac{\partial M_\beta^{(n)}}{R\partial\beta}+2\frac{\partial M_{\alpha\beta}^{(n)}}{R\partial\alpha}\right)\delta W^{(n)}-\right.$$
$$\left.-M_\beta^{(n)}\delta\frac{\partial W^{(n)}}{R\partial\beta}\right)Rd\alpha dt\Big|_\beta-\int_t M_{\alpha\beta}^{(n)}\delta W^{(n)}\Big|_\alpha\Big|_\beta dt-\int_t\int_\beta\int_\alpha\left(\frac{\partial N_\alpha^{(n)}}{R\partial\alpha}+\left(1+k_2^2\frac{h^2}{12}\right)\frac{\partial N_{\alpha\beta}^{(n)}}{R\partial\beta}\right)\delta U^{(n)}+$$
$$+\left(\frac{\partial N_{\alpha\beta}^{(n)}}{R\partial\alpha}+2\frac{\partial N_\beta^{(n)}}{R\partial\beta}\right)\delta V^{(n)}+\left(\frac{\partial^2 M_\alpha^{(n)}}{R^2\partial\alpha^2}+2\frac{\partial^2 M_{\alpha\beta}^{(n)}}{R^2\partial\alpha\partial\beta}+\frac{\partial^2 M_\alpha^{(n)}}{R^2\partial\beta^2}-\left(1+k_2^2\frac{h^2}{12}\right)k_2N_\beta^{(n)}\right)\delta W^{(n)}\Bigg)Rd\alpha d\beta dt \quad (16)$$

where

$$N_\alpha^{(n)}=\int_\gamma\sigma_{\alpha\alpha}^{(n)}d\gamma;\quad N_\beta^{(n)}=\int_\gamma\sigma_{\beta\beta}^{(n)}d\gamma;\quad N_{\alpha\beta}^{(n)}=\int_\gamma\sigma_{\alpha\beta}^{(n)}d\gamma;$$
$$M_\alpha^{(n)}=\int_\gamma\sigma_{\alpha\alpha}^{(n)}\gamma d\gamma;\quad M_\beta^{(n)}=\int_\gamma\sigma_{\beta\beta}^{(n)}\gamma d\gamma;\quad M_{\alpha\beta}^{(n)}=\int_\gamma\sigma_{\alpha\beta}^{(n)}\gamma d\gamma \quad (17)$$

Now determine the internal forces and moments by (17) under alternating loading, taking into account the stress expression (4):

$$N_\alpha^{(n)}=\int_\gamma\sigma_{\alpha\alpha}^{(n)}d\gamma=G_1\left[\left(a_{11}-a_{11\omega}^{(n)}\right)\frac{\partial U^{(n)}}{\partial\alpha}-\left(a_{12}-a_{12\omega}^{(n)}\right)\frac{\partial^2 W^{(n0}}{\partial\alpha^2}+\left(a_{13}-a_{13\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\beta}-\left(a_{14}-a_{14\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\beta^2}+\left(a_{15}-a_{15\omega}^{(n)}\right)W^{(n)}\right]$$
$$N_\beta^{(n)}=\int_\gamma\sigma_{\beta\beta}^{(n)}d\gamma=G_1\left[\left(a_{21}-a_{21\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\beta}-\left(a_{22}-a_{22\omega}^{(n)}\right)\frac{\partial^2 W}{\partial\beta^2}+\left(a_{23}-a_{23\omega}^{(n)}\right)W+\left(a_{24}-a_{24\omega}^{(n)}\right)\frac{\partial U}{\partial\alpha}-\left(a_{25}-a_{25\omega}^{(n)}\right)\frac{\partial^2 W}{\partial\beta^2}\right]$$
$$N_{\alpha\beta}^{(n)}=\int_\gamma\sigma_{\alpha\beta}^{(n)}d\gamma=G\left[\left(a_{31}-a_{31\omega}^{(n)}\right)\frac{\partial U^{(n)}}{\partial\beta}+\left(a_{32}-a_{32\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+\left(a_{33}-a_{33\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\alpha}+\left(a_{34}-a_{34\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}\right]$$
$$M_\alpha^{(n)}=\int_\gamma\sigma_{\alpha\alpha}^{(n)}\gamma d\gamma=G_1\left[\left(a_{41}-a_{41\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha^2}-\left(a_{42}-a_{42\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\beta^2}-\left(a_{43}-a_{43\omega}^{(n)}\right)W^{(n)}\right]$$
$$M_\beta^{(n)}=\int_\gamma\sigma_{\beta\beta}^{(n)}\gamma d\gamma=G_1\left[\left(a_{51}-a_{51\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\beta^2}-\left(a_{52}-a_{52\omega}^{(n)}\right)W^{(n)}-\left(a_{53}-a_{53\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha^2}\right]$$
$$M_{\alpha\beta}^{(n)}=\int_\gamma\sigma_{\alpha\beta}^{(n0}\gamma d\gamma=G\left[\left(a_{61}-a_{61\omega}^{(n)}\right)\frac{\partial U^{(n)}}{\partial\beta}+\left(a_{62}-a_{62\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+\left(a_{63}-a_{63\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\alpha}\right] \quad (18)$$

Where the coefficients are, for example, a_{1j} и $a_{1j\omega}$ are determined by the formulas:

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$$a_{11}=\frac{h}{R},\quad a_{12}=0,\quad a_{13}=\frac{h\mu}{R},\quad a_{14}=\mu k_2\frac{h^3}{12R^2},\quad a_{15}=\mu\left(h+k_2^2\frac{h^3}{12}\right)k_2,$$

$$a_{11\omega}=\frac{1}{R}\int_\gamma\omega^{(n)}d\gamma,\quad a_{12\omega}=\frac{1}{R^2}\int_\gamma\omega^{(n)}\gamma d\gamma,\quad a_{13\omega}=\frac{\mu}{R}\int_\gamma\omega^{(n)}d\gamma,$$

$$a_{14\omega}=\frac{\mu}{R^2}\int_\gamma\omega^{(n)}(\gamma-k_2\gamma^2)d\gamma,\quad a_{15\omega}=\mu k_2\int_\gamma\omega^{(n)}(1-k_2\gamma+k_2^2\gamma^2)d\gamma \tag{19}$$

Taking into account relations (4) and (19), the expressions of internal forces and moments, e.g, $N_\alpha^{(n)}$ и $M_\alpha^{(n)}$ can be represented as:

$$N_\alpha^{(n)}=G_1\left[\left(a_{11}-a_{11\omega}^{(n)}\right)\frac{\partial U^{(n)}}{\partial\alpha}-\left(a_{12}-a_{12\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha^2}+\left(a_{13}-a_{13\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\beta}-\right.$$
$$\left.-\left(a_{14}-a_{14\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\beta^2}+\left(a_{15}-a_{15\omega}^{(n)}\right)W^{(n)}+N_\alpha^{0(n-1)}+N_\alpha^{0(n-m)}\right] \tag{20}$$

Expressions of forces and moments of the type (20), rewrite in the following form

$$N_\alpha^{(n)}=\tilde N_\alpha^{(n)}-N_\alpha^{0(n-1)}+N_\alpha^{0(n-m)};\quad M_\alpha^{(n)}=\tilde M_\alpha^{(n)}-M_\alpha^{0(n-1)}+M_\alpha^{0(n-m)} \tag{21}$$

where

$$\tilde N_\alpha^{(n)}=\left(a_{11}-a_{11\omega}^{(n)}\right)\frac{\partial U^{(n)}}{\partial\alpha}-\left(a_{12}-a_{12\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\alpha^2}+\left(a_{13}-a_{13\omega}^{(n)}\right)\frac{\partial V^{(n)}}{\partial\beta}-\left(a_{14}-a_{14\omega}^{(n)}\right)\frac{\partial^2 W^{(n)}}{\partial\beta^2}+\left(a_{15}-a_{15\omega}^{(n)}\right)W^{(n)}$$

$$N_\alpha^{0(n-1)}=\left[-a_{11\omega}^{0(n-1)}\frac{\partial U^{(n)}}{\partial\alpha}+a_{12\omega}^{0(n-1)}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}-a_{13\omega}^{0(n-1)}\frac{\partial V^{(n)}}{\partial\beta}+a_{14\omega}^{0(n-1)}\frac{\partial^2 W^{(n)}}{\partial\beta^2}-a_{15\omega}^{0(n-1)}W^{(n)}\right];$$

$$N_\alpha^{0(n-m)}=\sum_{m=1}^{n-1}\left[a_{11\omega}^{0(n-m)}\frac{\partial}{\partial\alpha}\left(U^{0(n-m)}-U^{0(n-m-1)}\right)-a_{12\omega}^{0(n-m)}\frac{\partial^2}{\partial\alpha^2}\left(W^{0(n-m)}-\right.\right.$$
$$\left.-W^{0(n-m-1)}\right)-a_{13\omega}^{0(n-m)}\frac{\partial}{\partial\beta}\left(V^{0(n-m)}-V^{0(n-m-1)}\right)-a_{14\omega}^{0(n-m)}\frac{\partial^2}{\partial\beta^2}\left(W^{0(n-m)}-\right. \tag{22}$$
$$\left.\left.-W^{0(n-m-1)}\right)+a_{15\omega}^{0(n-m)}\left(W^{0(n-m)}-W^{0(n-m-1)}\right)\right]$$

For the sake of brevity, we will use the following notation:

$$\tilde a_{11}=\left(a_{11}-a_{11\omega}^{(n)}\right),\quad \tilde a_{21}=\left(a_{21}-a_{21\omega}^{(n)}\right),\quad ....,\quad \tilde a_{61}=\left(a_{61}-a_{61\omega}^{(n)}\right) \tag{23}$$

Note that relations (18) can be generalized using expressions of internal forces and moments for any n-loading, according to (21).

Now we substitute the variations of kinetic energy, potential energy and work of external forces into the Hamilton-Ostrogradsky variational principle:

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$$\int_t \left(\delta T^{(n)} - \delta \Pi^{(n)} + \delta A^{(n)}\right)dt = \int_\beta\int_\alpha \left\{ \rho h \frac{\partial U^{(n)}}{\partial t}\delta U^{(n)} + \left[\rho\left(h + k_2^2\frac{h^3}{12}\right)\frac{\partial V^{(n)}}{\partial t} - \rho k_2 \frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\beta}\right]\delta V^{(n)} + \right.$$

$$\left. + \left[\rho h \frac{\partial W^{(n)}}{\partial t} - \rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\alpha^2} + + \rho k_2 \frac{h^3}{12R}\frac{\partial^2 V^{(n)}}{\partial t\partial\beta} - \rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\beta^2}\right]\delta W^{(n)}\right\} R^2 d\alpha d\beta \Big|_t$$

$$+ \int_\beta \rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\alpha}\delta W^{(n)} R d\beta \Big|_t\Big|_\alpha - \int_\alpha\left(\rho k_2\frac{h^3}{12}\frac{\partial V^{(n)}}{\partial t} - \rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)} R d\alpha \Big|_t\Big|_\beta - \int_t\int_\beta\int_\alpha\left\{\rho h\frac{\partial^2 U^{(n)}}{\partial t^2} + \right.$$

$$+ N^{(n)}\left(P_1^{(n)}\right) + N^{(n)}\left(q_1^{(n)}\right) - \frac{G_1}{R}\left[\tilde{a}_{11}\frac{\partial^2 U^{(n)}}{\partial\alpha^2} - \tilde{a}_{12}\frac{\partial^3 W^{(n)}}{\partial\alpha^3} + \tilde{a}_{13}\frac{\partial^2 V^{(n)}}{\partial\beta\partial\alpha} - \tilde{a}_{14}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha} + \tilde{a}_{15}\frac{\partial W^{(n)}}{\partial\alpha}\right] +$$

$$+ \frac{G}{R}\left(1 + k_2^2\frac{h^2}{12}\right)\left[\tilde{a}_{31}\frac{\partial^2 U^{(n)}}{\partial\beta^2} + \tilde{a}_{32}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2} + \tilde{a}_{33}\frac{\partial^2 V^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{34}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2}\right]\Bigg]\delta U^{(n)} + \left[\rho\left(h + k_2^2\frac{h^3}{12}\right)\frac{\partial^2 V^{(n)}}{\partial t^2} - \right.$$

$$- \rho k_2\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta} + N^{(n)}\left(P_2^{(n)}\right) + N^{(n)}\left(q_2^{(n)}\right) + \frac{G}{R}\left[\tilde{a}_{31}\frac{\partial^2 U^{(n)}}{\partial\beta\partial\alpha} + \tilde{a}_{32}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta} + \tilde{a}_{33}\frac{\partial^2 V^{(n)}}{\partial\alpha^2} + \tilde{a}_{34}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta}\right] +$$

$$+ \frac{G_1}{R}\left[\tilde{a}_{21}\frac{\partial^2 V^{(n)}}{\partial\beta\partial\alpha} - \tilde{a}_{22}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha} + \tilde{a}_{23}\frac{\partial W^{(n)}}{\partial\alpha} + \tilde{a}_{24}\frac{\partial^2 U^{(n)}}{\partial\alpha^2} - \tilde{a}_{25}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha}\right]\Bigg]\delta V^{(n)} + \left[\rho h\frac{\partial^2 W^{(n)}}{\partial t^2} - \right.$$

$$- \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\alpha^2} + \rho k_2\frac{h^3}{12R}\frac{\partial^3 V^{(n)}}{\partial t^2\partial\beta} - \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\beta^2} + \frac{\partial}{R\partial\beta}\left(M^{(n)}\left(P_2^{(n)}\right) + M^{(n)}\left(q_2^{(n)}\right)\right) + Q^{(n)}\left(P_3^{(n)}\right) +$$

$$+ Q^{(n)}\left(q_3^{(n)}\right) + \frac{\partial}{R\partial\alpha}\left(M^{(n)}\left(P_1^{(n)}\right) + M^{(n)}\left(q_1^{(n)}\right)\right) + \frac{G_1}{R^2}\left[\tilde{a}_{41}\frac{\partial^4 W^{(n)}}{\partial\alpha^4} - \tilde{a}_{42}\frac{\partial^4 W^{(n)}}{\partial\beta^2\partial\alpha^2} - \tilde{a}_{43}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}\right] +$$

$$+ \frac{2G}{R^2}\left[\tilde{a}_{61}\frac{\partial^3 U^{(n)}}{\partial\beta^2\partial\alpha} + \tilde{a}_{62}\frac{\partial^4 W^{(n)}}{\partial\alpha^2\partial\beta^2} + \tilde{a}_{63}\frac{\partial^3 V^{(n)}}{\partial\alpha^2\partial\beta}\right] + \frac{G_1}{R^2}\left[\tilde{a}_{41}\frac{\partial^4 W^{(n)}}{\partial\alpha^2\partial\beta^2} - \tilde{a}_{42}\frac{\partial^4 W^{(n)}}{\partial\beta^4} - \tilde{a}_{43}\frac{\partial^2 W^{(n)}}{\partial\beta^2}\right] -$$

$$\left. - G_1 k_2\left(1 + k_2^2\frac{h^2}{12}\right)\left[\tilde{a}_{21}\frac{\partial V^{(n)}}{\partial\beta} - \tilde{a}_{22}\frac{\partial^2 W}{\partial\beta^2} + \tilde{a}_{23}W + \tilde{a}_{24}\frac{\partial U}{\partial\alpha} - \tilde{a}_{25}\frac{\partial^2 W}{\partial\beta^2}\right]\Bigg]\delta W^{(n)}\right\} R^2 d\alpha d\beta dt -$$

$$+ \int_t\int_\beta\left\{N^{(n)}\left(\varphi_1^{(n)}\right) + G_1\left[\tilde{a}_{11}\frac{\partial U^{(n)}}{\partial\alpha} - \tilde{a}_{12}\frac{\partial^2 W^{(n)}}{\partial\alpha^2} + \tilde{a}_{13}\frac{\partial V^{(n)}}{\partial\beta} - \tilde{a}_{14}\frac{\partial^2 W^{(n)}}{\partial\beta^2} + \tilde{a}_{15}W^{(n)}\right]\delta U^{(n)} + \right.$$

$$+ \left[N^{(n)}\left(\varphi_2^{(n)}\right) + G\left[\tilde{a}_{31}\frac{\partial U^{(n)}}{\partial\beta} + \tilde{a}_{32}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{33}\frac{\partial V^{(n)}}{\partial\alpha} + \tilde{a}_{34}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}\right]\delta V^{(n)} + \rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\alpha}\delta W^{(n)} + \right.$$

$$\left. + \left[-M^{(n)}\left(f_2^{(n)}\right) - G_1\left[-\tilde{a}_{41}\frac{\partial^2 W^{(n)}}{\partial\alpha^2} - \tilde{a}_{42}\frac{\partial^2 W^{(n)}}{\partial\beta^2} - \tilde{a}_{43}W^{(n)}\right]\right]\delta\frac{\partial W^{(n)}}{R\partial\beta}\right\} R d\alpha dt \Big|_\alpha +$$

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$$+\int_t\int_\alpha \Big\{\rho k_2 \frac{h^3}{12}\frac{\partial^2 V^{(n)}}{\partial t^2} - \rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta} + Q(f_3^{(n)}) + \frac{\partial M^{(n)}(f_1^{(n)})}{R\partial\alpha} + \frac{1}{R}\Big[G_1\Big[\tilde{a}_{51}\frac{\partial^3 W^{(n)}}{\partial\beta^3} -$$

$$-\tilde{a}_{52}\frac{\partial W^{(n)}}{\partial\beta} - \tilde{a}_{53}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta}\Big] + 2G\Big[\tilde{a}_{61}\frac{\partial^2 U^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{62}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta} + \tilde{a}_{63}\frac{\partial^2 V^{(n)}}{\partial\alpha^2}\Big]\delta W^{(n)} + \Big[N^{(n)}\big(f_1^{(n)}\big) +$$

$$+G\Big(1+k_2^2\frac{h^2}{12}\Big)\Big[\tilde{a}_{31}\frac{\partial U^{(n)}}{\partial\beta} + \tilde{a}_{32}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{33}\frac{\partial V^{(n)}}{\partial\alpha} + \tilde{a}_{34}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}\Big]\Big]\delta U^{(n)} + \Big[G_1\Big[\tilde{a}_{21}\frac{\partial V^{(n)}}{\partial\beta} - \tilde{a}_{22}\frac{\partial^2 W^{(n)}}{\partial\beta^2} +$$

$$+\tilde{a}_{23}W^{(n)} + \tilde{a}_{24}\frac{\partial U^{(n)}}{\partial\alpha} - \tilde{a}_{25}\frac{\partial^2 W^{(n)}}{\partial\beta^2}\Big] + N^{(n)}\big(f_2^{(n)}\big)\Big]\delta V^{(n)} - G_1\Big[\tilde{a}_{51}\frac{\partial^2 W^{(n)}}{\partial\beta^2} - \tilde{a}_{52}W^{(n)} - \tilde{a}_{53}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}\Big] -$$

$$-M^{(n)}\big(f_2^{(n)}\big)\Big]\delta\frac{\partial W^{(n)}}{R\partial\beta}\Big\}R\,d\alpha\,dt\Big|_\beta + \int_t\Big\{\Big[\tilde{a}_{61}\frac{\partial U^{(n)}}{\partial\beta} + \tilde{a}_{62}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{63}\frac{\partial V^{(n)}}{\partial\alpha}\Big] + M^{(n)}\big(\varphi_2^{(n)}\big)\Big]\delta W^{(n)}\Big\}dt\Big|_\alpha\Big|_\beta -$$

$$-\int_t M^{(n)}\big(f_1^{(n)}\big)\delta W^{(n)}dt\Big|_\beta\Big|_\alpha - \int_t G\Big[\tilde{a}_{61}\frac{\partial U^{(n)}}{\partial\beta} + \tilde{a}_{62}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{63}\frac{\partial V^{(n)}}{\partial\alpha}\Big]\delta W^{(n)}d\beta\,dt\Big|_\alpha \qquad (24)$$

From the variational equation (24) we obtain the following systems of equations of motion with appropriate boundary and initial conditions:

$$\rho h\frac{\partial^2 U^{(n)}}{\partial t^2} - \frac{G_1}{R}\Big[\tilde{a}_{11}\frac{\partial^2 U^{(n)}}{\partial\alpha^2} - \tilde{a}_{12}\frac{\partial^3 W^{(n)}}{\partial\alpha^3} + \tilde{a}_{13}\frac{\partial^2 V^{(n)}}{\partial\beta\partial\alpha} - \tilde{a}_{14}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha} + \tilde{a}_{15}\frac{\partial W^{(n)}}{\partial\alpha}\Big] +$$

$$+\frac{G}{R}\Big(1+k_2^2\frac{h^2}{12}\Big)\Big[\tilde{a}_{31}\frac{\partial^2 U^{(n)}}{\partial\beta^2} + \tilde{a}_{32}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2} + \tilde{a}_{33}\frac{\partial^2 V^{(n)}}{\partial\alpha\partial\beta} + \tilde{a}_{34}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2}\Big] + N^{(n)}\big(P_1^{(n)}\big) + N^{(n)}\big(q_1^{(n)}\big) = 0;$$

$$\rho\Big(h + k_2^2\frac{h^3}{12}\Big)\frac{\partial^2 V^{(n)}}{\partial t^2} - \rho k_2\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta} + \frac{G}{R}\Big[\tilde{a}_{31}\frac{\partial^2 U^{(n)}}{\partial\beta\partial\alpha} + \tilde{a}_{32}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta} + \tilde{a}_{33}\frac{\partial^2 V^{(n)}}{\partial\alpha^2} + \tilde{a}_{34}\frac{\partial^3 W^{(n)}}{\partial\alpha^2\partial\beta}\Big] +$$

$$+\frac{G_1}{R}\Big[\tilde{a}_{21}\frac{\partial^2 V^{(n)}}{\partial\beta\partial\alpha} - \tilde{a}_{22}\frac{\partial^3 W}{\partial\beta^2\partial\alpha} + \tilde{a}_{23}\frac{\partial W}{\partial\alpha} + \tilde{a}_{24}\frac{\partial^2 U}{\partial\alpha^2} - \tilde{a}_{25}\frac{\partial^3 W}{\partial\beta^2\partial\alpha}\Big] + N^{(n)}\big(P_2^{(n)}\big) + N^{(n)}\big(q_2^{(n)}\big) = 0;$$

$$\rho h\frac{\partial^2 W^{(n)}}{\partial t^2} - \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\alpha^2} + \rho k_2\frac{h^3}{12R}\frac{\partial^3 V^{(n)}}{\partial t^2\partial\beta} - \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\beta^2} + \frac{G_1}{R^2}\Big[\tilde{a}_{41}\frac{\partial^4 W^{(n)}}{\partial\alpha^4} -$$

$$-\tilde{a}_{42}\frac{\partial^4 W^{(n)}}{\partial\beta^2\partial\alpha^2} - \tilde{a}_{43}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}\Big] + \frac{2G}{R^2}\Big[\tilde{a}_{61}\frac{\partial^3 U^{(n)}}{\partial\beta^2\partial\alpha} + \tilde{a}_{62}\frac{\partial^4 W^{(n)}}{\partial\alpha^2\partial\beta^2} + \tilde{a}_{63}\frac{\partial^3 V^{(n)}}{\partial\alpha^2\partial\beta}\Big] + \frac{G_1}{R^2}\Big[\tilde{a}_{41}\frac{\partial^4 W^{(n)}}{\partial\alpha^2\partial\beta^2} - \qquad (25)$$

$$-\tilde{a}_{42}\frac{\partial^4 W^{(n)}}{\partial\beta^4} - \tilde{a}_{43}\frac{\partial^2 W^{(n)}}{\partial\beta^2}\Big] - G_1k_2\Big(1+k_2^2\frac{h^2}{12}\Big)\Big[\tilde{a}_{21}\frac{\partial V^{(n)}}{\partial\beta} - \tilde{a}_{22}\frac{\partial^2 W}{\partial\beta^2} + \tilde{a}_{23}W + \tilde{a}_{24}\frac{\partial U}{\partial\alpha} - \tilde{a}_{25}\frac{\partial^2 W}{\partial\beta^2}\Big] +$$

$$+\frac{\partial}{R\partial\alpha}\Big(M^{(n)}\big(P_1^{(n)}\big) + M^{(n)}\big(q_1^{(n)}\big)\Big) + \frac{\partial}{R\partial\beta}\Big(M^{(n)}\big(P_2^{(n)}\big) + M^{(n)}\big(q_2^{(n)}\big)\Big) + Q^{(n)}\big(P_3^{(n)}\big) + Q^{(n)}\big(q_3^{(n)}\big) = 0.$$

Boundary conditions for the parameter α:

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$$\left[\tilde{a}_{11}\frac{\partial U^{(n)}}{\partial\alpha}-\tilde{a}_{12}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}+\tilde{a}_{13}\frac{\partial V^{(n)}}{\partial\beta}-\tilde{a}_{14}\frac{\partial^2 W^{(n)}}{\partial\beta^2}+\tilde{a}_{15}W^{(n)}+N^{(n)}\left(\varphi_1^{(n)}\right)\right]\delta U^{(n)}+$$
$$+\left[\tilde{a}_{31}\frac{\partial U^{(n)}}{\partial\beta}+\tilde{a}_{32}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+\tilde{a}_{33}\frac{\partial V^{(n)}}{\partial\alpha}+\tilde{a}_{34}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+N^{(n)}\left(\varphi_2^{(n)}\right)\right]\delta V^{(n)}+$$
$$+\frac{1}{R}\left[\left[\tilde{a}_{41}\frac{\partial^3 W^{(n)}}{\partial\alpha^3}-\tilde{a}_{42}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha}-\tilde{a}_{43}\frac{\partial W^{(n)}}{\partial\alpha}\right]+\left[\tilde{a}_{61}\frac{\partial^2 U^{(n)}}{\partial\beta^2}+\tilde{a}_{62}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2}+\right.\right.$$
$$\left.\left.+\tilde{a}_{63}\frac{\partial^2 V^{(n)}}{\partial\alpha\partial\beta}\right]\right]\delta W^{(n)}-\left[-\tilde{a}_{41}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}-\tilde{a}_{42}\frac{\partial^2 W^{(n)}}{\partial\beta^2}-\tilde{a}_{43}W^{(n)}-M^{(n)}\left(f_2^{(n)}\right)\right]\delta\frac{\partial W^{(n)}}{R\partial\alpha}\bigg|_{\alpha}=0. \quad (26)$$

Nodal effect by the parameters α and β:

$$\left[\tilde{a}_{61}\frac{\partial U^{(n)}}{\partial\beta}+\tilde{a}_{62}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+\tilde{a}_{63}\frac{\partial V^{(n)}}{\partial\alpha}+M^{(n)}\left(\varphi_2^{(n)}\right)\right]\delta W^{(n)}\Big|_{\beta}\Big|_{\alpha}=0 \quad (27)$$

Initial conditions for the parameter t:

$$\rho h\frac{\partial U^{(n)}}{\partial t}\delta U^{(n)}\bigg|_{e}=0;\quad \left[\rho\left(h+k_2^2\frac{h^3}{12}\right)\frac{\partial V^{(n)}}{\partial t}-\rho k_2\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\beta}\right]\delta V^{(n)}\bigg|_{t}=0;$$
$$\left[\rho h\frac{\partial W^{(n)}}{\partial t}-\rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\alpha^2}+\rho k_2\frac{h^3}{12R}\frac{\partial^2 V^{(n)}}{\partial t\partial\beta}-\rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\beta^2}\right]\delta W^{(n)}\bigg\|_{e}=0. \quad (28)$$

Boundary and initial effects on the parameters t, α and β:

$$\int_{\beta}\rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\alpha}\delta W^{(n)}Rd\beta\bigg|_{t}\bigg|_{\alpha}=0;\quad \int_{\alpha}\left(\rho k_2\frac{h^3}{12}\frac{\partial V^{(n)}}{\partial t}-\rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)}Rd\alpha\bigg|_{t}\bigg|_{\beta}=0. \quad (29)$$

The Bubnov-Galerkin method [12] was used to solve boundary problems:

$$U=\sum_n U_n(\alpha,t)\cos\frac{n\pi\beta}{\beta_1},\ V=\sum_n V_n(\alpha,t)\sin\frac{n\pi\beta}{\beta_1},\ W=\sum_n W_n(\alpha,t)\cos\frac{n\pi\beta}{\beta_1}. \quad (30)$$

The system of differential equations (28) can be written in vector form

$$A_1\ddot{U}_n+A_2\ddot{U}_n^{II}+A_3U_n^{IV}+A_4U_n^{II}+A_5U_n^{I}+A_6U_n+F_n=0. \quad (31)$$

where $U_k=(W_k,U_k,V_k)^T$, $F_k=(Z_k,X_k,Y_k)^T$, A_i – matrix of the third order.

To solve boundary value problems (31), (30) and (25) the finite difference method of the second order of accuracy is applied. The following system of algebraic equations is obtained on the basis of using the central difference formulas [19,20]:

$$B_nU_{n,i-1}^{k+1}+C_nU_{n,i}^{k+1}+B_nU_{n,i+1}^{k+1}+\bar{A}_nU_{n,i+1}^{k+1}+\bar{B}_nU_{n,i-1}^{k}+\bar{C}_nU_{n,i}^{k}+\bar{D}_nU_{n,i+1}^{k}+\bar{A}_nU_{n,i+2}^{k}+B_nU_{n,i-1}^{k-1}+$$
$$+C_nU_{n,i}^{k-1}+B_nU_{n,i+1}^{k-1}+\tau^2F_{n,i}^{k}=0. \quad (32)$$

After approximation, the initial condition (28) will take the following form:

$$\left[\bar{M}_1U_{n,i-1}^{k+1}+\bar{M}_2U_{n,i}^{k+1}+\bar{M}_3U_{n,i+1}^{k+1}-\bar{M}_1U_{n,i+1}^{k+1}-\bar{M}_1U_{n,i-1}^{k-1}-\bar{M}_2U_{n,i}^{k-1}-\bar{M}_3U_{n,i+1}^{k-1}\right]\cdot t_0h\delta U_{n,i+1}^{k-1}=0. \quad (33)$$

The solution of the difference boundary value problem is carried out by the method of runs. It is assumed that the displacements and their velocities at the initial moment of time are given. It is assumed that the pipeline is clamped at $\alpha=0$ and $\alpha=1$. In vector form, the boundary conditions are expressed as follows:

$$U_{n,0}^{j}=0;\ A'U_{n,-1}^{j}=A'U_{n,1}^{j};\ U_{n,N}^{j}=0;\ A'U_{n,N+1}^{j}=A'U_{n,N-1}^{j}. \quad (34)$$

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Let us rewrite the system of difference equations (32) taking into account the boundary conditions (33) at as a result the system will look like

$$B_n U_{n,i-1}^{k+1} + C_n U_{n,i}^{k+1} + B_n U_{n,i+1}^{k+1} = b_{n,j}\,, \qquad (35)$$

where $b_{n,i} = \tau^2 F_{n,i}^k - (\bar{A}_n U_{n,i-2}^k + \bar{B}_n U_{n,i-1}^k + \bar{C}_n U_{n,i}^k + \bar{D}_n U_{n,i+1}^k + \bar{A}_n U_{n,i+2}^k + B_n U_{n,i-1}^{k-1} + C_n U_{n,i}^{k-1} + B_n U_{n,i+1}^{k-1})$.

From equation (35) we can derive a solution for the i-th equation

$$U_{n,i}^{K+1} = f_i - H_i U_{n,i+1}^{K+i}\,, \qquad (36)$$

where $f_i = (C_n - B_n H_{i-1})^{-1}(b_{n,i} - B_n f_{i+1})$, $H_i = (C_n - B_n H_{i-1})^{-1} B_n$.

In the reverse run, the remaining values of the displacement vector are determined $U_{n,i}^k$.

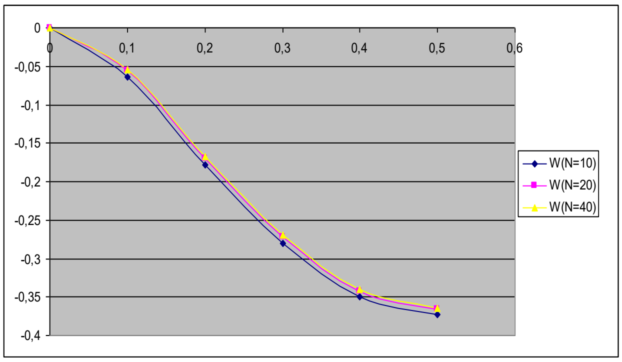
III. RESULTS

On the basis of the given algorithm, a computer realization of main pipeline problems solution under static loading by finite difference method has been considered.

Geometrical and mechanical characteristics of the cylinder: h=0.01 sm, μ=0.3, R=150 sm, *l*=1120 sm, β=2π, q=1, E=2-105 MPa. The obtained dimensionless results are given in the form of graphs and table.

TABLE I. CHARACTER OF CHANGES IN THE CALCULATED VALUES ALONG THE LENGTH (K) OF THE PIPELINE

k	$W\cdot 10^3$	$U\cdot 10^6$	M_1	M_2	N_1	N_2
0	0,0	0,0	-9,71608	-4,17511	-2,16607	-0,99421
0.1	-0,05403	-0,87381	-3,56012	-1,80509	-1,07511	-2,58409
0.2	-0,16721	0,06788	0,07746	-0,30733	-0,35142	-6,17013
0.3	-0,26960	2, 62508	2,15907	0,63612	-0,63503	-10,11210
0.4	-0,34052	5,70209	3,28106	1,24514	-1,78410	-13,10121
0.5	-0,36473	8,34110	3,53810	1,58013	-3,17805	-14,11012



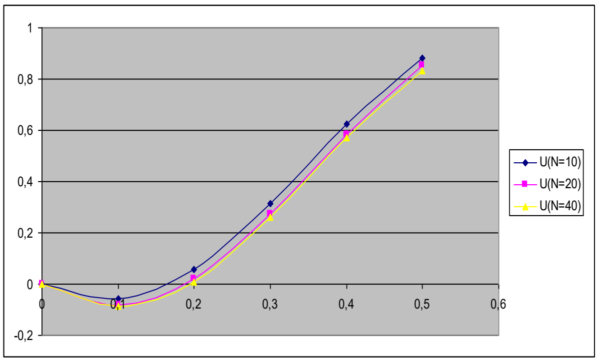


Fig. 1. The nature of the convergence of the calculated values at different. values of the grid spacing

IV. CONCLUSION

The paper gives a statement of the problem and mathematical models of pipeline deformation under repeated-variable loading. Based on the Hamilton-Ostrogradsky variational principle and the theory of small elastoplastic deformations, the system of differential equations of motion (equilibrium) under repeated-variable loading with appropriate boundary and initial conditions is derived. The finite difference method and matrix run is applied.

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