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Application of Boundary Integration in Time Dependent Problems

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ABSTRACT: In this paper, we discussed teh boundary integration method directly in the eigenvector space of the time domain, and the iterative application of Laplace transformation is not needed. Simply by applying the adjoint relationships of the symplectic orthogonality, an effective method for boundary condition is given. Based on this method, some typical examples are discussed, in which the whole character of total creep and relaxation of viscoelasticity is clearly revealed.

KEY WORDS: Boundary, Eigenvector, Laplace.

I. INTRODUCTION

The solution of Saint-Venant problem satisfies boundary conditions only in the sense of resultant equilibrium, therefore it is a Saint-Venant approximation. In fact, the exact solution should be divided into two parts, the solution of the Saint-Venant problem and the local solution [1-3]. The local solution decays with the distance from the boundary, which is termed by the Saint-Venant principle, and is often neglected. Based on the Sanit-Venant principle, in the discussion the effect of the boundary condition, an equivalent force system is usually selected to take the place of the actual load, with the distribution of the load neglected. Although the equivalent substitution is an approximate approach, the problem is simplified rationally, and Saint-Venant principle gained broad application in the analysis of the many complicated mechanical problems. Chirita [4] generalized Saint-Venant principle into linear viscoelasticity and obtained some solutions of the extension-bending-torsion problem and the flexure problem, in which the local effects of the ends are neglected and the evidence for the neglect is given.

The Hamiltonian system method can not be applied directly into viscoelasticity for the energy non-conservation. Xu etal [5] discussed plane problems of viscoelasticity in the Hamiltonian system on the basis of the correspondence principle and analytical inverse Laplace transform, and studied the decay property of non-zero eigenvectors, the creep and relaxation characters of two-dimensional viscoelastic materials and local effect near the boundary. Therefore solutions of combined problems of tension, torsion and bending can be obtained simply by the combinations of zero eigenvectors. In this paper, the boundary integral method is discussed directly in the time domain eigensolution space. Numerical results exhibit the important character of creep and relaxation of three-dimensional viscoelasticity, and the effects of boundary conditions, which show the efficiency of the direct method, especially for the complex boundary condition problems.

II. SOLUTION METHODBOUNDARY INTEGRATION

For three dimensional viscoelastioc probelms, the general solutions can be listed as follows:

$$u = J_{n+1}c_2 + J_{n-1}c_3 - a_7 \frac{d}{dr}\left(rJ_n c_1 + rJ_{n+1}c_2 + rJ_{n-1}c_3 \right) \tag{1}$$

$$v = -\frac{n}{r} a_7 J_n c_1 - \left(1 + na_7\right) J_{n+1}c_2 + \left(1 - na_7\right) J_{n-1}c_3 \tag{2}$$

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$$w = -\mu a_7 \left(J_n c_1 + rJ_{n+1} c_2 + rJ_{n-1} c_3 \right) \tag{3}$$

$$p_1 = 2G^* \mu a_7 \left[\left(\mu r J_{n-1} + nJ_n \right) c_1 + \left(n + a_8 \right) rJ_{n+1} c_2 - \left(n - a_8 \right) rJ_{n-1} c_3 \right] \tag{4}$$

$$p_2 = -2G^* \mu a_7 \left[nJ_n c_1 + \left(n + a_8 \right) rJ_{n+1} c_2 + \left(n - a_8 \right) rJ_{n-1} c_3 \right] \tag{5}$$

and

$$p_3 = \mu a_7 E^* \left[-\mu r J_n c_1 + \left(a_8 - 2 \right) rJ_n c_2 - \left(rJ_n + \mu r^2 J_{n-1} \right) c_3 \right] \tag{6}$$

in which $a_7 = (\lambda^* + G^*)/(2\lambda^* + G^*)$, $a_8 = 1/(2a_7)$, the Bessel function of order n $J_n = J_n\left(\mu r\right)$, c_1, c_2 and c_3 are integral constants. Simply by a variable substitution method, we can transform the inhomogeneous lateral conditions into the homogeneous ones. Introduce new variables

$$\bar{\psi}^* = \left\{ \bar{u}^*,\ \bar{v}^*,\ \bar{w}^*,\ \bar{p}_1^*,\ \bar{p}_2^*,\ \bar{p}_3^* \right\} = \bar{\psi} - \bar{\psi}^{**}, \tag{7}$$

where

$$\bar{\psi}^{**} = \left\{ \bar{u}^{**},\ \bar{v}^{**},\ \bar{w}^{**},\ \bar{p}_1^{**},\ \bar{p}_2^{**},\ \bar{p}_3^{**} \right\}, \tag{8}$$

Take the circle cylinder for example, the inhomogeneous boundary conditions are

$$\begin{cases} a_4 \dfrac{\partial \bar{u}}{\partial r} + 2a_1 G^* \left(\dfrac{\partial \bar{v}}{\partial \theta} + \bar{u} \right) + a_1 \bar{p}_3 = \bar{f}_r^0 \left(\theta, z \right) \\ \bar{p}_1 = \bar{f}_z^0 \left(\theta, z \right) \qquad\qquad\qquad\qquad \left(r = 1 \right). \\ G^* \left(\dfrac{\partial \bar{v}}{\partial r} + \dfrac{\partial \bar{u}}{\partial \theta} - \bar{v} \right) = \bar{f}_\theta^0 \left(\theta, z \right) \end{cases} \tag{9}$$

The solution vector is

$$\bar{\psi}^{**} = \left\{ 0,\ -a_3 \bar{f}_\theta^0,\ 0,\ r\bar{f}_z^0,\ 0,\ r\bar{f}_r^0 / a_1 + 2r \frac{\partial \bar{f}_\theta^0}{\partial \theta} \right\} \tag{10}$$

Introduce new dual variables

$$\bar{\psi}^* = \bar{\psi} - \bar{\psi}^{**} = \left\{ \bar{u},\ \bar{v} + a_3 \bar{f}_\theta^0,\ \bar{w},\ \bar{p}_1 - \bar{f}_z^0,\ \bar{p}_2,\ \bar{p}_3 - \frac{\bar{f}_r^0}{a_1} - 2\frac{\partial \bar{f}_\theta^0}{\partial \theta} \right\}. \tag{11}$$

Then the lateral boundary conditions become homogeneous, which are

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$$\begin{cases} a_4 \dfrac{\partial \overline{u}^*}{\partial r} + 2a_1 G^* \left(\dfrac{\partial \overline{v}^*}{\partial \theta} + \overline{u}^* \right) + a_1 \overline{p}_3^* = 0 \\ \overline{p}_1^* = 0 \\ \dfrac{\partial \overline{v}^*}{\partial r} + \dfrac{\partial \overline{u}^*}{\partial \theta} - \overline{v}^* = 0 \end{cases} \qquad (r = 1). \tag{12}$$

Thus we have

$$\overline{\mathbf{f}}^* = \frac{1}{r} \begin{Bmatrix} a_3 \overline{f}_z^0 \\ a_3 \dot{\overline{f}}_\theta^0 \\ (a_1 a_3 - 2a_3) \dfrac{\partial \overline{f}_\theta^0}{\partial \theta} + \overline{f}_r^0 \\ \overline{f}_r^0 + (2a_1 - a_3 a_6) \dfrac{\partial \overline{f}_\theta^0}{\partial \theta} - \dot{\overline{f}}_z^0 + r\overline{g}_1 \\ -\overline{f}_\theta^0 + \dfrac{\partial \overline{f}_r^0}{\partial \theta} + (a_3 a_4 + 2a_1) \dfrac{\partial^2 \overline{f}_\theta^0}{\partial \theta^2} + r\overline{g}_2 \\ r \dfrac{\partial \overline{f}_\theta^0}{\partial \theta} + r^2 \overline{g}_3 \end{Bmatrix}, \tag{13}$$

the inhomogeneous term can be developed in the time domain:

$$\mathbf{f} = \sum_{n=1}^{6} \left(a_n * \eta_n^{(\alpha)} + b_n * \eta_n^{(\beta)} \right). \tag{14}$$

Suppose η_p is a special solution:

$$\eta_p = \sum_{n=1}^{6} \left[A_n * \eta_n^{(\alpha)} + B_n * \eta_n^{(\beta)} \right]. \tag{15}$$

Therefore

$$\dot{A}_n = a_n \; ; \; \dot{B}_n = b_n . \tag{16}$$

The solutions are

$$A_n = \int_0^z a_n(\xi) d\xi \; ; \quad B_n = \int_0^z b_n(\xi) d\xi . \tag{17}$$

The end boundary can be expressed as

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$$\begin{cases} \int_{-1}^{1} \mathbf{p}_{-l} * \mathbf{q}_{j}^{(\beta)} dy = \int_{-1}^{1} \left[\sum_{n=1}^{6} a_n * (\mathbf{p}_{n}^{(\alpha)} * \mathbf{q}_{j}^{(\beta)}) + \sum_{n=1}^{6} b_n * (\mathbf{p}_{n}^{(\beta)} * \mathbf{q}_{j}^{(\beta)}) \right] dy \\ \int_{-1}^{1} \mathbf{q}_{l} * \mathbf{p}_{j}^{(\alpha)} dy = \int_{-1}^{1} \left[\sum_{n=1}^{6} a_n * (\mathbf{q}_{n}^{(\alpha)} * \mathbf{p}_{j}^{(\alpha)}) + \sum_{n=1}^{6} b_n * (\mathbf{q}_{n}^{(\beta)} * \mathbf{p}_{j}^{(\alpha)}) \right] dy \end{cases} (j = 1, 2, \cdots, 6). \quad (18)$$

III. NUMERICAL EXAMPLE

Suppose the boundary condition is

$$\sigma_r = -\sin\theta \ \ (r = 1). \quad (19)$$

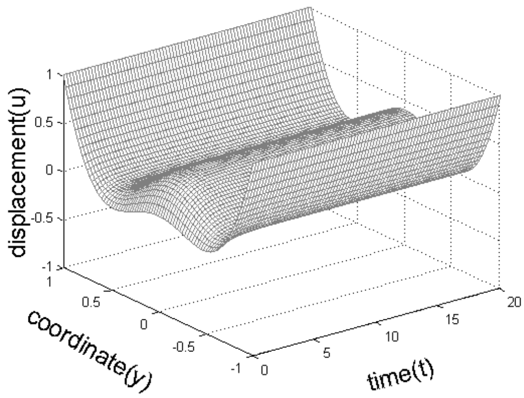

Fig 1.Displacement u

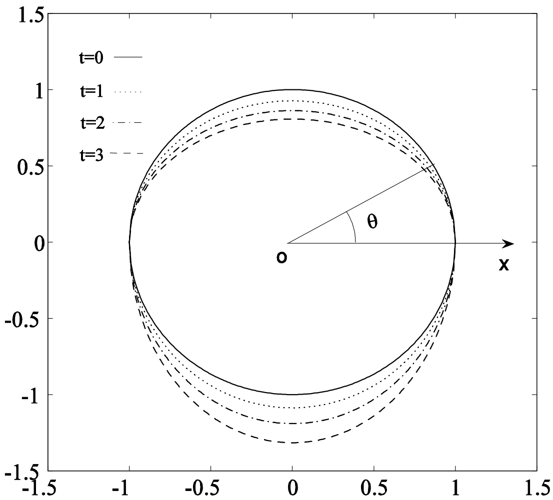

Fig 2. Shape evolution of circular cylinder at its free end

Fig. 1 shows creep of the Kelvin type model. We can see that the strain curves increase with time except for the elastic solution. The result indicates that there are some limits for these curves since the elastic term plays principal role,

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which is the peculiarity of Kelvin typemodel.Fig. 2 exhibits the creep phenomenon of the radial deformation of Maxwell type model.

IV. CONCLUSION AND FUTURE WORK

With the aid of Laplace integral transformation and the property of viscoelasticity, the Hamiltonian system is introduced in the research of three dimensional viscoelasticity. Based on this method, all Saint-Venant solutions and the local effect solutions are obtained from the zero eigenvectors and non-zero eigenvectors. By neglecting the local effect near the boundary, all solutions of Saint-Venant problems can be described approximately by the linear combinations of zero eigenvectors.

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