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On the Motion of Mechanical Systems with Non-Ideal Constrains

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ABSTRACT: Features of motion of mechanical systems with imperfect constrains are considered. Despite numerous studies, the dynamics of systems with conditional constrains currently has a number of poorly studied issues. These include the application of the method developed by P.Penleve [1] to the study of systems with non-ideal constrains. When using this method for controlled mechanical systems, it becomes necessary to develop a theory of extended equations of motion that allows us to rationally solve problems of synthesis and analysis of the dynamics of systems with conditional constrains, taking into account their imperfection and release.

KEYWORDS: constrain, velocity parameters, asymptotic stability, Lagrange multiplier.

I.INTRODUCTION

Let's analyse a mechanical system of material points $M_k\,(k=1,2,\dots,N)$, whose position in an inertial reference frame is determined by their Cartesian coordinates x_γ ($\gamma=1,2,\dots,3N$). Let the points of the system be affected by given forces $\overline{F}_k(X_\nu)$, and their motion is limited by joint and independent non-ideal constrains of the first and second kind (conditional)

$$f_\alpha(x_\nu,t)=0 \quad (\alpha=1,\dots,a), \tag{1}$$

$$\sum_{\nu=1}^{3N} a_{\rho\nu}\dot{x}_\nu + a_\rho = 0 \quad (\rho=1,\dots,b) \tag{2}$$

II. SIGNIFICANCE OF THE SYSTEM

Let among the constrains (1) first l, and among the constrains (2) first d be the constrains of the first kind. If we introduce generalized coordinates q_i and independent velocity parameters p_j, then taking into account only connections of the first kind, the variety of admissible states of the system can be represented in the following way:

$$x_\nu = a_\nu(q_i,t)\ (i=1,2,\dots,3N-l),$$

$$\dot{x}_\nu = b_\nu(q_i,p_j,t),$$

where: p_j ($j=1,2,\dots,3N-(l+d)$) – are independent velocity parameters.
Since bonds of the first kind are non-ideal, the condition holds for the forces of bond reactions for any virtual displacements

$$\sum_{\gamma=1}^{3N} R_\gamma \delta x_\gamma = \tau \neq 0.$$

In this case, the bond reaction forces can be decomposed into two components $\vec{R}_k^n$ (the bond force) and $\vec{R}_k^\tau$ (the friction force), such that

$$\sum_{k=1}^{N} \vec{R}_k^n \delta \vec{r}_k = 0,$$

and the vectors $\vec{R}_k^\tau \delta t$ are among the possible displacements of the system [1]. In this case, for the components of the vectors $\vec{R}_k^n$ and $\vec{R}_k^\tau$, and there are correlations

$$R_\nu^n = \sum_{\alpha=1}^{l} \lambda_\alpha \frac{\partial f_\alpha}{\partial x_\nu} + \sum_{\beta=1}^{d} u_\beta a_{\beta\nu},$$

$$R_\nu^\tau = \sum_{j=1}^{3N-(l+d)} \mu_j \frac{\partial (m_\nu \dot{x}_\nu)}{\partial p_j}.$$

The equations of motion of the system will be described by the equations

$$\frac{\partial S'}{\partial \dot{p}_j} = Q'_j + (R_j^\tau)' \quad (j = 1,...3N-(l+d)), \tag{3}$$

where

$$Q'_j = \sum_{\nu=1}^{3N} X_\nu \frac{\partial \dot{x}_\nu}{\partial p_j}, \; (R_j^\tau)' = \sum_{\nu=1}^{3N} R_\nu^\tau \frac{\partial \dot{x}_\nu}{\partial p_j}.$$

Adding to the equations (3) conditional constraints, having previously recorded them using generalized coordinates and speed parameters

$$f_m(q_i, t) = 0 \quad (m = l+1,...,a),$$
$$\varphi_s(q_i, p_j, t) = 0 \quad (s = d+1,...,b), \tag{4}$$

we obtain a complete system of equations for a controlled mechanical system.

III. LITERATURE SURVEY

It is acknowledged that compatible with the first kind of constrains, conditional constrains are implemented using external forces or forces of reactions of conditional constrains (control parameters). now we will consider the question of lowering the order of the system taking into account conditional constrains, as well as the problem of stable implementation of conditional constrains. Along with generalized coordinates and velocity parameters, we introduce parameters η_m, ζ_s that express the deviation from conditional constrains (continuous parametric release of the system). Often, the left part of the conditional relationships is taken as deviations.

$$f_m(q_i, t) = \eta_m \quad (m = l+1,...,a),$$
$$\varphi_s(q_i, p_j, t) = \zeta_s \quad (s = d+1,...,b), \tag{5}$$

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IV. METHODOLOGY

Using only constrains equations (5), the kinematic possible position of the system can be determined as follows:

$$q_i = A^*(q_\mu, \eta_m, t)\ (\mu = 1,2,\dots,n1),$$

$$\dot{q}_i = B_i^*(q_\mu, \eta_m, p_\nu, \zeta_s, \dot{\eta}_m, t)\,(\nu = 1,2,\dots,,,,,n2).$$

At the same time, it should be borne in mind that these ratios at $\eta_m = \dot{\eta}_m = \zeta_s = 0$ should not be released.
To account for geometric and kinematic constrain of the first kind from (1) and (2), we introduce generalized $q_1, q_2, \dots, q_k$ coordinates and independent velocity parameters $p_1, p_2, \dots, p_m$. According to this, the possible state of the system is defined as follows:

$$q_i = A_i(q_j, \eta_m, t),$$

$$\dot{q}_i = B_i(q_j, \eta_m, p_k, \zeta_s, \dot{\eta}_m, t). \qquad (6)$$

It is acknowledged that the Dalembert-Lagrange principle for a system freed from conditional constrains in generalized coordinates has the form:

$$\sum_{i=1}^{n}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} - Q_i - \Phi_i\right)\delta q_i = 0.$$

If we take into account, the conditions imposed by relations of the second kind (conditional constrains)

$$\sum_{i=1}^{n}\frac{\partial f_m}{\partial q_i}\delta q_i = \delta\eta_{im}, \quad \sum_{i=1}^{n}\frac{\partial \varphi_s}{\partial \dot{q}_i}\delta q_i = \delta\zeta_s,$$

then the Dalembert-Lagrange principle with multipliers takes the form:

$$\sum_{i=1}^{n}\left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} - Q_i - \Phi_i^\tau\right)\delta q_i = \sum_{m=l+1}^{a}\lambda_m\delta\eta_m + \sum_{s=d+1}^{b}\mu_s\delta\zeta_s.$$

Taking into account the introduced variables, the equations of motion of the system in the form of Appel equations can be written as follows:

$$\frac{\partial S}{\partial \dot{p}_k} = Q_k^p + \Phi_k^p,\ Q_s^p = \sum_{i=1}^{n} Q_i\frac{\partial B_i}{\partial p_k},\ \Phi_k^p = \sum_{i=1}^{n}\Phi_i^\tau\frac{\partial B_i}{\partial p_k},\ (k = 1,2,..,3N-(l+d)$$

$$\frac{\partial S}{\partial \ddot{\eta}_m} = Q_m^\eta + \Phi_m^\eta + \lambda_m,\ Q_m^\eta = \sum_{i=1}^{n} Q_i\frac{\partial B_i}{\partial \dot{\eta}_m},\ \Phi_m^\eta = \sum_{i=1}^{n}\Phi_i^\tau\frac{\partial B_i}{\partial \dot{\eta}_m},\ (m = l+1,\dots,a)$$

$$\frac{\partial S}{\partial \dot{\zeta}_s} = Q_s^\zeta + \Phi_s^\zeta + \mu_s,\ Q_s^\zeta = \sum_{i=1}^{n} Q_i\frac{\partial B_i}{\partial \zeta_s},\ \Phi_s^\zeta = \sum_{i=1}^{n}\Phi_i^\tau\frac{\partial B_i}{\partial \zeta_s}\ (s = d+1,\dots,b).$$

The equations of motion that we got are equations with respect to deviations and, in accordance with the relations (5) represent a closed system of equations with respect to variables $p_k, q_i, \eta_m, \zeta_s$. Where the multipliers $\lambda_{l+1}, \lambda_{l+2}, \dots, \lambda_{l+a}$; $\mu_{d+1}, \mu_{d+2}, \dots, \mu_{d+b}$ perform the role of control parameters. Usually, for a released system, the problems of stabilization relative to or optimal stabilization are considered. It is also possible to consider the issue of further lowering the order of the system, which is achieved by reducing conditional relationships to real ones, discussed in [2]. It should be noted that when the order of the system is further lowered, provided that there are non-ideal links among these links, it is necessary to use the rules for combining links, since in this case conditional links are also non-

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ideal. In fact, each point of the system, in addition to the active forces, is affected by two systems of bond reaction forces. This system of forces must also be decomposed into two components $(\vec{R}_k^{/})^n$ and $(\vec{R}_k^{/})^\tau$, and the elementary work of the forces of constrains, $(\vec{R}_k^{/})^n$ on virtual displacements caused by conditional constrains is zero, and the vectors $(\vec{R}_k^{/})^\tau \delta t$ are among the possible displacements of the system [4].

V. EXPERIMENTAL RESULTS

Task. Let's analyse the classical problem of A. Begen [2]. Keeping all the notation of the work [2], we consider the problem of the movement of a plate Σ, that is pivotally connected to a round disk Σ_1. The servomotor uses a special coupling to act on the disk Σ_1, and in such a way that there is a constant constrain between the angles $\alpha - \beta = \frac{\pi}{2}$ (in this case, the control parameter is the moment acting from the motor side on the disk).
When the conditional constrain is exactly executed

$$\alpha - \beta = \frac{\pi}{2}, \quad (7)$$

the motion of the plate and the control moment u applied to the disk are determined from the equations

$$M(b^2 + k^2)\ddot{\beta} - MRb\dot{\beta}^2 + Fa\sin\beta = 0, \quad (8)$$
$$[M(R^2 + b^2 + k^2) + I_1]\ddot{\beta} + F(R\cos\beta + a\sin\beta) = u.$$

In the work of V. I. Kirgetov [3], the system of equations (8) is obtained using the axiom of reducing conditional relations to real ones. The first equation of the system (7) it is a dynamic condition, that is, the reaction force of a conditional constraint assigned to a variable β is equal to zero. Enter the release parameter as discussed above $\eta = \alpha - \beta - \frac{\pi}{2}$ ($\eta -$ release parameter).

The equations of motion of the released system in variables β and η have the following form

$$[M(b^2 + k^2 + R^2 - 2bR\sin\eta) + I_1]\ddot{\beta} + [MR(R - b\sin\eta) + I_1]\ddot{\eta} -$$
$$- MbR\dot{\eta}(\dot{\eta} + 2\dot{\beta})\cos\eta + F[a\sin\beta + R\cos(\beta+\eta)] = 2u, \quad (9)$$
$$[MR(R - b\sin\eta) + I_1]\ddot{\beta} + (MR^2 + I_1)\ddot{\eta} + MbR\dot{\beta}^2\cos\eta +$$
$$+ RF\cos(\beta + \eta) = 2u.$$

Now we turn to equations (8), namely, to the equations for the exact fulfillment of the conditional constrain. The equation of motion of the plate has a partial solution $\beta = 0$, and taking into account the conditional coupling equation, we conclude that the pre-set motion is the position of the equilibrium

$$\alpha = \frac{\pi}{2}, \beta = 0.$$

It is easy to see that the first equation (8) has the first integral

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$$\frac{\dot{\beta}^2}{2}=[\frac{a_1(\cos\beta+2b_1\sin\beta)}{1+4b_1^2}+Ce^{2b_1\beta}],$$

where $b_1=\dfrac{Rb}{b^2+k^2}$, $a_1=\dfrac{Fa}{M(b^2+k^2)}$.

VI.CONCLUSION AND FUTURE WORK

The problem of stability of the equilibrium position is solved using the first integral, and it is not difficult to make sure that this particular solution is unstable. As the equation of motion shows, for the asymptotic stability of the equilibrium position in this case, it is sufficient to apply a dissipative force $-k\dot{\beta}$ over the variable β.
In this respect, when composing equations of motion for systems with non-ideal constraints of the first and second kind, it is necessary to know the law of friction of the system (friction forces) or generalized forces corresponding to the friction forces. In this case, it is convenient to use the Appel equations with multipliers of constraint in independent velocity parameters, which take into account only the first kind of constraints.

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