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Methods of Solving the Task of Action of Elastic Non-Stationary Waves in Cylindrical Layer

ZokirDjumaev, OlimaGadoeva , BakhriddinHamroev

Candidate of technical sciences, associate professor, department of “Mechanical engineering technology”, Bukhara Engineering-Technological Institute, Bukhara, Uzbekistan.
Assistant teacher of department “Construction of buildings and structures” Bukhara Engineering- Technological Institute, Bukhara, Uzbekistan.
Master student of department “Mechanical engineering technology” Bukhara Engineering-Technological Institute, Bukhara, Uzbekistan.

ABSTRACT: The article covers the development of method of solving the task of effect of non-stationary waves on N-layered cylindrical bodies (shell) located in limitless linear-elastic medium, as well as its algorithms. Loop systems of differential equations, as well as corresponding initial and boundary conditions are constructed. The obtained analytical results have theoretical and applied significance. Developed methodology is universal, and is fair in any rheological properties of the media.

KEYWORDS: methodology, N-layer cylinder, cylindrical bodies, displacement vector, polar coordinate system, unknown coefficients, Romberg method, theoretical, dynamic theory, block matrix. rheological properties.

1. INTRODUCTION.

In case of the sufficiently extended cavity and action directed perpendicular to longitudinal axis, the surrounding cavity of medium and lining are in conditions of the flat deformation, and tasks of determining stress state of massif and lining are reduced to the flat task of dynamic theory of elasticity [1, 2, 3, and 4]. In works [5, 6, and 7] tasks of stressed-deformable state of cylindrical bodies (shell), which are in limitless linear-elastic medium at propagation of longitudinal and transverse waves, are solved. Unlike other works, this work develops the methodology for solving and algorithm for task of interaction of non-stationary waves in layered cylindrical bodies. It is fair for any rheological properties of the media.

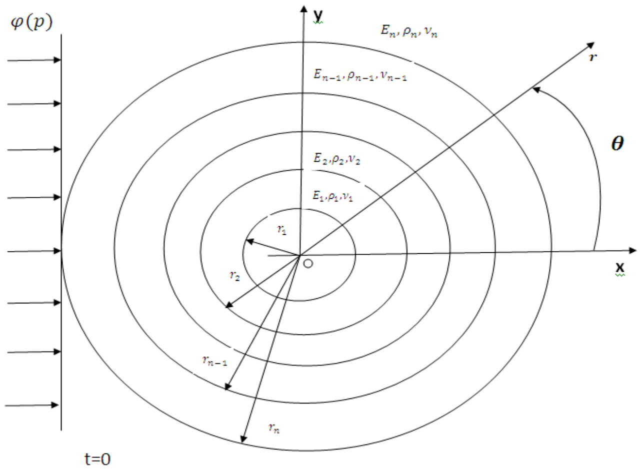

Fig. 1. Design diagram of layered cylindrical bodies in elastic medium

2. Formulation of the task.

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Non-stationary stress waves $\sigma_{xx}^{(i)}$ and $\sigma_{xy}^{(i)}$ are incident on the N-layer cylinder, the front of which is parallel to the longitudinal axis of the cylinder [1] (Fig. 1). It is required to determine the dynamic stress-strain state of the cylinder and its environment, caused by the falling stress pulse. Suppose that the time *t* is counted from the moment when the incident pulse touches the surface of the outer (N-1)[th] cylinder at the point $r = r_N$, $\theta = 0$. Peace is maintained until this moment. In accordance with the above, the task of finding the field of diffracted waves and the stress-strain state caused by the incident pulse [1]

$$\sigma_{xxn}^{(p)} = \sigma_0 H(t), \quad \sigma_{xyn}^{(p)} = \sigma_0 \frac{\nu_n}{1-\nu_n} H(t), \; t = t-(x+r_n)/C_{Pn}, \qquad (1)$$

where σ_0 is the amplitude of the incident waves; $H(t)$ is Heaviside unit function. First, we will find a solution for the plane step of a particular wave. General form of stress tensor is

$$\sigma_{ijn} = \sigma_{ijn}^{(p)} + \sigma_{ijn}^{(s)}$$

where $\sigma_{ijn}^{(p)}$ is voltage at incident waves, $\sigma_{ijn}^{(s)}$ is voltage of reflected waves.

In the polar coordinate system associated with the cylinder, the stresses and displacements in the incident wave $r = r_n$ have the form:

$$\sigma_{rrn}^0 = \sigma_0[(\varepsilon_n + 1)\cos 2\theta]H_0(z)/2$$

$$\sigma_{r\theta n}^0 = \sigma_0(\varepsilon_n - 1)\sin 2\theta H_0(z)/2;$$

$$\sigma_{\theta\theta n}^0 = \sigma_0[\varepsilon_n - (\varepsilon_n + 1)\cos 2\theta]H_0(z)/2; \; z = C_{pn}t - r_n + r_n\cos\theta, \quad \varepsilon_n = -\nu_n/(1-\nu_n)$$

where $H_0(z)$ is the Heaviside unit function; σ_0 is voltage at the front of the wave propagating in the x_1 direction; r_j is the radius of layered bodies (j=1,… .n); Cpj is the speed of the expansion wave, ν_j is Poisson coefficients, ρ_j is the density of media. In the absence of static body forces, the displacement vector $\vec{u}_j = [u_{rj}, u_{\theta j}, u_{zj}]^T$ in elastic medium is determined by the equation

$$(\lambda_j + 2\mu_j) grad\, dir\vec{u}_j - \mu_j \; rot\, rot\vec{u}_j = \rho_j (\partial^2 \vec{u}_j / \partial t^2) \qquad (2)$$

The displacement vector (u_j) is expressed through scalar (φ_j) and vector ($\vec{\psi}_j$) potentials [2]

$$\vec{u}_j = grad\varphi_j + rot\vec{\psi}_j \; ,$$

and equation (2) has the form

$$\nabla^2\varphi_j - c_{j1}^{-2}(\partial^2\varphi_j/\partial t^2) = 0, \qquad \nabla^2\psi_j - c_{j2}^{-2}(\partial^2\vec{\psi}_j/\partial t^2) = 0. \; (3)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2} + \frac{\partial^2}{\partial\theta^2}$ are differential operators in cylindrical coordinates.

At infinity r → ∞, the potentials of longitudinal and transverse waves at $j = n$ satisfy the Sommerfeld radiation condition [1]:

$$\lim_{r\to\infty} \varphi_n = 0, \; \lim_{r\to\infty} (\sqrt{r})^{\kappa} \left(\frac{\partial\varphi_n}{\partial r} + i\alpha_n \varphi_n \right) = 0, \qquad (4)$$

$$\lim_{r\to\infty} \psi_n = 0, \; \lim_{r\to\infty} (\sqrt{r})^{\kappa} \left(\frac{\partial\psi_n}{\partial r} + i\beta_n \psi_n \right) = 0.$$

At the contact of two bodies $r = r_j$ the equality of displacements and stresses is fulfilled (the condition of rigid contact)

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$$u_{rj}=u_{r(j+1)};\ \sigma_{rrj}=\sigma_{rr(j+1)}\ u_{\theta j}=u_{\theta(j+1)};\ \sigma_{r\theta j}=\sigma_{r\theta(j+1)}, \quad (5)$$

and on the free surface ($r = r_1$):

$$\sigma_{rr1}=0,\ \sigma_{r\theta 1}=0. \quad (6)$$

The task is solved under the following initial conditions:

$$\left.\frac{\partial\varphi_j}{\partial r}+\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right|_{t=0}=\frac{\partial}{\partial t}\left.\left(\frac{\partial\varphi_j}{\partial r}+\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right)\right|_{t=0}=0, \quad (7)$$
$$\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}-\frac{\partial\varphi_j}{\partial r}=\frac{\partial}{\partial t}\left.\left(\frac{1}{r}\frac{\partial\psi_j}{\partial\theta}-\frac{\partial\varphi_j}{\partial r}\right)\right|_{t=0}=0,$$

The stress field caused by forces (1) satisfies the wave equation (3), i.e. every cylindrical layer satisfies this. To solve the task formulated above, we use the t-integral Fourier transform in time.

$$\varphi_j^F(\Omega)=\int_{-\infty}^{\infty}\varphi_j(\tau)\exp(-i\Omega\tau)d\tau;\ \varphi_j(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_j^F(\Omega)\exp(i\Omega\tau)d\omega; \quad (8)$$
$$\psi_j^F(\Omega)=\int_{-\infty}^{\infty}\psi_j(\tau)\exp(-i\Omega\tau)d\tau;\ \psi_j(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi_j^F(\Omega)\exp(i\Omega\tau)d\omega,$$

where Ω is integral Fourier transform parameter; φ^F_j ψ^F_j is the image of the Fourier transform of the functions $\varphi_j(t)$ and $\psi_j(t)$, respectively. Using zero initial conditions (7), we obtain the depicted task (3). Then the solution of the equations of the problem depicted will have the form

$$\begin{pmatrix}\varphi_j^F(r,\theta,\Omega)\\ \psi_j^F(r,\theta,\Omega)\end{pmatrix}=\begin{pmatrix}\overline{\varphi}_j^F(r,\Omega)\\ \overline{\psi}_j^F(r,\Omega)\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}; \quad (9)$$

Here

$$\overline{\varphi}_j^F(r,\Omega)=\begin{cases}A_m H_m^{(1)}(\Omega r/C_{Pn})\quad at\quad r\ge r_n,\\ A_{mj}H_m^{(1)}(\Omega r/C_{Pj})+B_{mj}H_m^{(2)}(\Omega r/C_{Pj})\quad at\\ r_1\le 2\le r_n\quad (j=1,2,\dots n+1),\\ A_{m1}I_n(\Omega r/C_{Sn})\quad at\quad 0\le r\le r_1;\end{cases}$$
$$\psi_j^F(r,\Omega)=\begin{cases}C_{mj}\ H_m^{(1)}(\Omega r/C_{Sj})+L_{mj}H_m^{(2)}(\Omega r/C_{Sj})\\ at\quad r_1\le r\le r_n,\\ C_m H_m^{(1)}(\Omega r/C_{Sn})\quad at\quad r\ge r_n,\\ C_{m1}I_n(\Omega r/C_{S1})\quad at\quad 0\prec r\le r_1.\end{cases} \quad (10)$$

A_{m1}, A_{mj}, A_{mN}, B_{mj}, C_{mj}, C_{mn} coefficients are determined from the boundary conditions (4) - (6). The boundary conditions taking into account the incident waves (1) take the form:

$$a)\ \ \sigma^F_{rrn}+\sigma^{(i)F}_{rrn}=\sigma^F_{rr(n-1)},\quad b)\ \ \sigma^F_{r\theta n}+\sigma^F_{r\theta n}=\sigma^F_{r\theta(n-1)}, \quad (11)$$
$$c)\ \ u^F_{rn}+u^{(i)F}_{rn}=u^F_{r(n-1)},\quad d)\ \ u^F_{\theta n}+u^{(i)F}_{\theta n}=u^F_{\theta(n-1)},$$

where

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$$
\begin{aligned}
&a)\quad \sigma_{rrn}^{(i)}(\Omega)=\sigma_{01}^{(P)}\sum_{k=0}^{\infty}(-1)^{k}\in_{k} I_{k}\,(\Omega r/C_{Pn})\cos k\theta;\\
&b)\quad \sigma_{rrn}^{(F)}(\Omega)=\overline{\sigma}_{rrn}^{F}(\cos^{2}\theta+\in_{n}\sin^{2}\theta);\\
&c)\quad \sigma_{r\theta n}^{(F)}=-\overline{\sigma}_{rrn}^{F}\left[(1-E_{n})|2\right]\sin 2\theta;\\
&d)\quad u_{rm}^{F}=\overline{u}_{rm}^{F}\cos\theta;\ \partial)\quad u_{\theta m}^{F}=\overline{u}_{\theta m}^{F}\sin\theta;\quad \sigma_{01}^{(P)}=\sigma_{0}e^{-n\Omega/C_{Pn}}
\end{aligned}
\tag{12}
$$

Substituting (9) and (10) into the boundary conditions (4), (5), and (6), we obtain a system of complex algebraic equations with (4n + 3) unknowns in the form

$$
[Z]\{g\}=\{P\}, \tag{13}
$$

$$
[Z]=\left(\begin{array}{cc|cc}
[Z_1] & & & 0\\
 & [Z_2] & & \\
\hline
 & & [Z_{(N-1)}] & \\
0 & & & [Z_N]
\end{array}\right)
$$

where Z is a block matrix; $\lfloor Z_j \rfloor$ is a matrix of 4x4 order, the elements of which are the matters of Bessel and Hankel functions of the m[th] order of the first and second kind; {g} -vector-columns of unknown coefficients; *{P}-{0,0.....0,*P_{1n}, P_{2n}, $P_{\partial n}$, $P_{4n}\}^T$are the column vectors characterizing the falling loads. Let the stepped waves interact with a cylindrical hole at $r = r_1$ and a stress-free hole ($\sigma_{rr1}=0,\ \sigma_{r\theta 1}=0$). The only stress that does not reduce to zero at $r = r_1$ is the hoop stress $\sigma_{\theta\theta n}/\sigma_0$. Applying the Fourier transform to the equation of motion and boundary conditions [9], we obtain an expression for hoop stresses at

$$
\begin{aligned}
&\sigma_{rrn}=\sigma_0 H(t)\cos nt,\qquad \sigma_{r\theta n}=\tau_0 H(t)\sin\theta\\
&\sigma_{\theta\theta n}^{*}=\frac{\sigma_{\theta\theta n}(r_{01}\theta,t)}{\sigma}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Delta_n(r_0\Omega)e^{i\Omega t}}{\Omega_1[\Delta_1\Delta_2+\Delta_3\Delta_4]}d\Omega,\\
&\Delta_n(r_{01}\Omega)=(\Delta_3+\tau_0E_n)\lfloor 2\Omega H_{n-1}^{(1)}(\Omega)-((2n^2+2n)+\Omega^2)H_n^{(1)}(\Omega)\rfloor+\\
&+[\tau_0\Delta_2-\Delta_4]\left[2n(n+1)H_n^{(1)}((C_{Pn}/C_{Sn})\Omega)+\frac{2C_P n\Omega}{C_{S1}}H_{n-1}^{(1)}\left(\frac{C_{pn}}{C_{Sn}}\Omega\right)\right].
\end{aligned}
\tag{14}
$$

The $\Delta_k(k=1,2,3,4,5)$ expression is given in [10]. Improper integral (14) is solved numerically using the developed algorithms [10]. Practical calculation of (4) on a computer can be carried out as follows. Since numerical integration in infinite limits is unthinkable, then the integral (14) is replaced by

$$
\sigma_{\theta\theta n}^{*}=\frac{1}{2\pi}\int_{\omega_a}^{\omega_b}\frac{\Delta_1(r_{01}\Omega_1)}{\Omega_1[\Delta_2\Delta_3+\Delta_4\Delta_5]}e^{-i\Omega t}d\Omega. \tag{15}
$$

The values of the limits of integration of ω_a,ω_b are selected depending on the type of the incident pulse. The numerical values of the spectral density $\sigma_{rr}^{(i)F}(\Omega)$ from (12) of the final incident pulse differ significantly from zero only in a small frequency Ω range.The limits of ω_a,ω_b integration should be selected in accordance with this range and taking into account the required accuracy. In this case, the question remains about what error will be introduced by neglecting the contribution of integrals of the type (14) within the limits of integration from ∞ to ω_a and from ω_b to ∞ . Numerical summation of the infinite sum (12), of course, is also impossible. However, in [10] it was shown that for sufficiently large n (n is an order of the Bessel and Hankel functions) an asymptotic representation of the general term of this sumcan be constructed. As a result, it becomes possible either to estimate the error in the transition from an infinite sum to a finite sum, or to approximate summation of an infinite sum. In view of what was said above, let us keep the infinite sum in (12). The calculation by the considered method is reduced to the construction of two

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calculation algorithms: coefficients $Z_{ke}(\Omega)(k,e=1,2)$ (13) and integral (15). The first and second algorithms do not depend on the type of the mathematical model of the object. When calculating integral (15) by the Romberg method, it is necessary to calculate the integrand many times. The inverse Fourier transform for a certain image, the original of which is known in advance, showed that with an integration step length of 0.01, the procedure error does not exceed 0.3-0.5%. As an example, consider the diffraction of non-stationary waves by a cylindrical body. Let the internal boundary ($r=r_1$) be stress-free, and the condition of equality of stirring and stress (5) [1,2,4,10] is fulfilled at the contact with the surrounding medium. After the Fourier transform, we obtain the cylindrical Bessel equations, the solution of which is expressed (9) and (10). In our task, there will be six arbitrary constants, which are determined from the boundary conditions (6) and (11). The calculation results are given at $\theta=90^0$

$(\nu_1=0{,}2;\quad \nu_2=0{,}25;\quad r_0/r_1=0{,}5;\quad \eta=\rho_2/\rho_1=0{,}1)$.

When integrating the limit, the following value $\omega_a=10^{-4}, \omega_b=4, h=10^{-2}$ is assumed. The change in the circumferential stress $\sigma^*_{\theta\theta}(\theta=90^0, r=r_1)$ depending on τ is shown in Fig. 2.

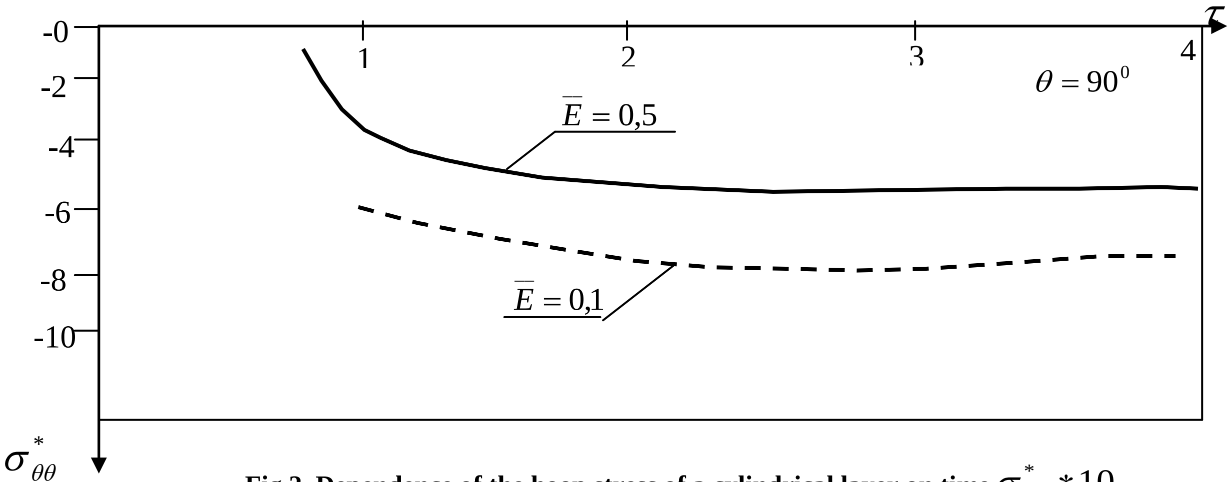

Fig 2. Dependence of the hoop stress of a cylindrical layer on time $\sigma^*_{\theta\theta}*10$

The difference between the stresses on the outer and inner surfaces reaches $\approx 15-20\%$, and the difference between the stresses on the middle and inner surfaces reaches $\approx 10\%(r_0/r_1=0{,}5)$.

III. CONCLUSION

- comparison of results for displacement waves with results for longitudinal waves shows that more stress occurs in longitudinal waves than displacement waves,

- calculations show that at $\tau=12\alpha/C_{P_1}$ the results of this research are close to the exact static value $\alpha^*_{\theta\theta}=8{,}13$. It can be seen that the maximum stress and displacement significantly depend on $\bar{\eta}$ and $\bar{E}$.

REFERENCES

1. Guz A.N., Kubenko V.D., Cherevenko M.A. Diffraction of elastic waves. “Nauka”, 1978, p. 308.
2. Pao Y.H., Mow C.C. diffraction of elastic waves and dynamic stress concentration. No. 4, Grane, Russak, 1973, p. 694.
3. Datta S.K. Tensional waves in an infinite elastic solid containing a penny – shaped crack.-z. answer. Math. And Phys., 1970, 21, No. 3, p. 343-351
4. MubarikovYa.N., Safarov I.I. About action of elastic wave on cylindrical shell. Bulletin of Academy of Science of UzSSR, series of technical sciences, 1987. No. 4, p. 34-40
5. Safarov I.I. Assessment of the seismic stress state of underground structures of the method of wave dynamics Collection of papers “Seismodynamics of buildings and structures”. Tashkent, Fan. 1988.
6. Filippov I.G., Egorychev O.A. Non-stationary vibrations and diffraction of waves in acoustic and elastic media. M.: Mechanical Engineering, 1977, p.304.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 7, Issue 11 , November 2020

7. Safarov I.I. Interaction of waves in multilayer cylindrical layers in infinite elastic medium. Proceedings of the VII all-union conference "Dynamics of the bases, foundations and underground structures" Dnepropetrovsk, 1989, p. 56-57
8. Safarov I.I., Jumaev Z.F. About the destruction of the tunnel at strong earth movements. International conference on earthquake engineering. St.Petersburg, 2000, p.71-78
9. Avliyakulov N.N., Safarov I.I. Modern problems of statics and dynamics of underground pipelines. Tashkent, Fan vatexnologiya. 2007, p.306.
10. Bozorov M.B., Safarov I.I., ShokinYu.I. Numerical simulation of oscillations of dissipative homogeneous and inhomogeneous mechanical systems. Novosibirsk: Publishing house of SB RAS. 1996, p. 189.
11. Rashidov T.R., Safarov I.I. et al. About two main methods of studying the seismic stress state of underground structures under the action of seismic waves. Tashkent: DAN. No. 6, 1989, p. 13-17.
12. Safarov I.I. Avliyakulov N.N. Methods for increasing the seismic resistance of underground plastic pipelines // Uzbek journal of oil and gas, 2005, No. 4, p.42–44.
13. Grace E., Matthews G.B. Bessel functions and their application to physics and mechanics. M.: Publishing house of foreign literature, 1953, p. 371
14. Safarov I.I. Oscillations and waves in dissipative inhomogeneous media and structures. Tashkent: "Fan", 1992.