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Algorithms for the synthesis of adaptive control systems with a reference model for objects with an input delay based on a simple filter

H.Z. Igamberdiyev, T.V.Botirov

Academician of the Republic of Uzbekistan, professor of Tashkent State Technical University, doctor of technical sciences, Tashkent, Uzbekistan,
Associate Professor, Department of Automation and Control, Navoi State Mining Institute, Navoi, Uzbekistan

ABSTRACT: The article presents the synthesis algorithms of adaptive control systems with a reference model for objects with input delay based on a simple filter. The synthesis procedure is based on the concept of forecasting a reference model using advanced inaccuracy. In this case, the structure of control devices uses simple filters and only blocks with concentrated delay, in contrast to the known configurations with distributed delay.

KEYWORDS: adaptive control, reference model, delays, simple filter.

I. INTRODUCTION

One of the modern tasks in the theory of automatic control is the development of an adaptive control system for dynamic objects with distributed delay. Chemical and energy systems are the most typical examples of delayed systems. Such delays not only limit the synthesis of systems with typical control laws, but can also act as a source of instability of closed systems. Since the practical value of any control law is its operability, there is a need to search for such an algorithm for the functioning of the system that would solve the problem posed to the specialist, and therefore, special methods of the theory of adaptive control are used.

II. RELATED WORK

The problem of controlling systems with a delay under uncertainty was solved by many authors in the class of adaptive systems [1 - 5]. Most of the proposed solutions for the synthesis of adaptive control algorithms for such systems were based on the use of distributed delay blocks [6-8], which are technically difficult to implement. In this article, to control systems "one input - one output" with a delay at the input, we propose the structure of a control device using a simple filtering scheme [9,10].

III. FORMULATIONOFTHEPROBLEM

Consider a class of controlled systems whose dynamics is described by a linear differential equation with input delay:
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$$\begin{aligned} \dot{x}(t) &= A_p x(t) + b_p u(t-\tau), \\ y(t) &= c_p^T x(t), \end{aligned} \tag{1}$$

where $x \in R^n$ – is the state vector, $u \in R$ – is the control input, $y \in R$ – is the measured output of the object. The constant matrix A_p and vectors b_p, cp of corresponding sizes are unknown, τ is the known delay time.
Denote by $W_0(s)$ the transfer function of system (1) and obtain:

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$$W_0(s)=\frac{y(s)}{u(s)}=c_p^T(sI-A_p)^{-1}b_pe^{-\tau s}=k_p\frac{B_0(s)e^{-\tau s}}{A_0(s)}, \tag{2}$$

$$\begin{aligned} A_0(s)&=s^n+a_{n-1}s^{n-1}+\dots a_1s^1+a_0,\\ B_0(s)&=s^m+b_{m-1}s^{m-1}+\dots b_1s^1+b_0, \end{aligned} \tag{3}$$

where n, m are the orders of polynomials ($m<n$); $a_i\left(i=\overline{0,n-1}\right), b_j\left(j=\overline{0,m-1}\right)$ - unknown coefficients of polynomials; kp - unknown gain of the object at high frequencies.
The reference model can be set in the form:

$$\begin{aligned} \dot{x}_m(t)&=A_mx_m(t)+b_mr(t-\tau),\\ y_m(t)&=c_m^Tx_m(t), \end{aligned} \tag{4}$$

where $x_m\in R^{n_1}$ is the state vector of the model, $r\in R$ is the defining influence, $y_m\in R$ is the measured output of the model. The constant matrix Am and vectors bm, cm of corresponding sizes are known. Accordingly, the transfer function of the reference model $Wm(s)$ is determined by the expression:

$$W_m(s)=\frac{y_m(s)}{r(s)}=c_m^T(sI-A_m)^{-1}b_me^{-\tau s}=k_m\frac{Z(s)e^{-\tau s}}{P(s)}, \tag{5.}$$

where k_m is the model gain, the polynomials $Z(s)$ and $P(s)$ have an object-like structure.
It is required to find such a control law $u(t)$ that provides the asymptotic zeroing of errors $e(t)=y(t)-ym(t)$ for arbitrary initial conditions and arbitrary bounded signals $r(t)$. Thus, the target condition for the control system takes the following form:

$$\lim_{t\to\infty}\left[e(t)=y(t)-y_m(t)\right]=0. \tag{6.}$$

IV. SOLUTION OF THE TASK

Suppose that all parameters of system (1) are known. To control a stable minimum-phase object with input delay (1), we use the open-loop control strategy for systems without delay [10]. We will use a filtering scheme of minimal complexity modified for the case of input delay.
Represent the transfer function of the object $W_0(s)$ in the form:

$$W_0(s)=\frac{k_p\dfrac{B_0(s)}{F(s)}e^{-\tau s}}{\dfrac{A_0(s)}{F(s)}}=\frac{\left(k_p+\sum\limits_{i=1}^{m}\dfrac{\beta_i}{s+\lambda_i}\right)e^{-\tau s}}{\Gamma(s)+\sum\limits_{i=1}^{m}\dfrac{\alpha_i}{s+\lambda_i}}, \tag{7}$$

where $F(s)$ is an arbitrary stable polynomial with real unequal roots:

$$F(s)=\prod_{i=1}^{m}(s+\lambda_i)=s^m+f_{m-1}s^{m-1}+\dots+f_1s^1+f_0, \tag{8}$$

here $\lambda_i>0, \lambda_i\neq\lambda_j\left(i=\overline{1,m},\ j=\overline{1,m},\ j\neq i\right)$.
The polynomial $\Gamma(s)$ from (7) has the following form:

$$\Gamma(s)=s^n+\gamma_Ns^N+\gamma_{N-1}s^{N-1}+\dots\gamma_1s^1+\gamma_0, \tag{9}$$

here $N=n^*-1$.

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The constant coefficients $\gamma_j,\ \alpha_i,\ \beta_i\ \left(j=\overline{0,N},\ i=\overline{1.m}\right)$ - are uniquely determined through the initial parameters of the object (2) and the parameters of the polynomial $F(s)$ (through the parameters $a_i\left(i=\overline{0,n-1}\right),\ b_j\left(j=\overline{0,m-1}\right),\ \lambda_k\left(k=\overline{1,m}\right)$).

Imagine the reference model in the form:

$$y_m(s)=\frac{e^{-\tau s}}{R(s)}u_r(s),\qquad u_r(s)=\frac{k_m R(s)Z(s)}{P(s)}r(s), \tag{10}$$

here $R(s)$ is an arbitrary stable polynomial of degree n^*:

$$R(s)=s^{n^*}+\rho_N s^N+\rho_{N-1}s^{N-1}+\ldots+\rho_1 s+\rho_0. \tag{11}$$

Proceeding from this, we introduce an arbitrary stable polynomial $L(s)$ with real unequal roots of the form
1

$$L(s)=\prod_{i=1}^{N}(s+\delta_i)=s^N+l_{N-1}s^{N-1}+\ldots+l_1 s^1+l_0, \tag{12}$$

here $\delta_i>0,\ \delta_i\neq\delta_j\ \left(i=\overline{1,m},\ j=\overline{1,m},\ j\neq i\right)$. Expanding the polynomial $\dfrac{R(s)-\Gamma(s)}{L(s)}$ into simple fractions we get:

$$\frac{R(s)-\Gamma(s)}{L(s)}=\eta_0+\sum_{i=1}^{N}\frac{\eta_i}{s+\delta_i}. \tag{13}$$

Obviously, as in the case of an object, the coefficients $\eta_i\ \left(i=\overline{0,N}\right)$ are uniquely determined through the initial parameters of this object $\gamma_j\ \left(j=\overline{0,N}\right)$ and the parameters of the polynomials $L(s)$ and $R(s)$. For simplicity, we set the polynomial $R(s)$ in the form:

$$R(s)=\frac{1}{T}(Ts+1)L(s),$$

where $T>0$ is an arbitrary time constant. Given the introduced notation, the equation of the perturbed motion of the system with respect to the error e takes the form:

$$\begin{aligned}R(s)=\frac{1}{T}(Ts+1)e(s)=\frac{1}{L(s)}\sum_{i=1}^{m}\left(\frac{\beta_i}{s+\lambda_i}e^{-\tau s}u(s)-\frac{\alpha_i}{s+\lambda_i}y(s)\right)+\\ +\frac{1}{L(s)}\left(k_p e^{-\tau s}u(s)-u_r(s)\right)+\left(\eta_0+\sum_{i=1}^{N}\frac{\eta_i}{s+\delta_i}\right)y(s).\end{aligned} \tag{14}$$

If the control law is chosen in the form:

$$\begin{aligned}u(s)=\frac{1}{k_p}\left[\sum_{i=1}^{m}\frac{\alpha_i}{s+\lambda_i}e^{\tau s}y_m(s)-\sum_{i=1}^{m}\frac{\beta_i}{s+\lambda_i}u(s)\right]+\\ \frac{1}{k_p}\left[1-\left(\eta_0+\sum_{i=1}^{N}\frac{\eta_i}{s+\delta_i}\right)\frac{k_l T}{Ts+1}\right]u_r(s),\end{aligned} \tag{15}$$

then, taking into account (7) and (13), the equation of the control device (15) will take the following form:

$$u(s)=\frac{1}{k_p}\frac{A_0(s)}{B_0(s)R(s)}u_r(s), \tag{16}$$

Since $A_0(s)$ and $B_0(s)$ are Hurwitz, the control law (15) will ensure exact correspondence of the transfer functions of the object and the reference model.

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In accordance with [10], for use in open control, we introduce the following filtering scheme for signals $\omega_{y_{mpr}}, \omega_u \in R^m, \ \omega_r \in R^N$ и $u_{rf}(t) \in R$:

$$\begin{aligned} &\dot{\omega}_{y_{mpr}}(t) = \Lambda_2 \omega_{y_{mpr}}(t) h_{2ym}(t+\tau|t), \ \ \omega_{y_{mpr}}(0) = 0, \\ &\dot{\omega}_u(t) = \Lambda_2 \omega_u(t) h_2 u(t), \ \ \omega_u(0) = 0, \\ &\dot{\omega}_r(t) = \Lambda_1 \omega_r(t) h_1 u_{rf}(t), \ \ \omega_r(0) = 0, \\ &\dot{u}_{rf} = -(1/T) u_{rf}(t) + k_l u_r(t), \ \ u_{rf}(0) = 0, \end{aligned} \tag{17}$$

where $\Lambda_1 \in R^{N\times N}, \Lambda_2 \in R^{m\times m}$ – are the diagonal matrices of constant coefficients of the form:

$$\Lambda_1 = diag(-\delta_i), \ \ i = \overline{1,N}, \qquad \Lambda_2 = diag(-\lambda_j), \ \ j = \overline{1,m},$$

$h_1 \in R^N, \ h_2 \in R^m$ unit vectors. Note that the total order of the filtration system (17) corresponds to $2m + N + 1 = n + m$.

This shows that the lead signal of the deterministic reference model $y_m(t + \tau|t)$, used in (17) to form a vector, can be easily obtained by passing the signal $r(t)$ through the transfer function of the reference model $W_m(s)$ without taking into account temporary lag.

Further, it can be argued that in accordance with (15) and (17) there exists such a block vector $\Theta^* \in R^{(m+m+N+1+1)}$:

$$\Theta^{*T} = \rho\left[\alpha^T, -\beta^T, -\eta^T, -\eta_0, 1\right], \ \ \rho = \frac{1}{k_p},$$

what kind of management

$$\begin{aligned} &u(t) = \Theta^{*T}\Omega_{mpr}(t) \\ &\Omega_{mpr}(t) = \left[\omega^T_{y_{mpp}}(t), \omega^T_u(t), \omega^T_r(t), u_{rf}(t), u_r(t)\right]^T, \end{aligned} \tag{18}$$

provides full coincidence of the transfer function of the object and the reference model.

We consider the synthesis of the control algorithm for the case when the excess of poles of the transfer function of the object n^*=1 ($L(s) = 1$). We set the law of adaptive control in the form:

$$u(t) - \Theta^T\Omega_{mpr}(t), \tag{19}$$

where $\Theta \in R^{(n+m+1)}$ is the vector of adjustable parameters.

Let $E_f = X_f - X_{fm}$ and $e = y - y_m$, , then we get the equation for the mistakes:

$$\begin{aligned} &\dot{E}_f(t) = A_{cf}E_f + B_u\left[u(t-\tau) - \Theta^{*T}\Omega_{mpr}(t-\tau)\right], \\ &e(t) = C_f^T E_f(t). \end{aligned} \tag{20}$$

The equation for the error $e(t)$, taking into account (19) and (20), will take the form:

$$\begin{aligned} &\dot{E}_{f1}(t) = A_{11}E_{f1}(t) + \hat{B}_{u1}\frac{1}{\rho}\left[\Theta(t-\tau) - \Theta^{*T}\right]\Omega_{mpr}(t-\tau), \\ &e(t) = C_{f1}^T E_{f1}(t), \end{aligned} \tag{21}$$

where $\hat{B}_{u1} = \hat{B}_{u1\rho}$. The transfer function of this system without taking into account the time delay is strictly positively valid and is defined as:

$$W_e(s) = C_{f1}^T (sI - A_{11})^{-1}\hat{B}_{u1}e^{-\tau s} = \frac{F(s)}{A_0(s)}e^{-\tau s} \tag{22}$$

Set the extended error signal as

$$e_a(t) = e(t) - y_a(t) = y(t) - y_m(t) - y_a(t), \tag{23}$$

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where y_a is the output of the auxiliary model, which is defined by the equation:

$$y_a(s) = W_a(s)u_a(s),$$
$$W_a(s) = \frac{L(s)}{R(s)} = \frac{1}{R(s)} = \frac{T}{Ts+1}, \tag{24}$$

where y_a is the control input of the auxiliary model, which will be defined below.
The new management goal is defined as:

$$\lim_{t\to\infty} e_a(t) = 0. \tag{25}$$

Obviously, if you require the fulfillment of target conditions (25) and

$$\lim_{t\to\infty} y_a(t) = 0, \tag{26}$$

this will fulfill the original control goal (6).
To represent the reference model in the state space, we carry out equivalent transformations with equation (24), after which we obtain

$$y_a(s) = \frac{k_l T}{Ts+1}\left[\tilde{\theta}^{*T} w_a(s) + u_a(s)\right], \tag{27}$$

here

$$\tilde{\theta}^{*T} = \left[-\alpha^T, \eta_0\right],$$
$$w_a^T(s) = \left[w_{\lambda y_a}^T(s), y_a(s)\right], \tag{28}$$

$$\dot{\omega}_{\lambda y_a}(t) = \Lambda_2 \omega_{\lambda ya}(t) + h_2{}_{ya}(t), \omega_{\lambda ya}(0) = 0. \tag{29}$$

From this we get:

$$y_a(s) = \frac{R(s) - \Gamma(s)}{R(s)} y_a - \frac{1}{R(s)} \sum_{i=1}^{m} \frac{\alpha_i}{s+\lambda_i} y_a(s) \frac{1}{R(s)}\left[u_a(s) - \tilde{\theta}^{*T}\omega_a(s)\right] \tag{30}$$

or

$$\left[\Gamma(s) + \sum_{i=1}^{m} \frac{\alpha_i}{s+\lambda_i}\right] y_a(s) = \left[u_a(s) - \tilde{\theta}^{*T}\omega_a(s)\right]. \tag{31}$$

Thus, the use of equation (17) for the auxiliary model ya leads to the control:

$$y_a(s) = k_p \frac{F(s)}{A_0(s)}\left[\rho u_a(s) - \rho\tilde{\theta}^{*T}\omega_a(s)\right]. \tag{32}$$

The transfer function of system (21) without taking into account the time delay is strictly positively real, then using the Kalman – Yakubovich lemma [11] there are $P = P^T > 0$ and $Q = Q^T > 0$ such that the following conditions are always satisfied:

$$A_{11}^T P + PA_{11} = -2Q$$
$$P\hat{B}_{u1} = C_{f1}. \tag{33}$$

If the algorithms for tuning the parameters $\Theta(t)$ and $\psi(t)$ are determined by the expressions:

$$\dot{\Theta}(t) = -\sin g(k_p)\tilde{\Phi} e_a(t)\Omega_{mpra}(t),$$
$$\dot{\psi}(t) = \Phi_\psi e_a(t)\left[\Theta(t-\tau)^T \Omega_{mpra}(t-\tau) - \Theta(t)^T \Omega_{mpra}(t)\right], \tag{34}$$

then the total time derivative of the Lyapunov function, taking into account (33), has the form

$$\dot{V}(t) = -E_a^T(t)QE_a(t) < 0. \tag{35}$$

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Thus, [12], the adaptive control (19) and the tuning algorithm (34) guarantee that $V\ (t)$, and therefore, the $E_a(t),\ e_a(t),\ \Theta(t)\, u\, \psi(t)$ signals are bounded, i.e. $E_a(t),\ e_a(t),\ \theta(t), \psi(t) \in L_\infty$. From (35) we establish that $E_a(t),\ e_a(t) \in L_2$. From the fulfillment of the $E_a(t) \in L_\infty \cap \in L_2\, u\, E_a(t) \in L_\infty$ conditions, the intermediate goal of the functioning of the system (26) follows. Wherein:

$$\lim_{t\to\infty} \Theta(t) = \Theta^\circ, \qquad \lim_{t\to\infty} \psi(t) = \psi^\circ, \tag{36}$$

Where Θ°, ψ° is the constant vector and number. This means that, in accordance with (21), all signals in a closed system are bounded. If the signal $r(t)$ is such that the object is identifiable [13,14], $\Theta^\circ = \Theta^*$ then in accordance with (21) we obtain the fulfillment of the original goal of the functioning of system (6).

V. CONCLUSION

Application of this scheme allows reducing the number of adjustable parameters in the adaptive controller to the number of unknown coefficients in the system. The proposed adaptive control structure contains only blocks with a concentrated delay, which simplifies the practical implementation of such systems.

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