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Adaptive Estimation Algorithms with Adjustment of Noise Covariance Matrices in Object and Measurement Interference

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ABSTRACT: Stable adaptive estimation algorithms are given with adjustment of noise covariance matrices in the object and measurement interference based on the Gauss-Newton recurrence-iteration procedure. Approximations of the functional under consideration are given, replacing the nonlinear least squares problem with the corresponding linear problem. Proposed algorithms allow adaptive filtering of linear object at unknown intensities of input signals, disturbances in object and noise of measurements.

KEYWORDS: adaptive estimation, matrices of noise and measurement interference covariances, adjustment algorithms.

I.INTRODUCTION

In the problem of linear filtration at the final time interval, the Kalman filter has proved to be an effective means both in stochastic conditions with stationary noise and in minimal staging [1-3]. Luenberger observers, built without taking into account the properties of perturbations, but only for the stability of the observation system, give significantly worse results both in computer modeling and in applied problems. The Kalman filter indicates a method of calculating the filter gain, well matched with all the properties of the object - the coefficients of the model equations and the matrices of perturbation covariances in the object and noise in the measurement.

In a number of applications, the least reliable information about the linear model is the properties of noise in the object and interference in the measurement. Particularly important is the ratio of dispersions ρ of these processes [2]. If the perturbation in the object is significantly less than the measurement noise $(\rho << 1)$, then the Kalman filter gain will be relatively small, which leads to smoothing of the measurements. On the contrary, if the measurement interference is small $(\rho >> 1)$, then the gain is relatively large and it is said that the filter "believes" the measurements. The RMS estimation error depends to a large extent on the nature of the smoothing of the filter, manifested in the number of recent measurements that significantly affect the current estimate. For multidimensional disturbances or noises, cases of large and small ρ can be combined in different ways.

It can be shown [2,3] that in the case of a stationary model with known coefficients, any stable Luenberger observer with a matrix from the object equation is a Kalman filter. Therefore, the use of the Kalman filter loses meaning in the absence of information about these matrices. Setting the tasks of evaluating linear systems with unknown matrices of disturbance covariances in the object and measurement noises can be very diverse. The specifics of the application may suggest the most appropriate mathematical model.

Methods of guaranteed estimation [5-7] provide the upper boundary of the quality indicator for all disturbances in the object and noise of measurement from a given class, and also minimize this boundary. Their advantage is the absence of random errors that can accompany adaptive tuning, as well as the ability to calculate the filter in advance. Disadvantages of guaranteed filtering are manifested in excessive conservativeness if the permissible set of parameter values is too wide.

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Methods of nonlinear filtering and Bayesian estimation of model parameters are very diverse [8]. Model parameters are evaluated using a suitable set of statistics generated from incoming measurements. Efficient numerical implementation is achieved by proper parameterization and finite-dimensional approximation, including filter banks.

In [1,8,9] discusses various algorithms for adaptive filtering of systems with unknown constant parameters. These parameters can be values of perturbation covariance matrices. The presented decision algorithms refer either to the case of a finite set of values of an unknown parameter, or to the case of the existence of a probabilistic distribution of an unknown parameter, which can be used in Bayesian formulas.

Algorithms for adaptive estimation of perturbation covariance matrices in linear stationary systems were studied in [4, 10]. In the case of scalar measurements and scalar disturbance in the object, an asymptotically accurate estimate of the ratio of disturbance dispersions in the object and measurement noise is obtained in [4]. A generalization to a multivariate case is presented in [10] assuming that the measurement noise covariance matrix is known.

Let the observation object be described by a linear model:

$$x_t = A_{t-1}x_{t-1} + B_t w_t,$$
$$y_t = C_t x_t + v_t, \quad 1 \le t \le T,$$

wherein x_t - is the dimension state vector n, y_t - is the dimension measurement vector m, w_t - is the perturbation in the dimension object l, v_t - is the measurement noise. Matrices A_t, B_t, C_t of respective dimensions are known. The measurement sequence $y^0 = (y_t)_{t=1}^T$ and the a priori Gaussian distribution of the initial vector x_0 with the mean $\bar{x}$ and the covariance matrix P are given. An estimate of the state vector x_T, as well as an accuracy characteristic of this estimate, must be found. Perturbation in object w_t and measurement noise v_t form a joint vector $\xi_t = col(w_t, v_t)$.

Thus, the model is determined by a set of vectors $\xi = col(w_t, v_t)_{t=1}^T$ aligned with the measurements $y = (y_t)_{t=1}^T$. Hereinafter, the consistency of the set ξ with the measurements y means that the linear object with the perturbations (w_t) and the measurement noise (v_t) of the set ξ has the same output as (y_t). All matched sets ξ form an affine space. To select one set and therefore one model, enter a quality measure. Let in an arbitrary model $R_T(\xi)$ - be a covariance matrix of the prediction error of the last dimension $\varepsilon_T = y_T - C_T\hat{x}_{T/T-1}$. The quality measure of this model can then be defined as

$$F(\xi) = tr(R_T(\xi)S),$$

where S - is some non-negative defined matrix with coefficients reflecting the weight of different components of vector ε_T. This quality indicator is selected for the following reasons [9-11]: in different stochastic models of the observed object, it gives a filter gain close to the optimal; there are effective algorithms for minimizing it; it takes into account the accuracy of evaluation at the last moment of time, which is important in application to management tasks.

Let $\xi = (\xi_t)_{t=1}^T$ - be a set of vectors in R^N. Then [11] among all normal distributions in R^N with zero mean, the maximum probability density on the set of ξ implementations of independent random variables with this distribution is achieved with a matrix of covariances

$$P = \frac{1}{T}\sum_{t=1}^{T}\xi_t\xi_t^*.$$

The functional $J(\xi)$ is completely defined by the covariance matrix

$$Q = \begin{pmatrix} Q_w & Q_{wv} \\ Q_{vw} & Q_v \end{pmatrix},$$

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the dimension of which is independent of T . Function F can be minimized by various methods, for which it will be necessary to calculate its partial derivatives over all elements ξ . We will use the Gauss-Newton iterative algorithm as one of the effective methods for minimizing the function of many variables [12-18].

The Gauss-Newton method, which allows the current point ξ_c to find the next point ξ_+, is based on the approximation of the function $F(\xi)$ in the vicinity of the current point ξ_c by its affine model

$$M_c(\xi) = F(\xi_c) + J(\xi_c)(\xi - \xi_c) = F(\xi_c) + J(\xi_c)\Delta\xi$$

and corresponding replacement of the nonlinear least squares problem with the linear least squares problem of the form [13-15]:

$$\|M_c(\xi)\|^2 / 2 = \|F(\xi_c) + J(\xi_c)\Delta\xi\|^2 / 2 = \varepsilon_c / 2, \quad (1)$$

where $\varepsilon_c = \|F(\xi_c) + J(\xi_c)\Delta\xi\|^2$ – is a misalignment of equation

$$J(\xi_c)\Delta\xi = -F(\xi_c), \quad (2)$$

here J – is the Jacobi matrix of the vector function $F = (f_1,..., f_m)$:

$$J = \begin{Vmatrix} \nabla^T f_1 \\ \dots \\ \nabla^T f_m \end{Vmatrix} \in R^{m\times n}, \quad \nabla f_u = \begin{Vmatrix} \partial f_u / \partial \xi_1 \\ \dots \\ \partial f_u / \partial \xi_n \end{Vmatrix} \in R^n,$$

In rare cases, the matrix $J(\xi_c)$ of equation (2) is square and non-degenerate; in these cases, its only solution

$$\Delta\xi = -J^{-1}(\xi_c)R(\xi_c), \quad \xi_+ = \xi_c + \Delta\xi \quad (3)$$

is the solution of problem (1) with zero non-binding [14].

More often there are cases when matrix $J(\xi_c) \in R^{m\times n}$ is a matrix of general form, but the condition of compatibility of equation (2), having the form:

$$J(\xi_c)J^+(\xi_c)F(\xi_c) = F(\xi_c) \quad (4)$$

wherein $J^+(\xi_c) \in R^{n\times m}$ – is pseudo-inverse to matrix $J(\xi_c)$. Note [12-14] that if equation (2) is rewritten in equivalent form:

$$J(\xi_c)\xi = J(\xi_c)\xi_c - F(\xi_c),$$

that condition of its compatibility

$$J(\xi_c)J^+(\xi_c)[J(\xi_c)\xi_c - F(\xi_c)] = J(\xi_c)\xi_c - F(\xi_c),$$

given property $JJ^+J = J$, it is equivalent to condition (4).

Then the solution of equation (2) is:

$$\Delta\xi = -J^+(\xi_c)F(\xi_c) + [I - J^+(\xi_c)J(\xi_c)]y, \qquad \xi_+ = \xi_c + \Delta\xi \quad (5)$$

This solution is the only one if the condition is met along with (4)

$$J^+(\xi_c)J(\xi_c) = I,$$

and is recorded as

$$\Delta\xi = -J^+(\xi_c)F(\xi_c), \quad \xi_+ = \xi_c + \Delta\xi . \quad (6)$$

In other cases, the choice of a single solution from (5) is subject to some additional requirements; such is, for example, the requirement [14-16]: find

$$\min \|\Delta\xi\|, \qquad \Delta\xi = -J^+(\xi_c)F(\xi_c) + [I - J^+(\xi_c)J(\xi_c)]y . \quad (7)$$

The solution of problem (2), (7) is called normal; it is also recorded as (7).

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The most real cases are when the matrix $J(\xi_c)$ has a general form and condition (4) is not fulfilled, so that the solution of equation (2) does not exist. It is well known [13-18] that even in these cases the ratio (5) provides a solution to the problem (1), and the ratio (6) provides a solution to the problem (1), (7); for equation (2), they serve as general and normal pseudo-solutions, respectively. They, in the Gauss-Newton method, determine the main step of the iterative procedure, usually written in the form:

$$\Delta\xi = -[J^T(\xi_c)J(\xi_c) + \alpha I]^{-1} J^T(\xi_c)F(\xi_c), \qquad \xi_+ = \xi_c + \Delta\xi ,$$

where $\alpha > 0$ – is the regularization parameter, I – is the unit matrix.

The basis is usually taken as a more general and brief entry (6), as well as better adapted for the formulation of recurring procedures.

Proposed algorithms allow adaptive filtering of linear object at unknown intensities of input signals, disturbances in object and noise of measurements.

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