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# **Researching the Forced Oscillations of Tractor Trailer When Braking is in Process**

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**ABSTRACT:** The stability and controllability of tractor trailer during transportation of agricultural goods is mostly determined by its mechanical oscillations. Existing methodological principles do not concern the issues of arising and occurrence of the trailer forced oscillations during braking. Therefore, important task was development the method for studying forced oscillations of tractor trailer in a mode of its braking. The tangible 2PTS-4-793-03A two-axle trailer equivalent to it with dynamic model having four degrees of freedom were replaced and vertical movements of the sprung and unsprung bodies of trailer under the influence of exciting and elastic forces, as well as resistance forces were studied. Methods of general dynamics of wheeled vehicles were applied. Formulas of the kinetic and potential energies of system and the dissipative function of power dissipation and generalized forces. Using the second-order Lagrange equations, we obtained a system of equations for trailer's forced oscillations during braking. It was revealed that in general case, the oscillations of the front and rear parts of the sprung skeleton of the trailer are interconnected. Case was shown when the vertical oscillations of the front and rear parts of trailer are independent.

**KEY WORDS:** tractor trailer, mechanical system, dynamic model of trailer, roughness of supporting surface, Lagrange equations, system of differential equations of trailer during braking.

## **I. INTRODUCTION**

Despite the availability of extensive literature on dynamics of tractor trailer [1, 2], the issues of its forced oscillations in the process of transporting agricultural goods have not been sufficiently studied.

Smoothness of trailer, safety and quality of the goods being transported are strongly influenced by linear and angular oscillations of the framework to the vertical and transverse axes. Arousing such oscillations of force and its amplitudes noticeably increase during emergency braking of the trailer. Therefore, the study of the emergence and occurrence of vertically linear and transverse-angular oscillations of the trailer is an important scientific task.

## **II. MATERIALS AND METHODS**

The development was carried out on basis of methods of the general dynamics of wheeled vehicles [3, 4], tractor and road trains [5, 6], and studies of small oscillations of mobile machines [7, 8].

## **III. RESULTS AND DISCUSSION**

Let's replace the tangible 2PTS-4-793-03A brand two-axle trailer (Figure a) equivalent to it with dynamic model (Figure b). Gross trailer weight  $m_{\Pi}$  ( $m_{\Pi} = G_{\Pi} / g$ ,  $G_{\Pi}$  - trailer weight;  $g$  - acceleration of gravity) consists of the total amount of bodies of sprung part  $m_{\Pi\Pi}$  and supporting wheels  $m_F$ . Body fraction  $m_{\Pi\Pi}$ , attributable to the front and rear axles of the trailer we denote by  $m_{1p}$  and  $m_{2p}$ . Moment of inertia of sprung part (base) of the trailer relative to transverse axis  $O_{\Pi}Y$ , passing through the center of body  $O_{\Pi}$  is equal to  $J_y$ . The front and rear axles of the 2PTS-4-793-03A trailer have the same springs and wheels. Therefore, in figure b) the total stiffness  $2C_{p1}, 2C_{\Pi\Pi}$  (N/m) tires and springs of front and rear axles, their total resistance coefficients  $2K_{p1}, 2K_{\Pi\Pi}$  (kg/s) with noted with identical symbols.

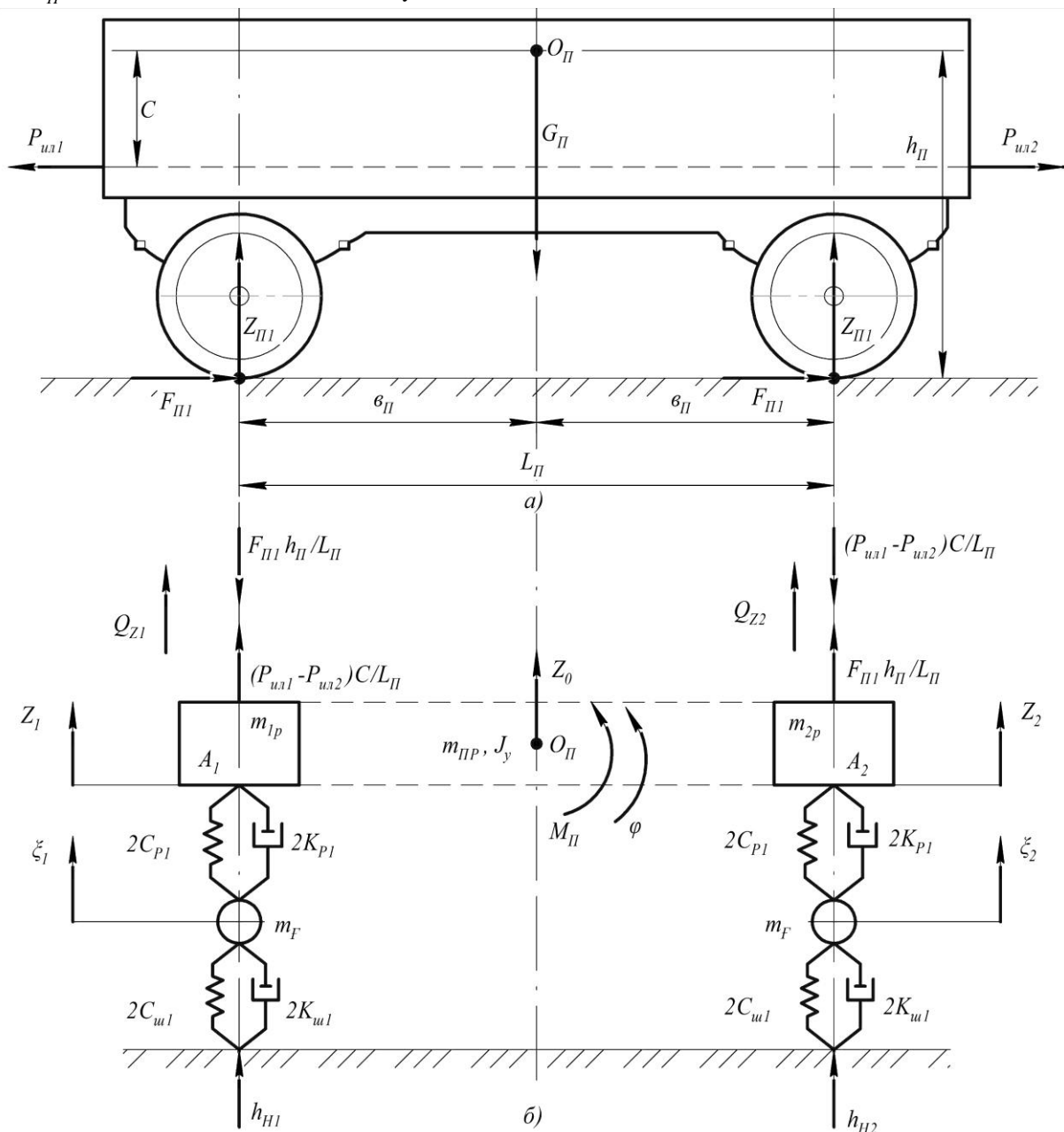
When braking is in process, the trailer is affected by the weight  $G_{II}$ , traction forces  $P_{ua1}, P_{ua2}$  and ordinary  $Z_{II1}$ , brake  $F_{II1}$  support surface reactions.

Under the influence of couplers  $P_{ua1}, P_{ua2}$  and brake  $F_{II1}$  forces there are arising forces  $Q_{Z1}, Q_{Z2}$ , influencing on front and rear axis of the trailer.

The height of the roughness of bearing surface under the front  $h_{H1}$  and rear  $h_{H2}$  wheels is function of time. Moreover, the signal caused by roughness of the surface enters the rear wheel with  $t_o$  delay relative to front wheel. Consequently,

$$h_{H1} = h(t); \quad h_{H2} = h(t - t_o); \quad t_o = \frac{L_{II}}{V},$$

where  $L_{II}$  - trailer base;  $V$  - movement velocity.



The following generalized forces act on a trailer (mechanical system): exciting (disturbing) forces, elastic forces and resistance (friction) forces.

Disturbing forces arising from roughnesses  $h_{H1}, h_{H2}$  the reference plane on the trailer wheels are functions of time. Under their action, compelled, first not established, and then steady-state oscillations of the system arise.

Elastic forces arising as a result of elastic deformations of springs and tires, directed opposite to the deviation of the system relative to its initial position, restore it.

Resistance forces arising on springs and tires reduce the mechanical power of the oscillatory motion of the system, turning it into heat. The resistance forces of the system are directed in direction opposite to the oscillation speed and are proportional to some extent to the speed.

To simplify the analysis of forced oscillations of the trailer frame, let us assume that: 1) the trailer moves uniformly and rectilinearly; 2) the profile of the bearing surface under the right and left wheels is the same; 3) the input signal causing the oscillations coincides with the path profile (the trailer wheels maintain constant point contact with the supporting surface); 4) the path profile is a stationary random distance function.

Calculated dynamic model of trailer has four degrees of freedom:  $Z_1$  and  $Z_2$  – vertical transition of sprung bodies  $m_{1p}, m_{2p}$  over front and rear wheels;  $\xi_1$  и  $\xi_2$  – transition of unsprung body  $m_F$ . Vertical coordinates  $Z_1$  and  $Z_2$ ,  $\xi_1$  and  $\xi_2$  counted from the position of the statistical equilibrium of the system.

With small angular oscillations of the trailer frame, when  $tg\varphi \approx \varphi$ , angle of rotation of the sprung body relative to the transverse axis  $OY$ , passing through resiliency center  $O_{II}$ , lets determine base on the following formula

$$\varphi = (Z_2 - Z_1) / L_{II}.$$

Vertical movement of the center of the sprung body of the trailer  $Z_o$  is related with the vertical transitions  $Z_1$  and  $Z_2$  of the following dependencies:

$$Z_o = (Z_1\epsilon_{II} + Z_2\epsilon_{II}) / L_{II} \text{ and}$$

$$Z_o = \frac{(Z_1 + Z_2)\epsilon_{II}}{L_{II}}.$$

Kinetic power of the system:

$$T = \frac{1}{2} m_{IIp} \dot{Z}_o^2 + \frac{1}{2} J_y \dot{\varphi}^2 + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2.$$

Considering  $\dot{Z}_o = \frac{(\dot{Z}_1 + \dot{Z}_2)\epsilon_{II}}{L_{II}}$ ,  $\dot{\varphi} = \frac{\dot{Z}_2 - \dot{Z}_1}{L_{II}}$  let's not the following formula:

$$\begin{aligned} T &= \frac{1}{2} m_{IIp} \frac{\epsilon_{II}^2}{L_{II}^2} (\dot{Z}_1 + \dot{Z}_2)^2 + \frac{1}{2} J_y \frac{(\dot{Z}_2 - \dot{Z}_1)^2}{L_{II}^2} + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2 = \\ &= \frac{m_{IIp} \epsilon_{II}^2}{2L_{II}^2} (\dot{Z}_1^2 + 2\dot{Z}_1\dot{Z}_2 + \dot{Z}_2^2) + \frac{J_y}{2L_{II}^2} (\dot{Z}_2^2 - 2\dot{Z}_1\dot{Z}_2 + \dot{Z}_1^2) + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2 = \frac{m_{IIp} \epsilon_{II}^2}{2L_{II}^2} \dot{Z}_1^2 + \\ &+ \frac{m_{IIp} \epsilon_{II}^2}{L_{II}^2} \dot{Z}_1\dot{Z}_2 + \frac{m_{IIp} \epsilon_{II}^2}{2L_{II}^2} \dot{Z}_2^2 + \frac{J_y}{2L_{II}^2} \dot{Z}_2^2 - \frac{J_y}{L_{II}^2} \dot{Z}_1\dot{Z}_2 + \frac{J_y}{2L_{II}^2} \dot{Z}_1^2 + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2 = \\ &= \left( \frac{m_{IIp} \epsilon_{II}^2}{2L_{II}^2} + \frac{J_y}{2L_{II}^2} \right) \dot{Z}_1^2 + \left( \frac{m_{IIp} \epsilon_{II}^2}{2L_{II}^2} + \frac{J_y}{2L_{II}^2} \right) \dot{Z}_2^2 + \left( \frac{m_{IIp} \epsilon_{II}^2}{L_{II}^2} - \frac{J_y}{L_{II}^2} \right) \dot{Z}_1\dot{Z}_2 + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2 = \\ &= \frac{(J_y + m_{IIp} \epsilon_{II}^2)}{2L_{II}^2} \dot{Z}_1^2 + \frac{1}{2} \cdot \frac{(J_y + m_{IIp} \epsilon_{II}^2)}{L_{II}^2} \dot{Z}_2^2 + \frac{(m_{IIp} \epsilon_{II}^2 - J_y)}{L_{II}^2} \dot{Z}_1\dot{Z}_2 + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2 \end{aligned}$$

$$\text{and } T = \frac{1}{2} m_1 \dot{Z}_1^2 + \frac{1}{2} m_2 \dot{Z}_2^2 + m_3 \dot{Z}_1\dot{Z}_2 + \frac{1}{2} m_F \dot{\xi}_1^2 + \frac{1}{2} m_F \dot{\xi}_2^2, \tag{1}$$

where  $m_1 = \frac{J_y + m_{IIp} \epsilon_{II}^2}{L_{II}^2}$ ;  $m_2 = \frac{J_y + m_{IIp} \epsilon_{II}^2}{L_{II}^2}$ ;  $m_3 = \frac{m_{IIp} \epsilon_{II}^2 - J_y}{L_{II}^2}$  are the given bodies.

The potential power of the system is equal to the work of the elastic forces of the suspension and tires, which are functions of linear deflections. Deflections of elastic elements (springs, tires) counted from the equilibrium position are equal: for springs  $Z_1 - \xi_1$  и  $Z_2 - \xi_2$ ; for tires  $\xi_1 - h_{H1}$  and  $\xi_2 - h_{H2}$ .

Formula of potential power of the system has the form:

$$\begin{aligned} \Pi &= \frac{1}{2} 2C_{P1}(Z_1 - \xi_1)^2 + \frac{1}{2} 2C_{P1}(Z_2 - \xi_2)^2 + \frac{1}{2} 2C_{III}(\xi_1 - h_{H1})^2 + \frac{1}{2} 2C_{III}(\xi_2 - h_{H2})^2 \\ \text{and } \Pi &= C_{P1}(Z_1 - \xi_1)^2 + C_{P1}(Z_2 - \xi_2)^2 + C_{III}(\xi_1 - h_{H1})^2 + C_{III}(\xi_2 - h_{H2})^2. \end{aligned} \quad (2)$$

The dissipative power dissipation function is determined through the resistance forces by the below given formula:

$$\begin{aligned} \Phi &= \frac{1}{2} 2K_{P1}(\dot{Z}_1 - \dot{\xi}_1)^2 + \frac{1}{2} 2K_{P1}(\dot{Z}_2 - \dot{\xi}_2)^2 + \frac{1}{2} 2K_{III}(\dot{\xi}_1 - \dot{h}_{H1})^2 + \frac{1}{2} 2K_{III}(\dot{\xi}_2 - \dot{h}_{H2})^2 \text{ and} \\ \Phi &= K_{P1}(\dot{Z}_1 - \dot{\xi}_1)^2 + K_{P1}(\dot{Z}_2 - \dot{\xi}_2)^2 + K_{III}(\dot{\xi}_1 - \dot{h}_{H1})^2 + K_{III}(\dot{\xi}_2 - \dot{h}_{H2})^2. \end{aligned} \quad (3)$$

Generalized forces for the front and rear supports of trailer frame:

$$Q_{Z1} = \frac{(P_{u1} - P_{u2})C - F_{III}h_{II}}{L_{II}}; \quad Q_{Z2} = \frac{F_{III}h_{II} - (P_{u1} - P_{u2})C}{L_{II}}. \quad (4)$$

Let's draw the Lagrange equations of the second kind for this system as in the following:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial \Pi}{\partial q} + \frac{\partial \Phi}{\partial \dot{q}} = Q_q, \quad (5)$$

where  $q$  – generalized coordinates.

For the system being considered  $q_1 = Z_1$ ;  $q_2 = Z_2$ ;  $q_3 = \xi_1$ ;  $q_4 = \xi_2$ .

Taking the corresponding derivatives of the formulas for the kinetic (1) and potential (2) powers, the dissipative function (3) and substituting them in the Lagrange equations (5) taking into account the generalized forces (4), we derive the equations of the trailer forced oscillations during braking:

$$\begin{aligned} m_1 \ddot{Z}_1 + m_3 \ddot{Z}_2 + 2C_{P1}(Z_1 - \xi_1) + 2K_{P1}(\dot{Z}_1 - \dot{\xi}_1) &= \frac{(P_{u1} - P_{u2})C - F_{III}h_{II}}{L_{II}}; \\ m_2 \ddot{Z}_2 + m_3 \ddot{Z}_1 + 2C_{P1}(Z_2 - \xi_2) + 2K_{P1}(\dot{Z}_2 - \dot{\xi}_2) &= \frac{F_{III}h_{II} - (P_{u1} - P_{u2})C}{L_{II}}; \\ m_F \ddot{\xi}_1 + 2C_{P1}(\xi_1 - Z_1) + 2C_{III}(\xi_1 - h_{H1}) + 2K_{P1}(\dot{\xi}_1 - \dot{Z}_1) + 2K_{III}(\dot{\xi}_1 - \dot{h}_{H1}) &= 0; \\ m_F \ddot{\xi}_2 + 2C_{P1}(\xi_2 - Z_2) + 2C_{III}(\xi_2 - h_{H2}) + 2K_{P1}(\dot{\xi}_2 - \dot{Z}_2) + 2K_{III}(\dot{\xi}_2 - \dot{h}_{H2}) &= 0. \end{aligned} \quad (6)$$

Let's reveal brackets on the left side of the first equation of the system (6):

$$m_1 \ddot{Z}_1 + 2K_{P1} \dot{Z}_1 + 2C_{P1}Z_1 + m_3 \ddot{Z}_2 - 2K_{P1} \dot{\xi}_1 - 2C_{P1} \xi_1 = \frac{(P_{u1} - P_{u2})C - F_{III}h_{II}}{L_{II}}.$$

Let's divide both parts of this equation into  $m_1$ :

$$\ddot{Z}_1 + \frac{2K_{P1}}{m_1} \dot{Z}_1 + \frac{2C_{P1}}{m_1} Z_1 + \frac{m_3}{m_1} \ddot{Z}_2 - \frac{2K_{P1}}{m_1} \dot{\xi}_1 - \frac{2C_{P1}}{m_1} \xi_1 = \frac{(P_{u1} - P_{u2})C - F_{III}h_{II}}{L_{II}m_1}.$$

Let's input the following designations:  $\eta_1 = \frac{m_3}{m_1}$ ;  $\omega_1^2 = \frac{2C_{P1}}{m_1}$ ;

$$2h_1 = \frac{2K_{P1}}{m_1}; \quad F_1(t) = \frac{(P_{u1} - P_{u2})C - F_{III}h_{II}}{L_{II}m_1}.$$

Then.

$$\ddot{Z}_1 + 2h_1 \dot{Z}_1 + \omega_1^2 Z_1 + \eta_1 \ddot{Z}_2 - 2h_1 \dot{\xi}_1 - \omega_1^2 \xi_1 = F_1(t). \quad (7)$$

Similarly, from the second equation of system (6) we have:

$$m_2 \ddot{Z}_2 + m_3 \ddot{Z}_1 + 2C_{P1}Z_2 - 2C_{P1}\xi_2 + 2K_{P1}\dot{Z}_2 - 2K_{P1}\dot{\xi}_2 = \frac{F_{III}h_{II} - (P_{u1} - P_{u2})C}{L_{II}}. \Rightarrow$$

$$\ddot{Z}_2 + \frac{m_3}{m_2} \ddot{Z}_1 + \frac{2C_{P1}}{m_2} Z_2 - \frac{2C_{P1}}{m_2} \xi_2 + \frac{2K_{P1}}{m_2} \dot{Z}_2 - \frac{2K_{P1}}{m_2} \dot{\xi}_2 = \frac{F_{II1}h_{II} - (P_{u1} - P_{u2})C}{L_{II}m_2} \Rightarrow$$

$$\eta_2 = \frac{m_3}{m_2}; \quad \omega_2^2 = \frac{2C_{P1}}{m_2}; \quad 2h_2 = \frac{2K_{P1}}{m_2}; \quad F_2(t) = \frac{F_{II1}h_{II} - (P_{u1} - P_{u2})C}{L_{II}m_2} \Rightarrow$$

$$\ddot{Z}_2 + 2h_2\dot{Z}_2 + \omega_2^2 Z_2 + \eta_2\ddot{Z}_1 - 2h_2\dot{\xi}_2 - \omega_2^2 \xi_2 = F_2(t). \tag{8}$$

From the third equation of system (6) we will have:

$$m_F \ddot{\xi}_1 + 2C_{P1}\dot{\xi}_1 - 2C_{P1}Z_1 + 2C_{III}\xi_1 - 2C_{III}h_{H1} + 2K_{P1}\dot{\xi}_1 - 2K_{P1}\dot{Z}_1 +$$

$$+ 2K_{III}\dot{\xi}_1 - 2K_{III}\dot{h}_{H1} = 0. \Rightarrow$$

$$\ddot{\xi}_1 + \frac{2C_{P1}}{m_F} \dot{\xi}_1 - \frac{2C_{P1}}{m_F} Z_1 + \frac{2C_{III}}{m_F} \xi_1 - \frac{2C_{III}}{m_F} h_{H1} + \frac{2K_{P1}}{m_F} \dot{\xi}_1 - \frac{2K_{P1}}{m_F} \dot{Z}_1 + \frac{2K_{III}}{m_F} \dot{\xi}_1 - \frac{2K_{III}}{m_F} \dot{h}_{H1} = 0. \Rightarrow \omega_{P1}^2 = \frac{2C_{P1}}{m_F};$$

$$\omega_{F1}^2 = \frac{2C_{III}}{m_F}; \quad 2h_{P1} = \frac{2K_{P1}}{m_F}; \quad 2h_{F1} = \frac{2K_{III}}{m_F}. \Rightarrow$$

$$\ddot{\xi}_1 + \omega_{P1}^2 \xi_1 - \omega_{P1}^2 Z_1 + \omega_{F1}^2 \xi_1 - \omega_{F1}^2 h_{H1} + 2h_{P1}\dot{\xi}_1 - 2h_{P1}\dot{Z}_1 + 2h_{F1}\dot{\xi}_1 - 2h_{F1}\dot{h}_{H1} = 0. \Rightarrow$$

$$\ddot{\xi}_1 + (2h_{F1} + 2h_{P1})\dot{\xi}_1 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_1 - 2h_{P1}\dot{Z}_1 - \omega_{P1}^2 Z_1 = 2h_{F1}\dot{h}_{H1} + \omega_{F1}^2 h_{H1}. \tag{9}$$

From the fourth equation of system (6) we will have:

$$m_F \ddot{\xi}_2 + 2C_{P1}\dot{\xi}_2 - 2C_{P1}Z_2 + 2C_{III}\xi_2 - 2C_{III}h_{H2} + 2K_{P1}\dot{\xi}_2 - 2K_{P1}\dot{Z}_2 +$$

$$+ 2K_{III}\dot{\xi}_2 - 2K_{III}\dot{h}_{H2} = 0. \Rightarrow$$

$$\ddot{\xi}_2 + \frac{2C_{P1}}{m_F} \dot{\xi}_2 - \frac{2C_{P1}}{m_F} Z_2 + \frac{2C_{III}}{m_F} \xi_2 - \frac{2C_{III}}{m_F} h_{H2} + \frac{2K_{P1}}{m_F} \dot{\xi}_2 - \frac{2K_{P1}}{m_F} \dot{Z}_2 + \frac{2K_{III}}{m_F} \dot{\xi}_2 - \frac{2K_{III}}{m_F} \dot{h}_{H2} = 0. \Rightarrow$$

$$\ddot{\xi}_2 + \omega_{P1}^2 \xi_2 - \omega_{P1}^2 Z_2 + \omega_{F1}^2 \xi_2 - \omega_{F1}^2 h_{H2} + 2h_{P1}\dot{\xi}_2 - 2h_{P1}\dot{Z}_2 + 2h_{F1}\dot{\xi}_2 - 2h_{F1}\dot{h}_{H2} = 0. \Rightarrow$$

$$\ddot{\xi}_2 + (2h_{F1} + 2h_{P1})\dot{\xi}_2 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_2 - 2h_{P1}\dot{Z}_2 - \omega_{P1}^2 Z_2 = 2h_{F1}\dot{h}_{H2} + \omega_{F1}^2 h_{H2}. \tag{10}$$

Summarizing formulas (7), (8), (9) and (10) we obtain a system of differential equations describing the forced oscillations of the trailer during braking with respect to the longitudinally vertical plane and the transverse axis:

$$\ddot{Z}_1 + 2h_1\dot{Z}_1 + \omega_1^2 Z_1 + \eta_1\ddot{Z}_2 - 2h_1\dot{\xi}_1 - \omega_1^2 \xi_1 = F_1(t);$$

$$\ddot{Z}_2 + 2h_2\dot{Z}_2 + \omega_2^2 Z_2 + \eta_2\ddot{Z}_1 - 2h_2\dot{\xi}_2 - \omega_2^2 \xi_2 = F_2(t);$$

$$\ddot{\xi}_1 + (2h_{F1} + 2h_{P1})\dot{\xi}_1 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_1 - 2h_{P1}\dot{Z}_1 - \omega_{P1}^2 Z_1 = 2h_{F1}\dot{h}_{H1} + \omega_{F1}^2 h_{H1};$$

$$\ddot{\xi}_2 + (2h_{F1} + 2h_{P1})\dot{\xi}_2 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_2 - 2h_{P1}\dot{Z}_2 - \omega_{P1}^2 Z_2 = 2h_{F1}\dot{h}_{H2} + \omega_{F1}^2 h_{H2}. \tag{11}$$

It can be seen from (11) that, in the general case, the oscillations of the front and rear parts of the sprung skeleton of the trailer are interconnected, since  $\ddot{Z}_1$  and  $\ddot{Z}_2$  in the first and second equations participate simultaneously.

Consider the case where the oscillations of the front and rear of the trailer are independent. It is possible with  $\varepsilon = 1$ , that means in the event  $\rho^2 = \sigma_{II}^2$  or  $m_{II}\sigma_{II}^2 = J_y$ .

In the case if  $m_3 = 0$ ;  $\eta_1 = \eta_2 = 0$ ;  $m_1 = m_2 = \frac{m_{II}\sigma_{II}^2 + m_{II}\sigma_{II}^2}{4\sigma_{II}^2} = \frac{m_{II}}{2}$ ;

$$\omega_1^2 = \omega_2^2 = \frac{4C_{P1}}{m_{II}}; \quad 2h_1 = 2h_2 = \frac{4K_{P1}}{m_{II}}.$$

In this case, the system of differential equations (11) takes a simplified form:

$$\ddot{Z}_1 + \frac{4K_{P1}}{m_{II}} \dot{Z}_1 + \frac{4C_{P1}}{m_{II}} Z_1 - \frac{4K_{P1}}{m_{II}} \dot{\xi}_1 - \frac{4C_{P1}}{m_{II}} \xi_1 = F_1(t);$$

$$\ddot{Z}_2 + \frac{4K_{P1}}{m_{II}} \dot{Z}_2 + \frac{4C_{P1}}{m_{II}} Z_2 - \frac{4K_{P1}}{m_{II}} \dot{\xi}_2 - \frac{4C_{P1}}{m_{II}} \xi_2 = F_2(t);$$



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$$\begin{aligned}\ddot{\xi}_1 + (2h_{F1} + 2h_{P1})\dot{\xi}_1 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_1 - 2h_{P1}\dot{Z}_1 - \omega_{P1}^2 Z_1 &= 2h_{F1}\dot{h}_{H1} + \omega_{F1}^2 h_{H1}; \\ \ddot{\xi}_2 + (2h_{F1} + 2h_{P1})\dot{\xi}_2 + (\omega_{F1}^2 + \omega_{P1}^2)\xi_2 - 2h_{P1}\dot{Z}_2 - \omega_{P1}^2 Z_2 &= 2h_{F1}\dot{h}_{H2} + \omega_{F1}^2 h_{H2}.\end{aligned}\quad (12)$$

In effort to assess the possibility of such a simplification, the trailer frequency ratio is calculated  $\Omega_1, \Omega_2$  to partial  $\omega_1, \omega_2$ , т.е.  $\Omega_1 / \omega_1$  и  $\Omega_2 / \omega_2$ .

These frequencies are determined by studying the trailer's natural oscillations.

## IV. CONCLUSION

The forced oscillations of the tractor trailer during braking are described by a system of second-order differential equations. Moreover, the first and second equations correspond to the vertical movements of the sprung, the third and fourth unsprung parts of the trailer.

To simplify the solution of the obtained differential equations moving front ( $Z_1, \xi_1$ ) и задней ( $Z_2, \xi_2$ ) parts of the trailer can be represented as harmonic (deterministic) functions.

## REFERENCES

- [1] Zakin Y.Kh. Applied theory of the movement of the road train. Transport, 255 p. Moscow, 1967.
- [2] Rashidov N.R. Multi-link trains of tractor. Uzbekistan, 368 p. Tashkent, 1981.
- [3] Barsky I.B., Anilovich V.Ya., Kutkov G.M. The dynamics of the tractor. Machine Engineering, 280 p. Moscow, 1973.
- [4] Tractors: Theory. Training for university students in special. "Cars and tractors" / Guskov V.V., Velev N.N., Atamanov Yu.E. and others. Under the general edition. Guskova V.V. Machine Engineering, 376 p. Moscow, 1988.
- [5] Tractor trains / Artemyev P.P., Atamanov Yu.E., Bogdanov N.V. et al. ed. Guskova V.V. Machine Engineering, 183 p. Moscow, 1982.
- [6] Rozanov V.G. Braking of car and road train. Mashgiz, 244 p. Moscow, 1964.
- [7] Dynamics of the road - bus system / Ed. Khachaturova A.A. Machine Engineering, 536 p. Moscow, 1976.
- [8] Rotenberg R.V. Car suspension. Machine Engineering, 392 p. Moscow, 1972.