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Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati by AGHM

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ABSTRACT: In continuation to the study on formulations of Arithmetic–Geometric Mean (abbreviated as *AGM*), Arithmetic–Harmonic Mean (abbreviated as *AHM*) and Geometric-Harmonic Mean (abbreviated as *GHM*), which have been found to be a technique of evaluating the value of parameter from observed data containing the parameter itself and random error, an attempt has here been made on formulating of one more formulation of average termed as Arithmetic–Geometric-Harmonic Mean (abbreviated as *AGHM*) with an attempt to derive that this formulation can be a technique of determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *AGHM* and the derivation of the technique along with numerical application.

KEYWORDS: AGHM, numerical data, parameter, random error, determination of parameter.

I. INTRODUCTION

A lot of research had already been done on developing definitions / formulations of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean, Geometric Mean & Harmonic Mean which are called Pythagorean means [4 , 5 , 14 , 18]. A lot of definitions / formulations have already been developed among which some are arithmetic mean. geometric mean, harmonic mean, quadratic mean, cubic mean, square root mean, cube root mean, general *p* mean and many others [6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14 , 15 , 16 , 17, 18 , 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized *f* - Mean. [7 , 8*]*. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized *f* - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11*]*. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15, 16 , 17] as well as the different formulations of average from the first principles [19].

In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots , x_n$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N \qquad (1.1)$$

[21 , 22 , 23 , 24 , 25 , 26 , 27, 28 , 29].

The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 – 52] cannot provide appropriate value of the parameter μ [21 , 22 , 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 ,22 , 23 , 24 , 25 , 26, 27 , 28 , 29 , 30 , 53 ,54 , 55 , 56, 57 , 58 , 59 , 60]. The methods, developed in this studies, for determining the appropriate value of the parameter from observed data containing the parameter itself and random error involve huge computational tasks. Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. However, the appropriate value of the parameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. In order to obtain such value of parameter, three methods have already been developed which involves lesser computational tasks than those involved in the earlier methods as well as which can be applicable in the case of finite set of data [61 , 62 , 63 , 64]. The methods developed are based on the concepts of Arithmetic-Geometric Mean (abbreviated as *AGM*) [61 , 62, 65 , 66], Arithmetic-Harmonic Mean(abbreviated as *AHM*) [63] and Geometric-Harmonic Mean (abbreviated as *GHM*) [64] respectively.

In continuation to the study on formulations of Arithmetic–Geometric Mean (abbreviated as *AGM*), Arithmetic–Harmonic Mean (abbreviated as *AHM*) and Geometric-Harmonic Mean (abbreviated as *GHM*), which have been found to be a technique of evaluating the value of parameter from observed data containing the parameter itself and random error, an attempt has here been made on formulating of one more formulation of average termed as Arithmetic–Geometric-Harmonic Mean (abbreviated as *AGHM*) with an attempt to derive that this formulation can be a technique of determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *AGHM* and the derivation of the technique along with numerical application.

II. ARITHMETIC-GEOMETRIC-HARMONIC MEAN (*AGHM)*

Let a_0, g_0 & h_0 be respectively the *AM* , the *GM* & the *HM* of n numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots , x_N$$

Then, $h_0 \leq g_0 \leq a_0$

From the inequality of Pythagorean means [4 , 5] namely

$$AM > GM > HM$$

it follows that

$$h_0 \leq g_0 \leq a_0$$

provided $x_1 , x_2 , \ldots\ldots , x_N$ are not all equal.

Let $\{\mathbf{a}^{///}_n\}$, $\{\mathbf{g}^{///}_n\}$ & $\{h^{///}_n\}$ be three sequences respectively defined by

$$a^{///}_n = 1/3\,(a^{///}_{n-1} + g^{///}_{n-1} + h^{///}_{n-1}) , \qquad (2.1)$$

$$\mathbf{g}^{///}_n = (a^{///}_{n-1}\, g^{///}_{n-1}\, h^{///}_{n-1})^{1/3} \qquad (2.2)$$

$$\& \; h^{///}_n = \{1/3\,(a^{///}_{n-1}{}^{-1} + g^{///}_{n-1}{}^{-1} + h^{///}_{n-1}{}^{-1})\}^{-1} \qquad (2.3)$$

where the square cube takes the <u>principal value</u>..

For $n = 1$, we have

$$h^{///}_1 \leq g^{///}_1 \leq a^{///}_1$$

Since $a^{///}_1$, $g^{///}_1$ & $h^{///}_1$ are respectively the *AM* , the *GM* & the *HM* of

$$a_0 , \; g_0 \; \& \; h_0$$

therefore, each of $a^{///}_1$, $g^{///}_1$ & $h^{///}_1$ lies between the maximum a_0 and the minimum h_0 of a_0, g_0 & h_0 ..

Therefore,

$$h_0 \leq h^{///}_1 \leq g^{///}_1 \leq a^{///}_1 \leq a_0$$

By the similar logic, we have for $n = 2$ that

$$h_0 \leq h^{///}_1 \leq h^{///}_2 \leq g^{///}_2 \leq a^{///}_2 \leq a^{///}_1 \leq a_0$$

Proceeding with the same logic, one can obtain at the n^{th} step that

$$h_0 \leq h^{///}_1 \leq h^{///}_2 \leq \ldots\ldots \leq h^{///}_n \leq h^{///}_{n+1} \leq g^{///}_{n+1} \leq a^{///}_{n+1} \leq a^{///}_n \leq \ldots\ldots \leq a^{///}_2 \leq a^{///}_1 \leq a_0$$

This inequality implies that the values of $a^{///}$, $\mathbf{g}^{///}_n$ & $h^{///}_n$ have been increasing starting from h_0 and have been decreasing starting from a_0 .

This means that the values of $\mathbf{a}^{///}_n$, $\mathbf{g}^{///}_n$ & $h^{///}_n$ will be more and more close as n becomes more and more large.

Thus, there exists a finite real number M_{AGH} such that

$\{\mathbf{a}^{///}_n\}$, $\{\mathbf{g}^{///}_n\}$ & $\{h^{///}_n\}$ converges to M_{AGH} as n approaches infinity.

This common converging point M_{AGH} can be termed / named / regarded as the Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) of the *N* numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots , x_N$$

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III. *AGHM* AS A TECHNIQUE OF EVALUATION OF μ

If the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N)$$

where

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

are the random errors associated to

$$x_1 , x_2 , \ldots\ldots , x_N$$

respectively which assume positive real values and negative real values in random order.
In this case,

$$A(x_1 , x_2 , \ldots\ldots , x_N) \to \mu \text{ as } N \to \infty$$

where $A(x_1 , x_2 , \ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i$

On the other hand, if the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

are composed of some parameter μ and random errors then the observations can also be expressed as

$$x_i = \mu \, \varepsilon_i^{/} \quad , \quad (i = 1 , 2 , \ldots\ldots , N)$$

where

$$\varepsilon_1^{/} , \varepsilon_2^{/} , \ldots\ldots , \varepsilon_N^{/}$$

are the random errors associated to

$$x_1 , x_2 , \ldots\ldots , x_N$$

respectively which assume positive real values in $(0 , 1)$ and in $(1 , \infty)$ in random order.
In this case,

$$G(x_1 , x_2 , \ldots\ldots , x_N) \to \mu \quad \text{as} \quad N \to \infty$$

where $G(x_1 , x_2 , \ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i \right)^{1/N}$

Again since the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

consist of μ and random errors,
therefore, the reciprocals

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

are composed of μ^{-1} and random errors different from the respective random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

provided $x_1 , x_2 , \ldots\ldots , x_N$ are all different from zero.
In this case thus

$$x_i^{-1} = \mu^{-1} + \varepsilon_i^{//} \quad , \quad (i = 1 , 2 , \ldots\ldots , N)$$

where

$$\varepsilon_1^{//} , \varepsilon_2^{//} , \ldots\ldots , \varepsilon_N^{//}$$

are the random errors associated to

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

respectively which assume positive real values and negative real values in random order.
In this case,

$$H(x_1 , x_2 , \ldots\ldots , x_N) \to \mu \text{ as } N \to \infty$$

where $H(x_1 , x_2 , \ldots\ldots , x_N) = \left(\frac{1}{N} \sum_{i=1}^{N} x_i^{-1} \right)^{-1}$

This implies that the common converging value of

$$A(x_1 , x_2 , \ldots\ldots , x_N) , \; G(x_1 , x_2 , \ldots\ldots , x_N) \; \& \; H(x_1 , x_2 , \ldots\ldots , x_N) \text{ as } N \to \infty$$

is the value of μ.
It is to be noted that a finite set of observed values may not be sufficient for obtaining the common converging value.
In order to obtain the value of μ, in this case, let us write

$$A(x_1 , x_2 , \ldots\ldots , x_N) = A_0 ,$$
$$G(x_1 , x_2 , \ldots\ldots , x_N) = G_0$$
$$\& \; H(x_1 , x_2 , \ldots\ldots , x_N) = H_0$$

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and then define the three sequences $\{A_n\}$, $\{G_n\}$ & $\{H_n\}$ respectively.by

$$A_{n+1} = 1/3\,(A_n + G_n + H_n) \ ,$$
$$G_{n+1} = (A_n . G_n . H_n)^{1/3}$$
$$\& \ H_{n+1} = \{1/3\,(A_n^{-1} + G_n^{-1} + H_n^{-1})\}^{-1}$$

Then, the three sequences $\{A_n\}$, $\{G_n\}$ & $\{H_n\}$ converge to acommon real number which is the *AGHM* of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

Now, from the model described by equation (1.1), it follows that

$$A_0 = \mu + \delta_0 \ , \ G_0 = \mu + d_0 \ \& \ H_0 = \mu + e_0$$

for some real numbers δ_0 , d_0 , e_0 .
Since $A_0 > G_0 > H_0$
therefore $\delta_0 > d_0 > e_0$
Thus $A_1 = \mu + \delta_1$ where $\delta_1 = 1/3\,(\delta_0 + d_0 + e_0)$
Here, $\delta_1 < 1/3\,(\delta_0 + \delta_0 + \delta_0)$, since $d_0 < \delta_0$ & $e_0 < \delta_0$
i.e. $\delta_1 < \delta_0$
In general, $A_{n+1} = \mu + \delta_{n+1}$ where $\delta_{n+1} = 1/3\,(\delta_n + d_n + e_n)$
Now, $\delta_{n+1} = 1/3(\delta_n + d_n + e_n) < 1/3\,(\delta_n + \delta_n + \delta_n)$, since $d_n < \delta_n$ & $e_n < \delta_n$
i.e. $\delta_{n+1} < \delta_n$
This implies that the value of A_n moves to be closer and closer to μ as n goes to be larger and larger.
Thus, the converging point (value) of the sequence $\{A_n\}$ is very close to μ.
Again the three sequences $\{A_n\}$, $\{G_n\}$ & $\{H_n\}$ converge to the same point (value).
Therefore, the *AGHM* of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

is that value which is very closet to μ.

IV. NUMERICAL EXAMPLE: APPLICATION TO NUMERICAL DATA

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

A. Annual Maximum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the values (in Degree Celsius) of *AM*, *GM* & *HM* have been found as follows:

$$AM = 37.2093023255814 \ ,$$
$$GM = 37.1922871485760$$
$$\& \ HM = 37.17539890356262$$

[61 ,62 , 63 , 64].
Here the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Evaluation of Value of μ (the central tendency of annual maximum)
Let us write

$$A_0 = 7.363414634146341463414634146341463415,$$
$$G_0 = 7.259717619457618560870961635129 7$$
$$\& \ H_0 = 7.154393380282352520984974470756 9$$

In this case the iterations give the values which are given in the following table (**Table – 1**):

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Table – 1

n	**Term of Sequence $\{A_n\}$, $\{G_n\}$ & $\{H_n\}$**	**Value**
0	A_0	37.2093023255814
	G_0	37.1922871485760 76781925812747586
	H_0	37.175398903562627634836294491501
1	A_1	37.192329459240034805587369079696
	G_1	37.192326883784773277226087433254
	H_1	37.192324308332441617854668614447
2	A_2	37.192326883785749900222708375799
	G_2	37.192326883785690452815011296956
	H_2	37.192326883785631005407314219677
3	A_3	37.192326883785690452815011297477
	G_3	37.192326883785690452815011297446
	H_3	37.192326883785690452815011297413
4	A_4	37.192326883785690452815011297445
	G_4	37.192326883785690452815011297445
	H_4	37.192326883785690452815011297441
5	A_5	37.192326883785690452815011297444
	G_5	37.192326883785690452815011297444
	H_5	37.192326883785690452815011297441

The digits in A_n , G_n & H_n which are agreed, have been underlined in the above table.
The *AGHM* of the observed values given in the above table
is the common limit of these two sequences which is
37.192326883785690452815011297441
Thus the value of μ, the central tendency of annual maximum of surface air temperature at Guwahati, obtained by *AGHM*, is 37.192326883785690452815011297441 Degree Celsius.

B. Annual Minimum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the values (in Degree Celsius) of *AM*, *GM* & *HM* have been found as follows:

$$AM = 7.3634146341463414634146341463415 \ ,$$
$$GM = 7.2597176194576185608709616351297$$
$$\& \ HM = 7.1543933802823525209849744707569$$

[61 ,62 , 63 , 64].
In this case also, the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Determination of Value of μ (the central tendency of annual minimum)
In this case the iterations give the values which are given in the following table (**Table – 2**):

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Table – 2

n	Term of Sequence $\{A_n\}$, $\{G_n\}$ & $\{H_n\}$	Value
0	A_0	7.3634146341463414634146341463415
	G_0	7.2597176194576185608709616351297
	H_0	7.1543933802823525209849744707569
1	A_1	7.259175211295437515090190084076
	G_1	7.2586735811751601863075880738685
	H_1	7.2581719135850851422025166245462
2	A_2	7.2586735686852276145334315941636
	G_2	7.2586735571288657683004939856174
	H_2	7.258673545572503902182631426563
3	A_3	7.2586735571288657616721856687813
	G_3	7.2586735571288657555393158774539
	H_3	7.258673557128865749406446086127
4	A_4	7.2586735571288657555393158774541
	G_4	7.2586735571288657555393158774539
	H_4	7.2586735571288657555393158774538
5	A_5	7.2586735571288657555393158774539
	G_5	7.2586735571288657555393158774538
	H_5	7.2586735571288657555393158774538
	A_6	7.2586735571288657555393158774538
	G_6	7.2586735571288657555393158774538
	H_6	7.2586735571288657555393158774538

The digits in A_n, G_n & H_n which are agreed, have been underlined in the above table.
The *AGHM* of the observed values given in the above table
is the common limit of these three sequences which is
7.2586735571288657555393158774538
Thus the value of μ, the central tendency of annual minimum of surface air temperature at Guwahati, obtained by *AGHM*, is 7.2586735571288657555393158774538 Degree Celsius.

V. CONCLUSION

In the methods developed so far, for determining the value of parameter from observed data containing the parameter itself and random error, a finite set of observed data may not be sufficient for obtaining the value of the parameter. However, the applications of *AGM*, *AHM* & *GHM* [61, 62, 63, 64] can yield the value of the parameter even if the set of observed data is small. Similarly, the application of *AGHM* can also yield the value of the parameter even if the set of observed data is small. The application of *AGHM* has also the same merit as that of the applications of *AGM*, *AHM* & *GHM* in determining the value of parameter in such situation.

It seems that the application of *AGHM* can yield the value which is closest to the actual value of the parameter in this situation among the respective values yielded by *AM*, *GM*, *HM*, *AGM*, *AHM*, *GHM* & *AGHM* respectively. It is thus a problem for the researchers, at this stage, to make study on finding out the information on whether this is true.

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(Dr. Dhritikesh Chakrabarty with his mother Late Kanak Prova Chakrabarty)

Dr. Dhritikesh Chakrabarty joined the Department of Statistics of Handique Girls' College, Gauhati University, as a Lecturer on December 09, 1987 and has been serving the institution continuously since then. Currently he is in the position of Associate Professor (& Ex Head) of the same Department of the same College. He had also been serving the

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