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# Construction of a mathematical model of a variable frequency electric pump drive

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**ABSTRACT:** The work shows that the best option for regulating the performance of a pumping unit is to control the speed of the motor. Further presented a mathematical model of a frequency-controlled electric drive of an induction motor, which allows one to study energy processes with scalar frequency control and determine the influence of the supply frequency and slip on the operation mode of the induction motor.

**KEY WORDS:** Pump, Induction Motor, Frequency Control, Mathematical Model, Energy Losses, Equivalent Circuit.

## I. INTRODUCTION

In connection with the depletion of non-renewable energy resources and an increase in their cost, the problem of energy conservation is currently becoming increasingly important.

The development of energy-saving policies is possible only on the basis of the widespread introduction of new technology with the regulation and automation of production processes. One of the most important sectors in the world is water management, in which pumped plants are the most energy-intensive process.

The main task of increasing the energy efficiency of pumping units is to improve the technological process using modern scientific and technical achievements, which are also associated with increased productivity and easier work for maintenance personnel.

Despite a fairly large number of studies on this issue, the task of energy conservation in the technology of regulation and control of pumping units is not sufficiently developed. This is because the object of study - pumping units - is a complex dynamic system with its inherent properties. Therefore, the formulation and solution of scientific and scientific-technical problems in this area are extremely relevant, and are important.

As you know, the energy loss in the process depends on the flow rate determined by the technology requirement and the pressure loss on the equipment of the pumping unit, which are determined by the hydraulic resistance of the circuit elements. To organize a technological process with minimal energy losses, it is necessary, first of all, to reduce the pressure loss.

This nature of the relationship of the parameters requires the installation of throttle control elements in the system - control valves (pressure valves) or the use of an adjustable electric drive. Throttle control of pump performance is aimed at solving technological problems and practically does not take into account the energy aspects of pulp transport. As our calculations showed, energy losses during throttle control of pump performance during the process become quite large, reaching 45 percent or more of the unit's rated power.

**The main disadvantages of regulating the operation of pumping units by throttling the flow are [1, 2]:**

- loss of electricity in the pumping unit;
- increased wear of pumps, valves, electric motors;
- low efficiency of the pumping unit;
- the need for constant monitoring by duty personnel.

## II. RELATED WORK

This paper [3] presents a development of a model of a set of multistage centrifugal electro pumps including two 4 stage stainless steel centrifugal pumps, each coupled to a 4 kW three-phase induction motor, connected to a hydraulic application running under two control strategies including constant speed and variable speed methods. This paper [4] derives the mathematical models of doubly fed adjustable-speed pumped storage units to be utilized in the power

system analysis. It adopts the improved induction machine model with ac excitation on the wire-wound rotor as well as the field-oriented control theory for the study and analysis of DFASPSU models. This paper [5] gives some considerations concerning modeling of doubly-fed machines for analytical studies, especially on long term dynamics of a power system in the time range of beyond 10 seconds in which the doubly-fed machines are operating and being controlled. In addition the mathematical models are briefly presented first for short-term analysis (transient stability and dynamic stability analyses, up to 10 seconds) and then for long term analysis (frequency control analysis). This paper [6] presents application of differential equations in modelling and simulation tests of water supply network pumps drive system together with powered water supply pipelines. The description is preceded with an analysis of practically applicable mathematical models representing drive systems with induction motors and PWM converter. In paper [7] shows solving the problem of energy saving is possible when improving a variable speed electric drive based on induction motors which should be designed and manufactured for special energy-saving technologies. Here, the energy-saving effectiveness evaluation mathematical model has been presented. The possibility of improvement of the induction motor without cross-section geometry changing has been considered on the proposed model basis.

### III. PUMP PERFORMANCE REGULATION

With the advent of a reliable controlled AC electric drive, prerequisites have been created for the development of a fundamentally new technology with smooth regulation of the operating parameters of the pumping unit without unproductive energy costs and with wide opportunities to improve the accuracy and efficiency of the technological criteria for the operation of supply systems. At the same time, the characteristics of pipelines become the geometrical place of the operating points of the pump installation, and not the characteristics of the pumps as in the case of regulating the supply of pumping units with a constant speed (Fig. 1) [1-2, 8-9].

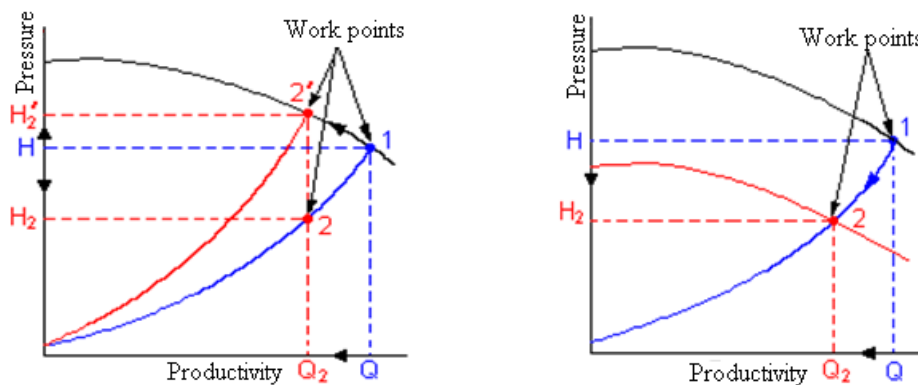


Fig. 1. The operating points of the pumps when regulating performance by throttling and changing the speed.

We can notice that during throttling, the energy of the flow of a substance held back by a valve or valve is simply lost without doing any useful work. The use of an adjustable electric drive of the pump unit will allow you to simply set the required pressure or flow rate, which will provide not only energy savings, but also a reduction in water losses.

In this regard, we consider the basic methods for controlling the speed of electric drives with induction motors, which consist of two groups:

- a) parametric methods (changing the active resistance of the rotor, the total resistance of the stator or rotor, switching the number of pairs
- b) regulation when the engine is powered from a separate energy source (engine power from a variable frequency source, cascade installations with the introduction of additional emf into the secondary circuit of the engine).

Since the engine speed is determined by the expression

$$n_2 = n_1(1 - s) = \frac{60 f_1}{p} \cdot (1 - s), \quad (1)$$

then the rotation speed can be regulated in three ways: by changing the number of pole pairs ( $P$ ), by changing the slip, and by changing the frequency ( $f_1$ ) of the supply network.

As our studies, have shown, as well as the works [10-14], a frequency-controlled electric drive is the most rational means of controlling the speed of an induction motor, in particular, regulating the performance of a pump installation.

With the frequency regulation of the speed of induction motors of pulp pumping pumps, a number of problems arise related to both the behavior of the motor and the choice of the law of frequency regulation [2]. In this case, the control of an induction squirrel-cage motor is carried out by changing the amplitude and frequency of the supply voltage. When developing a system for the frequency control of an induction motor of a pulp pump, and taking into account the main dependences of the motor operation and its parameters, it is necessary to choose a control law that, under the conditions of the pump drive, will ensure stable operation of the motor in a given control range with the least possible losses. In accordance with the chosen law, it is necessary to determine the simplest scheme that allows this law to be implemented.

**IV. FREQUENCY CONTROL MODEL OF ON INDUCTION MOTOR**

To obtain the basic ratios of an induction motor with frequency regulation, we use the T-shaped equivalent circuit (Fig. 2.). The following well-known assumptions should be made: steel saturation is not taken into account; the effect of current displacement and the active resistance of the magnetization circuit are not taken into account; phase windings of the machine are considered symmetrical; a symmetrical sinusoidal voltage is applied to the stator.

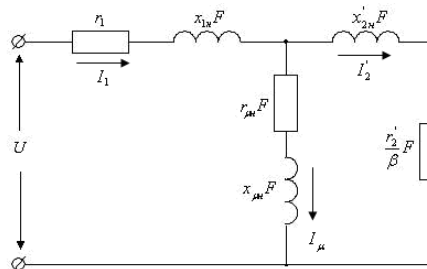


Fig. 2. Induction motor phase equivalent circuit

In the equivalent circuit of an induction motor, the following notation:  $r_1$  and  $r_2'$  - stator winding resistance and reduced rotor winding phase resistance;  $x_{1s}'$ ,  $x_{2r}'$  and  $x_{\mu\mu}'$  - rated inductive resistances of the stator winding, reduced resistance of the rotor winding and the magnetizing circuit;  $\sigma = 1 + \frac{x_{\mu\mu}'^2}{x_s x_r}$  - dispersion coefficient at rated frequency;  $x_s = x_{1s}' + x_{\mu\mu}'$  - stator loop inductance at rated frequency;  $x_r = x_{2r}' + x_{\mu\mu}'$  - rotor loop inductance at rated frequency;  $f$  и  $f_n$  - actual and rated frequency of stator current;  $U$  and  $U_n$  - actual and nominal value of voltage supplied to the stator;  $I$  and  $I_1$  - actual and nominal value of stator current;  $I_2'$  and  $I_\mu$  - reduced rotor current and magnetization current;  $F = f / f_n$  - relative control frequency;  $\gamma = U / U_n$  - relative voltage applied to the stator;  $i = I / I_1$  - relative stator current;  $\beta$  - absolute slip parameter;  $s$  - relative slip;  $p$  - the number of pairs of motor poles;  $m$  - the number of phases of the stator of an induction motor.

Inverters controlled by current or voltage are distinguished, depending on the frequency converter circuit, in particular on the construction of an intermediate DC circuit, as well as on the control method.

Based on the equivalent circuit, we find the active, reactive and impedances of a frequency-controlled induction motor

$$r_o = r_1 + \frac{\frac{r_2'}{\beta} x_{\mu\mu}'^2 F}{\left[ \frac{r_2'}{\beta^2} \right]^2 + x_{2r}'^2}, \tag{2}$$

$$x_o = \frac{\left(\frac{r_2'}{\beta^2}\right)^2 + r_2'^2 \sigma}{\left(\frac{r_2'}{\beta^2}\right)^2 + x_r^2} x_s F, \quad (3)$$

$$Z = \sqrt{r_o'^2 + x_o'^2}. \quad (4)$$

The critical moment of the motor

$$M_{kf} = \frac{U_n^2 pm x_\mu^2 \left(\frac{\gamma}{F}\right)^2}{2\pi f_n} \frac{1}{\sqrt{\left(\frac{r_1^2}{F^2} + x_s^2\right)\left(\frac{r_1^2}{F^2} + x_s^2 \sigma^2\right) + \frac{r_1}{F} x_s (1-\sigma)}}, \quad (5)$$

critical absolute motor slip in general

$$\beta_k = \frac{r_2'}{x_r} \frac{\sqrt{\left(\frac{r_1}{F}\right)^2 + x_s^2}}{\sqrt{\left(\frac{r_1}{F}\right)^2 + x_s^2 \sigma^2}}, \quad (6)$$

correction factor

$$\varepsilon = \frac{r_2'}{x_r} \frac{\left(\frac{r_1}{F}\right)^2 x_s^2 (1-\sigma)}{\sqrt{\left(\frac{r_1^2}{F^2} + x_s^2\right)\left(\frac{r_1^2}{F^2} + x_s^2 \sigma^2\right)}}, \quad (7)$$

as well as

$$K_1 = \frac{pm}{2\pi f_{1n}}. \quad (8)$$

The basic values of the engine recorded in general form are functions of the parameters of the equivalent circuit and the adjustable coordinates  $\gamma$ ,  $\beta$ ,  $F$  and  $i$ . The formation of mechanical induction motors with frequency control is subordinated to the tasks of ensuring the required overload capacity and stiffness of characteristics in the entire range of speed control [3].

The determining factors for an induction motor are losses in the stator and rotor copper, losses in the stator steel due to hysteresis and eddy currents, as well as mechanical losses.

Stator copper losses

$$P_{m1} = m r_1 I_1^2 = m r_1 U_n^2 \left(\frac{\gamma}{F}\right)^2 \frac{\left(\frac{r_2'}{\beta}\right)^2 + x_r^2}{D^2}, \quad (9)$$

losses in copper rotor

$$P_{m2} = m r_2' I_2'^2 = m r_2' U_n^2 \left(\frac{\gamma}{F}\right)^2 \frac{x_\mu}{D^2}. \quad (10)$$

Total copper losses

$$p_{m\Sigma} = mU_n^2 \left(\frac{\gamma}{F}\right)^2 \frac{1}{D^2} \left[ \left(\frac{r_2'}{\beta}\right)^2 r_1 + r_1 x_r'^2 + r_2' x_\mu'^2 \right]. \quad (11)$$

St  
eel losses

$$p_c = \frac{U_n^2 x_\mu'^2 \gamma^2}{c_1^2 f_{1n} F} \frac{\left(\frac{r_2'}{\beta}\right)^2 + x_2'^2}{D^2} [k_r + k_{em} f_{1n} F], \quad (12)$$

where the index "r" and "em" denote the values related to hysteresis and eddy currents.

The total power losses in copper and steel from (1) and (2) will be

$$\Delta P_\Sigma = U_n^2 \gamma^2 \frac{x_\mu'^2 r_2'^2 k_r + x_\mu'^2 r_2'^2 k_{em} f_{1n} F + x_\mu'^2 x_2'^2 \beta^2 k_r + x_\mu'^2 x_2'^2 \beta^2 k_{em} f_{1n} F + \dots}{c_1^2 f_{1n} F} \rightarrow \dots \rightarrow \frac{+ mc_1^2 f_{1n} r_2'^2 r_1 + mc_1^2 f_{1n} r_2'^2 r_1 x_r'^2 \beta^2 + mc_1^2 f_{1n} r_2' x_\mu'^2 \beta^2}{\beta^2 D^2}. \quad (13)$$

In the steady state, the total loss in the engine depends on the load, magnetic flux and frequency and can be expressed in relative units by the formula [2]

$$\Delta P = \rho i_2^2 + \theta \varphi^2 + a_3 i_\mu^2 + a_{mex} F^n, \quad (14)$$

where  $\rho$  is the coefficient of variable losses in the stator and rotor copper from the reduced rotor current,  $\rho = 0,28 \div 0,32$ ;  $\theta = k_r F + k_{em} F^2$  - loss coefficient in stator steel from hysteresis and eddy currents;  $k_r + k_{em} = 0,19 \div 0,22$ ; the ratio  $k_r/k_{em}$  at 50 Hz is 0.12 - 0.6 - lower values refer to sheets of high alloy steel with a thickness of 0.35 mm, large ones to light alloy steel with a thickness of 0.5 mm;  $a_3$  is the fraction of electric losses caused by the magnetizing current at rated mode,  $a_3 = 0,02 \div 0,06$ ;  $a_{mex} = \Delta P_{mex} / \Delta P_n$  - ratio of mechanical losses to nominal losses at rated speed;  $n = 1 \div 1,5$ ;  $i_2 = I_2 / I_{2n}$  - rotor relative current;  $\varphi = \Psi / \Psi_n$  - relative flow;  $i_\mu = I_\mu / I_{\mu n}$  - relative magnetization current.

Efficiency of an induction motor

$$\eta = 1 - \frac{\Delta P}{\mu v + \Delta P}, \quad (15)$$

where  $\mu = M / M_n$  - relative torque of an induction motor;  $v = \omega / \omega_n$  - relative speed.

The power factor obtained from the equivalent circuit has the form

$$\cos \varphi = \left[ 1 + \frac{\left[ \frac{r_2'^2}{\beta^2} F + \frac{r_2'}{\beta^2} F \frac{x_1}{x_\mu} + \dots \right]}{\left[ \frac{r_2'^2}{\beta^2} \frac{r_1}{x_\mu} - \frac{r_2' x_2'}{\beta x_\mu} F + x_\mu \frac{r_2'}{\beta} F + \frac{r_2'}{\beta} F x_1 + x_\mu r_1 + \dots \right]} \right]^{-1/2} \rightarrow \dots \rightarrow \frac{\left[ x_2' F + \frac{x_2'^2}{x_\mu} F \right]^2}{\left[ 2r_1 x_2' + x_2' \frac{r_2'}{\beta} F + x_2' \frac{r_2'}{\beta} F \frac{x_1}{x_\mu} + r_1 \frac{x_2'^2}{x_\mu} \right]^2}. \quad (16)$$



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## V.CONCLUSION AND FUTURE WORK

In order to thus, reducing the rotational speed in accordance with the technological load allows not only to save energy consumption by eliminating hydraulic losses, but also to obtain an economic effect by increasing the efficiency of the pump itself - converting mechanical energy into hydraulic energy.

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