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Coc-b-closed and Coc-b-open Functions

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ABSTRACT: study some separation properties using coc-b-open sets, that are similar to the well know separation axioms, and we prove some relations between them..

KEYWORDS: coc-b-opensets,separationaxioms,coc-b-closedsets

I.INTRODUCTION

In this work, we have studied important types in topological spaces namely,coc-b-connected,coc-b-compact,coc-b-lindelof,coc-b-paracompact and spaces .

In [5] D. Andrijevic introduced the concept of b-open set in topology. In [6] E Ekici and M .Caldas introduced the concept of b-lindel of space. In [24] T .Noiri, A.AL-Omari and Mohd . Salmi Md. Noorani studied some important generalization of b-lindelof space

well known that the effects of the investigation of properties of closed bounded sets, In [19] R. Engleking studied the characterizations of continuity provided that the continuous image of connected space is connected . Several properties of connected space in [18,11].

II. THE MAIN RESULT

Definition (2.1.1):

Let $f: X \rightarrow Y$ be a function of a space X into a space Y then f is called an coc-b-closed function if $f(A)$ is an coc-b-closed set in Y for every closed set A in X .

Example (2.1.2):

The constant function is an coc-b-closed .

Proposition (2.1.3): A function $f: X \rightarrow Y$ is an coc-b-closed iff $\overline{f(A)}^{b-coc} \subseteq f(\overline{A})$ for all $A \subseteq X$.

Proof:

Suppose that $f: X \rightarrow Y$ is an coc-b-closed function , let $A \subseteq X$, since $\overline{A}$ is closed set in X .

Then $f(\overline{A})$ is coc-b-closed set in Y, since $A \subseteq \overline{A}$ then $f(A) \subseteq f(\overline{A})$ hence $\overline{f(A)}^{b-coc} \subseteq \overline{f(\overline{A})}^{b-coc}$.

But $\overline{f(\overline{A})}^{b-coc} = f(\overline{A})$. Therefor $\overline{f(A)}^{b-coc} \subseteq f(\overline{A})$.

Conversely, let F be a closed set of X , then $F = \overline{F}$ by hypothesis $\overline{f(F)}^{b-coc} \subseteq f(\overline{F})$ hence $\overline{f(F)}^{b-coc} \subseteq f(F)$, thus $f(F)$ is an coc-b-closed set in Y . Therefore $f: X \rightarrow Y$ is an coc-b-closed function.

Proposition (2.1.4): Let $f: (X, \tau) \rightarrow (Y, \tau')$ be a function and $f(\overline{A}) = \overline{f(A)}^{b-coc}$ for each set A of X . Then f is coc-b-closed and coc-b-continuous function .

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Proof:
By proposition (2.1.3) f is an coc-b-closed function .
Now to prove that f is coc-b-continuous . Let $F \subseteq X$, then $f(\overline{F}) = \overline{f(F)}^{b-coc}$.
By proposition (1.1.14) (3) $\overline{f(F)}^{b-coc} \subseteq \overline{f(F)}$. Hence $f(\overline{F}) \subseteq \overline{f(F)}$,then f is coc-b-continuous function .

Proposition (2.1.5):
Let $f: X \to Y$ and $g: Y \to Z$ be function then if f is a closed and g is an coc-b-closed then $g \circ f$ is a coc-b-closed function.
Proof: Clear

Proposition (2.1.6):
Let $f: X \to Y$ be a coc-b-closed function .Then the restriction of f to a closed subset F of X is an coc-b-closed of F into Y.
Proof:
Since F is a closed subset in X .Then the inclusion function $i/_F: F \to X$ is a closed function . Since $f: X \to Y$ is an coc-b-closed function . Then by proposition (2.1.5) $f \circ i/_F: F \to Y$ is an coc-b-closed function . But $f \circ i/_F = f/_F$ is an coc-b-closed function.

Definition (2.1.7):
Let $f: X \to Y$ be a function of a space X into a space Y then f is called an coc-b-open function if $f(A)$ is an coc-b-open set in Y for every open set A in X .

Example (2.1.8):
Let $X = \{1,2,3\}$ and $Y = \{4,5\}$, $\tau = \{X, \emptyset, \{1\}\}$, $\tau' = \{\emptyset, Y, \{4\}\}$,$f: (X,\tau) \to (Y,\tau')$ $\ni$ $f(1) = f(2) = 4$ $f(3) = 5$. The open sets in X is $\emptyset, X, \{1\}$.
$f(\emptyset) = \emptyset$, $\emptyset$ is coc-b-open set in Y. $f(X) = \{f(1), f(2), f(3)\} = \{4,5\} = Y$ is coc-b-open set in Y . $f(\{1\}) = \{f(1)\} = \{4\}$, $\{4\}$ is coc-b-open set in Y.
Then f coc-b-open function .

Proposition (2.1.9):
A function $f: X \to Y$ is coc-b-open if and only if $f(A^\circ) \subseteq (f(A))^{\circ b-coc}$ for all $A \subseteq X$.
Proof:
Suppose that $f: X \to Y$ is an coc-b-open function, let $A \subseteq X$, since A° open in X . Then $f(A^\circ)$ coc-b-open in Y, since $A^\circ \subseteq A$ then $f(A^\circ) \subseteq f(A)$ hence $\left(f(A^\circ)\right)^{\circ b-coc} \subseteq (f(A))^{\circ b-coc}$ but $\left(f(A^\circ)\right)^{\circ b-coc} = f(A^\circ)$. Then $f(A^\circ) \subseteq (f(A))^{\circ b-coc}$.
Conversely, let A be open in X , then $A^\circ = A$ since $f(A^\circ) \subseteq (f(A))^{\circ b-coc}$ then $f(A) \subseteq (f(A))^{\circ b-coc}$ then $f(A) = (f(A))^{\circ b-coc}$. Hence $f(A)$ is coc-b-open in Y . Therefore $f: X \to Y$ is an coc-b-open function .

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Vol. 6, Issue 10 , October 2019

Remark (2.1.10):

Every closed (open) function is an coc-b-closed (an coc-b-open) function, but the converse not true in general as the following example show:
Let $X = \{1,2,3\}$, $Y = \{4,5\}$, $\tau = \{\emptyset, X, \{3\}\}$ be a topology on X and τ' be indiscrete topology on Y . Let $f: X \to Y$ be a function defined by $f(1) = f(2) = 4$, $f(3) = 5$. Then f is an coc-b-closed (an coc-b-open) function , but is not a closed (an open) function .

Proposition(2.1.11):

A bijective function from a topological (X, τ) into a topological $(Y, \tau')f: X \to Y$ is an coc-b-closed function iff f is an coc-b-open function.
Proof:
By using the fact $f(U^c) = \left(f(U)\right)^c$ $\forall\, U \subseteq X$ the proof is complete.

Proposition (2.1.12):

Let $f: X \to Y$ be bijective function from a topological space (X, τ) into a topological space (Y, τ') .Then :
(1) f is an coc-b-open function ifff^{-1} is an coc-b-continuous .
(2) f is an coc-b-closed function iff f^{-1} is an coc-b-continuous .
Proof:
By using the fact $(f^{-1})^{-1}(A) = f(A)$ $\forall$ $U \subseteq X$ the proof is complete .

Definition (2.1.13):

Let X and Y are topological space .Then a function $f: X \to Y$ is called an coc-b-homeomorphism if:
(1) f is bijective .
(2) f is an coc-b-continuous .
(3) f is an coc-b-closed (coc-b-open).
It is clear that every homeomorphism is an coc-b-homeomorphism .

Now , we introduce the definition of coc'-b-closed (coc'-b-open) function and some propositions about it .

Definition (2.1.14):

Let $f: X \to Y$ be a function of a topological space (X, τ) into a topological space (Y, τ') then :
(1)f is called an coc'-b-closed function if $f(A)$ is an coc-b-closed set n Y for every coc-b-closed set A in X .
(2)f is called an coc'-b-open function if $f(A)$ is an coc-b-open set in Y for every coc-b-open set A in X .

Example (2.1.15):

(1)The constant function is an coc'-b-closed function .
(2) Let X and Y be finite sets and f be function from a space X into a space Y .Then f is an coc'-b-open function .

ISSN: 2350-0328
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Vol. 6, Issue 10 , October 2019

Proposition (2.1.16):
A function $f:(X,\tau)\to(Y,\tau')$ is coc'-b-closed iff $\overline{f(A)}^{b-coc} \subseteq f\left(\overline{A}^{b-coc}\right)$ for all $A \subseteq X$.
Proof:
Suppose that $f:X\to Y$ is an coc'-b-closed function .
Let $A \subseteq X$,since $\overline{A}^{b-coc}$ is coc-b-closed set in X . Then $f\left(\overline{A}^{b-coc}\right)$ is coc-b-closed set in Y , since $f(A) \subseteq f\left(\overline{A}^{b-coc}\right)$ then $\overline{f(A)}^{b-coc} \subseteq f\left(\overline{A}^{b-coc}\right)$.
Conversely, let A be a coc-b-closed set of X ,then $A=\overline{A}^{b-coc}$ hypothesis $\overline{f(A)}^{b-coc} \subseteq f\left(\overline{A}^{b-coc}\right)$,hence $\overline{f(A)}^{b-coc} \subseteq f(A)$, thus $f(A)$ is an coc-b-closed set in Y .Therefore $f:X\to Y$ is an coc'-b-closed function.

Proposition (2.1.17):

Let X ,Y and Z be space and $f:X\to Y$, $g:Y\to Z$ be function then:
(1) If f and g are coc'-b-closed function ,then $g\circ f$ is coc'-b-closed function .
(2) If $g\circ f$ is coc'-b-closed function ,f is coc'-b-continuous and onto ,then g is coc'-b-closed function.
(3) If $g\circ f$ is coc'-b-closed function ,g is coc'-b-continuous and one-to-one ,then f is coc'-b-closed function.
Proof:
(1) Let F be a coc-b-closed set in X ,then $f(F)$ is an coc-b-closed set in Y .Thus $g\big(f(F)\big)$ is an coc-b-closed set in Z .But $(g\circ f)(F)=g\big(f(F)\big)$. Hence $g\circ f$ is coc'-b-closed function .
(2) Let F be a coc-b-closed set in Y ,then by proposition (1.2.10) $f^{-1}(F)$ is coc-b-closed set in X. Thus $g\circ f\big(f^{-1}(F)\big)$is coc-b-closed set in Z , since f is onto then $g\circ f\big(f^{-1}(F)\big)=g(F)$, hence $g(F)$ is coc-b-closed set in Z .Thus g is coc'-b-closed .
(3) Let F be a coc-b-closed set in X , then $g\circ f(F)$ is coc-b-closed set in Z ,then by proposition (1.2.10) $g^{-1}\big(g\circ f(F)\big)$ is coc-b-closed set in Y. Since g is one-to-one ,then $g^{-1}\big(g\circ f(F)\big)=f(F)$,hence $f(F)$is coc-b-closed set in Y. Then f is coc'-b-closed .

Proposition (2.1.18):
Let X ,Y and Z be space and $f:X\to Y$,$g:Y\to Z$ be function. Then:
(1) If f and g are coc'-b-open function . Then $g\circ f$ is coc'-b-open function .
(2) If $g\circ f$ is coc'-b-open function , f is coc'-b-continuous and onto then g is coc'-b-open .
(3) If $g\circ f$ is coc'-b-open function , g is coc'-b-continuous and one-to-one, then f is coc'-b-open .
Proof: Similar to proof proposition (2.1.17).

Proposition (2.1.19):
A function $f:(X,\tau)\to(Y,\tau')$ is coc'-b-open function if and only if $f(A^{\circ b-coc}) \subseteq \big(f(A)\big)^{\circ b-coc}$ for all $A \subseteq X$.

ISSN: 2350-0328
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Vol. 6, Issue 10 , October 2019

Proof:
Suppose that $f: X \to Y$ is an coc'-b-open function. Let $A \subseteq X$, since $A^{\circ b-coc}$ is coc-b-open in X .Then $f(A^{\circ b-coc})$ is coc-b-open in Y . hence $f(A^{\circ b-coc}) = \left(f(A^{\circ b-coc})\right)^{\circ b-coc} \subseteq (f(A))^{\circ b-coc}$.
Conversely, let A coc-b-open in X ,since $f(A^{\circ b-coc}) \subseteq (f(A))^{\circ b-coc}$,then $f(A) \subseteq (f(A))^{\circ b-coc}$,then $f(A) = (f(A))^{\circ b-coc}$ .Hence $f(A)$ is coc-b-open in Y. Therefore $f: X \to Y$ is an coc'-b-open function.

Proposition(2.1.20):

A bijective function $f: X \to Y$ is an coc'-b-closed function if and only if f is an coc'-b-open function.
Proof:
By using the fact $f(U^c) = (f(U))^c \ \forall \ U \subseteq X$ .The proof is complete.

Proposition (2.1.21):

Let $f: X \to Y$ be bijective function from a space X into a space Y then:
(1) f is an coc'-b-open function iff f^{-1} is an coc'-b-continuous.
(2)f is an coc'-b-closed function iff $f^{-1}$is an coc'-b-continuous.
Proof:
By using the fact $(f^{-1})^{-1}(A) = f(A) \ \ \forall A \subseteq X$ .The proof is complete.

Definition (2.1.22):

Let X and Y be space then a function $f: X \to Y$ is called an coc'-b-homeomorphism if:
(1)f is bijective .
(2) f is an coc'-b-continuous .
(3) f is an coc'-b-closed (coc'-b-open).
It is clear that every coc'-b-homeomorphism is an coc-b-homeomorphism .

Proposition (2.1.23):

Let $f: X \to Y$ be bijective function , then the following statements are equivalent:
(1) f is coc'-b-homeomorphism .
(2) $f\left(\overline{A}^{b-coc}\right) = \overline{f(A)}^{b-coc} \ \ \forall \ A \subseteq X$.
Proof:
(1)→(2) obvious

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Vol. 6, Issue 10 , October 2019

(2)→(1) Let $f\left(\overline{A}^{b-coc}\right) = \overline{f(A)}^{b-coc}$ since $f\left(\overline{A}^{b-coc}\right) \subseteq \overline{f(A)}^{b-coc}$ by proposition (1.2.11) then f is coc'-b-continuous and since $\because \overline{f(A)}^{b-coc} \subseteq f\left(\overline{A}^{b-coc}\right)$ by proposition (2.1.16) then f is coc'-b-closed and since f bijective then f is coc'-b-homeomorphism .

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