



Processing wave transformation using the spline wavelet method

J.U. Juraev

Assistant of Samarkand State University, Uzbekistan Samarkand

ABSTRACT: The article is devoted to the application of the spline wavelet transform to sequences of strongly correlated random variables.

The data processing technique using spline wavelets allows weakening their correlation, which was shown in previously published works [1,2]. Also, studies were previously conducted that prove the effectiveness of applying a linear discrete wavelet transform (DWT) to sequences of random variables on real data in the framework of the problems of queuing theory. This article examines the question of the efficiency of using quadratic fast DWT based on splines, formulates the corresponding theorem and provides data from numerical experiments.

KEYWORDS: spline wavelets; decorrelation; discrete wavelet transform; time series.

I. INTRODUCTION

It is known that for a Gaussian random process, the best decorrelation method is the transition to the Karunen – Loev basis [3]. But the high computational complexity and the lack of fast conversion algorithms makes it difficult to use in time series processing tasks. Today, the use of discrete wavelet transforms (DWT) is widely studied in research centers around the world. From the entire spectrum of wavelet bases, in this paper we study spline bases of varying degrees, as potentially the most effective in the framework of the problem of weakening the correlation of the data sequence.

II. SPLINE WAVELET FUNCTIONS

Let $[a, b]$ -be an arbitrary segment, $m \geq 1$ -be a natural number, n_0 -be an integer such, that $2^{n_0} < 2m + 1 < 2^{n_0+1}$ and k -be an integer such, that $2^k < 2m - 1$. Consider the family $\Delta = \{\Delta_n, n = n_0, n_0 + 1, \dots\}$ of partitions of a segment $[a, b]$ with a step $h = h_n = (b - a) / 2^n$. Denote by $S(\Delta_n, m, k)$ the set of splines the degree m of defect k , defined on the grid Δ_n . On each partition, we consider the space of splines $L_n = S(\Delta_n, m - 1, 1)$. Then for each $k \geq n_0$ space $S(\Delta_n, m - 1, 1)$ can be represented as a direct sum $L_k = L_{n_0} \otimes W_{n_0+1} \otimes W_{n_0+2} \otimes \dots \otimes W_k$, where W_k the orthogonal complement of space L_{k-1} is denoted by to space. The wavelet basis is obtained as the union of the basis in L_{n_0} and all bases in spaces $W_n, n_0 \leq n \leq k$.

For $i \geq 0$ such, that the entire segment $[x_i^{n-1}, x_{i+2m-1}^{n-1}]$ is contained in the $[a, b]$ function $\psi_{i,n}(x) \in W_n$, we will search by the formula

$$\psi_{i,n}(x) = \sum_{j=2i}^{2i+3m-2} \alpha_j \phi_{j,n-1}$$

Where $\phi_{j,n-1}$ is the normalized B-spline. Odds α_j are found from the condition

$$(\psi_{i,n}(x), \phi_{k,n-1}) = 0, k = i - m + 1, i - m + 2, \dots, i + 2m - 2$$

The totality of the constructed wavelet functions is obtained by shifting a single function according $\psi_{0,n}$ to the formula

$$\psi_{i,n}(x) = \psi_{0,n}(2^{n-n_0} x - i(b - a) / 2^{n_0-1})$$

The semi-orthogonal spline wavelets and the construction algorithm are described in $\psi_{i,n}(x)$ more detail in the works of I. Blatov. [four]. Figure 1 shows the graphs of the central spline wavelet functions for $m = 2, m = 3$ and $m = 4$

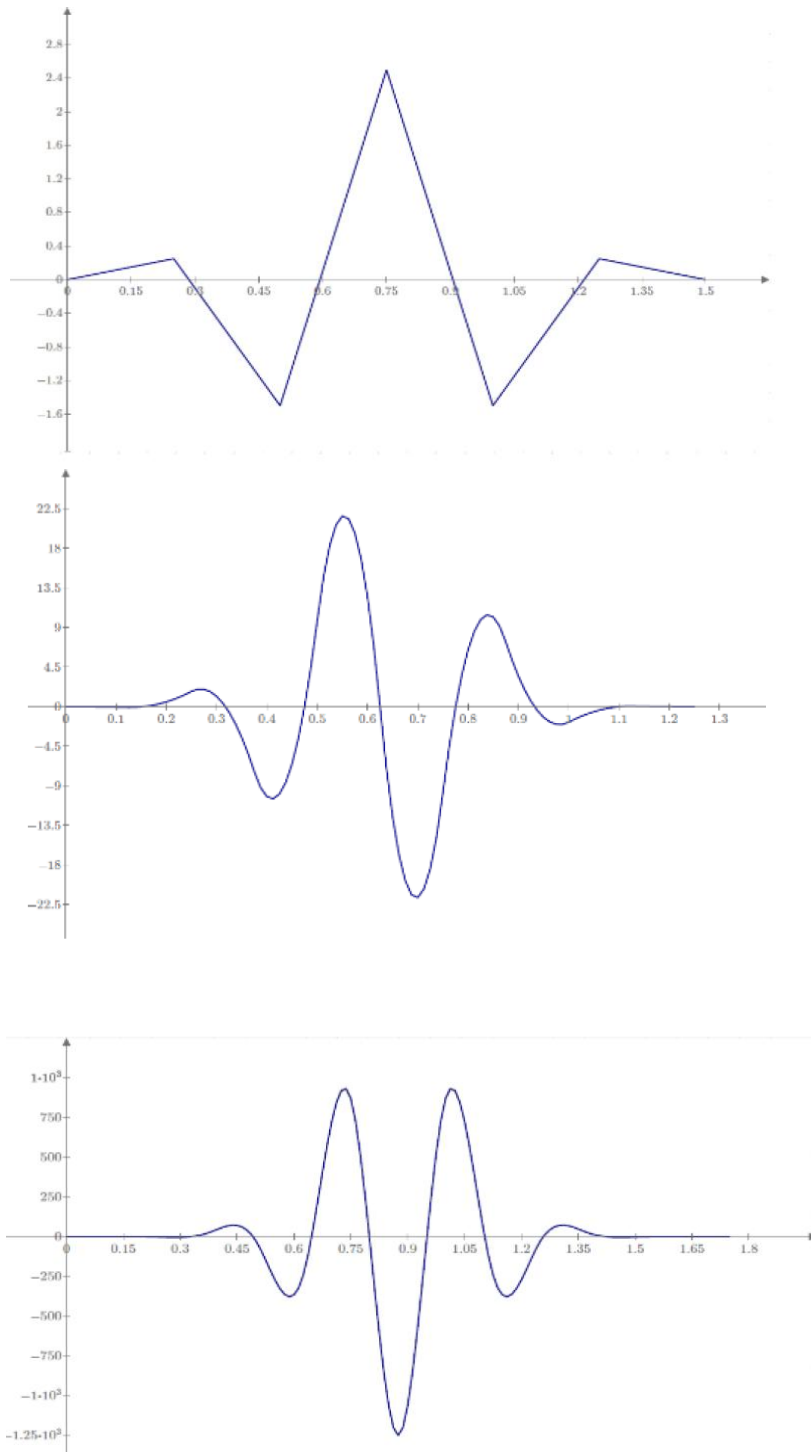


Fig.1 Spline wavelet functions - linear, quadratic and cubic (from left to right).

3. The concept of fast fiberboard based on splines The discrete wavelet transform is implemented by the Mall pyramidal algorithm [3]. Fast fiberboard is a linear transformation that processes a numerical vector of length N that is a multiple of some power of 2, converting it to another vector of the same length. A discrete wavelet transform is reversible and, in the general case, orthogonal. But for wavelet transforms in the class of finite orthogonal wavelets, there is no possibility of applying fast economical algorithms that exist for piecewise polynomial functions. And for piecewise polynomial functions there are no orthogonal systems with a finite number of supports. The use of finite semi-orthogonal spline wavelets makes it possible to avoid these shortcomings.

3.1. Brief algorithm

The direct conversion task is to find a set of coefficients

$$\{d_{0j}, -m+1 \leq j \leq 2^{n_0} - 1\} \cup \bigcup_{i=1}^{k-n_0} \{c_{ij}, -m+1 \leq j \leq 2^{n_0+i-1} - m\}$$

for a given function $f = \{f_{ij}\}, 0 \leq i \leq 2^k - 1, 1 \leq j \leq s$. The task of the inverse wavelet transform is to restore all the values of the $f_{ij}, 0 \leq i \leq 2^k - 1, 1 \leq j \leq s$. function $\{f_{ij}\} \in \tilde{S}(\Delta_k, m-1, 1)$ from a given set of coefficients, if

$$f = \sum_{j=-m+1}^{2^{n_0}-1} d_{0j} \phi_{j,n_0} + \sum_{i=1}^{k-n_0} \sum_{j=-m+1}^{2^{n_0+i-1}-m} c_{ij} \psi_{j,n_0+i}$$

The fast-fiberboard algorithm is described in more detail in the work of I. Blatov [4].

III. DECORRELATION PROPERTIES

Let $X = \{X_i\}, 0 \leq i \leq n, n = 2^k$ -be a sequence of random variables. For each implementation of the sequence, we denote by the $PX(t) \in S(\Delta, 2, 1), t \in [0, n]$ interpolation spline of the first degree (continuous polyline) constructed by points

$(t_i, X_i), 0 \leq i \leq n$, where $t_{i=i}$. Consider $PX(t)$ as a random process with $t \in [0, n]$. We $K(t, \tau), t, \tau \in [0, n]$ denote by its correlation function, and by the $K \in \{k_{ij}\}$ correlation matrix of the sequence X . Suppose that the correlation matrix has the X form $Y_i = (TPX)_i, 0 \leq i \leq n$, chara T cteristic of a self-similar process $S(\Delta, 2, 1)$.

Where The correlation matrix $\tilde{K} = \{\tilde{k}_{ij}\}, \tilde{k}_{ij} = \text{cov}(Y_i, Y_j), 0 \leq i, j \leq n$ of a sequence of random variables has the

form where is the Gram matrix $K = \{k_{ij}\}, k_{ij} = \text{cov}(X_i, X_j)$ of We introduce the matrix

$$\hat{K} = \{\hat{k}_{ij}, 0 \leq i, j \leq n\}, \hat{k}_{ij} = \text{cov}((PX, \chi_i), (PX, \chi_j)) = \int_0^n \int_0^n K(t, \tau) \chi_i(t) \chi_j(\tau) dt d\tau,$$

The $K(t, \tau) = \text{cov}(PX(t), PX(\tau))$.

correlation matrix of a sequence of Y random variables has the form $\tilde{K} = \Gamma^{-1} \hat{K} \Gamma^{-1}$, where is the Gram Γ matrix of the wavelet basis. Then the following theorem holds:

Theorem. For matrix \hat{K} elements, the estimates ($p \geq 2$)

$$\left| \hat{k}_{ij}^{pp+s} \right| \leq C \cdot 2^{(1-\alpha)(k-p)} \cdot 2^{-(3/2-\alpha)s} \frac{1}{\left(1 + \left| j - \frac{i}{2^s} \right| \right)^{2+\alpha}}, \left| \hat{k}_{ij}^{p+sp} \right| \leq C \cdot 2^{(1-\alpha)(k-p)} \cdot 2^{-(3/2-\alpha)s} \frac{1}{\left(1 + \left| i - \frac{j}{2^s} \right| \right)^{2+\alpha}}$$

An estimate of the elements of the matrix \hat{K} follows from the theorem. It also follows from the above that the matrix \hat{K} is pseudo rarefied, i.e. there is a sufficient number of small modulo elements.

IV. NUMERICAL EXPERIMENT ON DECORRELATION OF TIME SERIES

To conduct a numerical experiment, sequences of random variables were obtained that characterize the signal in terms of the processing time of data packets in the system. A comparison of time series decorrelation methods was carried out as follows. Direct conversion was applied to the original data sequences.

From the obtained conversion coefficients, new sequences were compiled, and then the inverse transformation was performed. As a metric, the value of the sum of the modules of the correlation coefficients was used. The algorithm for calculating the correlation coefficients for time series is described in the work of I.V. Kartashevsky [5]. The results obtained at the end of experiments on five data sequences are presented in Table 1.

Table 1. Comparison of experimental results with other types of transformations

Conversion type	1	2	3	4	5
Source sample	208,277	420,396	17,751	35,882	16,007
Fourier Transform	22,619	24,799	5,044	8,497	3,805
Dobeshi Wavelet Transform	22,490	18,204	5,455	7,351	5,532
Spline-based fiberboard (m=2)	20,640	30,468	2,377	4,124	3,147
Spline-based fiberboard (m=3)	18,707	30,811	2,110	4,009	3,025

V. CONCLUSION

In conclusion, it must be said that the use of linear and quadratic spline wavelet functions for decorrelation of a sequence of strongly correlated random variables is potentially more effective than some of the orthogonal transformations studied in this paper. The prospect of using fiberboard based on splines is determined by its flexibility, good decorrelation properties and the existence of fast calculation algorithms.

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JURAYEVJONIBEK UKTAMOVICH

**Samarkand State University,
Assistant**

Samarkand State University, 140104, Uzbekistan

Author of over 19 scientific-methodical works, including 2 educational manual and 2 certificates on registration of software.

He participated in 2 international, 15 republican scientific conferences.

Jurayev J.U. graduated from the Applied Mathematics and Informatics Department in 2008, in 2010 received a Master of Science degree in Applied Mathematics and Information Technology.

In the field of scientific activity is engaged in research of a new scientific direction on the development of scientific and methodological foundations, methods and algorithms for increasing the reliability of information using mechanisms for extracting logical and semantic links of elements of electronic documents

He actively participates in reports at the scientific and methodological seminar. He reads lectures, conducts practical and laboratory classes in the fields of programming, computer networks, information security. Introduces modern systems of information and pedagogical technologies in the educational process.