ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 11 , November 2019

Robust Algorithms for Estimating the State of Control Objects Based on Advanced Observers of the Kalman Type

Sevinov J.U., YusupovYo.A., Mamirov U.F.

Professor, Department of Information processing and control systems, Faculty Electronic and Automatic, Tashkent State Technical University Uzbekistan,
senior teacher, Fergana branch of the Tashkent University of Information Technologies, Fergana, Uzbekistan,
PhD on technical sciences, Department of Information processing and control systems, Faculty Electronic and Automatic, Tashkent State Technical University Uzbekistan

ABSTRACT: Robust algorithms for estimating the state of control objects based on advanced observers of the kalman type are presented. An estimator with exponential weighting of data is considered, and the Riccati equation includes a shift by analogy with the case of a linear quadratic controller. For the case of a nonlinear observer, an adaptation algorithm is given. The problem of synthesizing an observer with a noisy output is also considered.

KEY WORDS: extended robust Kalman-type observer, construction algorithms, non-linear adaptation algorithm.

I.INTRODUCTION

To generate feedback in automatic control systems, information on the state vector of the control object is required. In the case when all state variables are available for measurement, for a controlled system, you can choose feedback that provides the desired dynamic properties of a closed system. In practice, usually the measured variables of an object are only individual components of the state vector or linear combinations of these components. The installation of additional sensors, on the one hand, leads to an increase in the cost of the system, and, on the other hand, measuring devices add extra dynamics to the control system, which complicates the synthesis of the control system. Thus, the need arises for solving the observation problem, namely, the task of obtaining current information about the state vector of dynamic systems from measured variables [1-8].

The observation problem, which is the fundamental task of the theory of automatic control, is solved on the basis of the theory of state observers. In many cases, only after solving the observation problem can we begin to solve the control problem, namely, the synthesis of feedback [4,5,9].

II.FORMULATION OF THE PROBLEM

Consider a dynamic object described by the equation:

$$\dot{x}(t) = A(t)x(t) + B(t)u'(t) + w(t)\,, \qquad (1)$$

where x, u' and w – are state, control, and noise vectors of dimensions n, m and n, respectively, and A and B – are matrices of dimensions $n \times n$ and $n \times m$, respectively. The real input of the object u' – is determined by the expression:

$$u'(t) = G\{u(t)\}, \qquad (2)$$

where u is the desired input, G is some operator.

When receiving state estimates, only signal u can be used. The vector of dimension $l \times 1$ of the sensor output has the form:

$$y(t) = L\{z(t)\} + w = [L_1(z_1), L_2(z_2), \ldots, L_l(z_l)]^T + v\,, \qquad (3)$$

where

$$z(t) = C(t)x(t)\,, \qquad (4)$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

and v – interference measurements in dimension $l \times 1$, a $C(t)$ - $l \times n$ matrix dimension.

In real cases, there is only approximate knowledge about the operators G and L, namely $\hat{G}$ and $\hat{L}$. Then the extended Kalman filter [10, 11], based on such knowledge about the object, can be described using the equation:

$$\dot{\hat{x}}(t) = A\hat{x} + B\hat{G}\{u\} + K\left(y - \hat{L}\{\hat{z}\}\right), \tag{5}$$

where $\hat{X}$ and K mean, respectively, a state estimate and a Kalman gain.

Let $\tilde{x} = x - \hat{x}$ denote the estimation error. It is necessary to determine the conditions when the observer will be asymptotically stable (i.e., $\tilde{x} \to 0$ at $t \to \infty$) in the absence of interference [2,5].

Suppose that A, B, C are non-stationary, and G, L – are non-stationary unambiguous non-linearities without memory. At the disposal of the designer are «estimates» $\hat{G}$ and $\hat{L}$. In addition, even if the L will be continuous and differentiable everywhere. If $L'_i(\sigma) = \partial L_i(\sigma, t)/\partial\sigma$ – is designated, then the extended Kalman filter will be described by equation (5), and the Kalman gain will be described by the expressions [9, 10]:

$$K(t) = \sum(t)\hat{L}'(\hat{z})W^{-1}(t), \tag{6}$$

$$\hat{L}'(\hat{z}) = \mathrm{diag}\left[\hat{L}_i(\hat{z}_1), \ldots, \hat{L}_l(\hat{z}_l)\right]C \ , \tag{7}$$

where $\hat{z} = C\hat{x}$ a W – $l \times l$ – is a positive-definite dimensional matrix corresponding to the covariance matrix of noise interference in the sensors. Here $\Sigma(t)$ – is the Riccati matrix of the extended Kalman filter corresponding to the expression:

$$\dot{\Sigma} = A\Sigma + \Sigma A^T - \Sigma C^T \hat{L}'(\hat{z}) W^{-1} \hat{L}'(\hat{z}) C \Sigma + V. \tag{8}$$

Let us assume that $\Sigma(0) = \Sigma_0 > 0$, a $V(t)$ – positive definite matrix of size $n \times n$.
The estimation error will be determined by the equation:

$$\dot{\hat{x}}(t) = A\hat{x} + B\left[G\{u\} - \hat{G}\{u\}\right] - K\left[L\{z\} - \hat{L}\{\hat{z}\}\right].$$

When analyzing the stability of the extended Kalman filter, we will use the Lyapunov method [10, 11]. Matrix $\sum^{-1}(t)$ plays the role of a Riccati matrix for a linearly quadratic controller in the formation of the Lyapunov function. Since it is non-stationary, it is necessary to prove that it is bounded.
Suppose $\left[A(t)\ C(t)\right]$ is completely observable and there are constants β and γ, such that $0 < \gamma \le L'_i(\sigma) \le \beta$ for $\sigma \in (-\infty, \infty)$, $i = 1, 2, \ldots, l$.

For any given trajectory $\hat{x}(t)$, the Riccati equation (8) corresponds to the error covariance equation for the Kalman-Bucy filter for a linear non-stationary system with matrix $A(t)$ and observation matrix $L'(t)\ C(t)$ [7-11]. The grammar of observability for this system is limited as follows: for some $\theta > 0$:

$$\begin{aligned} \gamma^2 F_0(t, t-\theta) &\le F(t, t-\theta) \\ &\overset{\Delta}{=} \int_{t-\theta}^{t} \Phi^T(t,\tau) C^T(\tau)[L'(\tau)]^2 C(\tau)\Phi(t,\tau)d\tau \\ &\le \beta^2 F_0(t, t-\theta), \end{aligned}$$

where $\Phi(t,\tau)$ – fundamental matrix:

$$F_0(t, t-\theta) \overset{\Delta}{=} \int_{t-\theta}^{t} \Phi^T(t,\tau) C^T(\tau) C(\tau)\Phi(t,\tau)d\tau$$

is grammianom observability $\left[A(t)\ C(t)\right]$. Therefore, since $\left[A(t)\ C(t)\right]$ is received entirely uniformly observed, this applies to $\left[A(t)\ L'C(t)\right]$. In addition, $\Sigma(t) > 0$ (since $V > 0$) and $\Sigma(t)$ are uniformly bounded above and below for all $t \ge 0$. This allows the use of a quadratic function $\Sigma^{-1}(t)$ Lyapunov for stability studies.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

Suppose that for a system described by equations (l) - (4) (for $w = v = 0$), $\hat{G} = G$, $\hat{L} = L$, W – is diagonal, and $\hat{L}_i$ – is positively and uniformly bounded above and below. Then following [6,12] it can be shown that the estimate obtained using equations (5) - (8) is globally asymptotically stable, i.e. $\tilde{x} \to 0$ at $t \to \infty$, if

$$\inf_{(-\infty,\infty)} (L'_i) \geq \left(\frac{1}{2}\right) \sup_{(-\infty,\infty)} (L'_i) \text{ for } i = 1,2,\ldots,l .$$

III.SOLUTION OF THE TASK

Consider the use of an extended constant-gain Kalman filter for stationary systems. Here, when calculating the gains ((6), (8)), the constant L' is used. In particular, if we use $L' = I$, then the equation for the gain will take the form [2,4,10]:

$$K = \Sigma C^T W^{-1} . \tag{9}$$

Consider the estimator with exponential data weighting. Here algebraic Riccati equation includes a «α – shift» by analogy with the case of the linear quadratic regulator. For this purpose, the matrix of A is added to member αI (where α - non-negative scalar) and eigenvalues for the closed loop real parts are smaller – α . The algebraic Riccati equation will have the form:

$$(A + \alpha I)\Sigma + \Sigma(A + \alpha I)^T - \Sigma C^T W^{-1} C \Sigma + V = 0 . \tag{10}$$

Thus, equations (9) and (10) are the same as for the steady state of the Kalman – Bucy filter for a linear object [4, 10].
Suppose that for a system described by equations (1) - (4) at $w = v = 0$, A, B, C, V, W – are stationary, $G = \hat{G}$ and $L = \hat{L}$. Then the estimator defined by expressions (5), (9) and (10) is globally asymptotically stable [6,12-14], if a

$$L'_i > (1 - \sigma_E)/2, \quad i = 1, 2, \ldots, l ,$$

where

$$\sigma_E = \lambda_M^{-1} [(V + 2\alpha\Sigma)^{-1} \Sigma C^T W^{-1} C \Sigma]$$

λ_M – denotes the largest eigenvalue.

Let us now consider dynamical systems described by nonlinear equations:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + d(x(t), u(t), \sigma(t), t), \\ &x(x_0) = x_0 , \end{aligned} \tag{11}$$

where $t \in R^n, x(t) \in R^n$ – state, $u(t) \in R^m$ – control, $\sigma(t) \in R^p$ – undefined parameter, A – constant matrix.

The equations for the output are determined by the expression

$$y(t) = Cx(t), \tag{12}$$

where $y(t) \in R^q$ – is the output, the matrix C is constant.

Consider the task of synthesizing an observer based on information about the input and output and certain information about the structure of the system (11) so that the output of the observer $\hat{x}$ is close to state x.
We assume that in system (11), (12) the pair (A, C) is observable.

It follows that there exists a matrix $\overline{A}$ such that $\lambda(\overline{A}) \subset C$ for $\overline{A} \overset{\Delta}{=} A + KC$. In addition, there is a solution $P > 0$ of the Lyapunov equation $\overline{A}^T P + P\overline{A} + Q = 0,\ Q > 0$ [15-19],

a) there is a function $h(\cdot) : R^n \times R^m \times \Sigma \times \mathrm{R} \to \mathrm{R}^q$ such that for all $(x, u, \sigma, t) \in R^n \times R^m \times \Sigma \times \mathrm{R}$:

$$Pd(x, u, \sigma, t) = C^T h(x, u, \sigma, t) ;$$

b) there are unknown constants $\beta \in (0, \infty)^r$ and a well-known function $\rho(\cdot) : R^m \times (0, \infty)^r \times R \to R_+$ such that for all

$$(x, u, \sigma, t) \in R^n \times R^m \times \Sigma \times \mathrm{R} ,$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 11 , November 2019

$$\|h(x,u,\sigma,t)\| \le \rho(u,\beta,t).$$

v) for each $(u,t)\in R^m \times R$, the function $\rho(u,\cdot,t):(0,\infty)^r \to R_+$ belongs to C^1 and is convex.

Thus, for any $\beta_1,\beta_2 \in (0,\infty)^r$, the inequality

$$\rho(u,\beta_1,t)-\rho(u,\beta_2,t) \le \frac{\partial\rho}{\partial\beta}(u,\beta_2,t)(\beta_1-\beta_2),$$

Consider the following observer:

$$\dot{\hat{x}}(t)=\overline{A}\hat{x}(t)-Ky(t)+p(\hat{x}(t),y(t)\,\hat{\beta}(t),t), \tag{13}$$

where $\hat{x}(t)\in R^n$ assessed condition.

Then, adaptation parameter $\hat{\beta}(t)$ can be determined in accordance with the following adaptation algorithm [2,4,7]:

$$\left.\begin{array}{l}\hat{\beta}(t)=L_1\|\alpha(\hat{x},y)\|\dfrac{\partial^T\rho}{\partial\beta}(u,\hat{\beta},t)-L_2\hat{\beta}(t)\\ \hat{\beta}(t_0)=\hat{\beta}_0,\end{array}\right\},$$

where $\alpha(x,y)=C\hat{x}-y$. Here L_1,L_2 – are constant, positive definite matrices.

Let $e(t)\overset{\Delta}{=}\hat{x}(t)-x(t)$ denote an error. If we subtract (13) from (11), we can obtain the following equation for the error:

$$\dot{e}(t)\overset{\Delta}{=}\overline{A}\hat{x}(t)-Ky(t)+p(\hat{x}(t),\,y(t),u(t),\,\hat{\beta}(t),\,t)-Ax(t)-d(x(t),u(t),\,\sigma(t),\,t).$$

Given $\overline{A}=A+KC$ and (12), we can write

$$\left.\begin{array}{l}\dot{e}(t)=\overline{A}e(t)+p(\hat{x}(t),\,y(t),u(t),\,\beta(t),\,t)-d(x(t),u(t),\,\sigma(t),\,t)\\ e(t_0)=e_0=\hat{x}(t_0)-x_0\end{array}\right\}.$$

Consider the observer (13) for function $p(\cdot)$ defined by the expression

$$p(\hat{x},y,\hat{\gamma})=-P^{-1}C^T\hat{\gamma}(C\hat{x}-y). \tag{14}$$

Here $\hat{y}(t)$ – is the parameter corresponding to the expression

$$\left.\begin{array}{l}\hat{\gamma}(t)=l_1\|C\hat{x}(t)-y(t)\|^2-l_2\hat{\gamma}(t)\\ \hat{\gamma}(t_0)=\hat{y}_0\end{array}\right\},$$

where $l_1,l_1>0$.

Recall that $e(t)=\hat{x}(t)-x(t)$. The equation for the error at p determined using (14) will have the form

$$\left.\begin{array}{l}\dot{e}(t)=\overline{A}e(t)-P^{-1}C^T\hat{\gamma}(t)[C\hat{x}(t)-y(t)]-d(x(t),u(t),\sigma(t),t)\\ e(t_0)=e_0=\hat{x}(t_0-x_0)\end{array}\right\}.$$

Now consider the case of a noisy exit.
Let

$$y(t)=Cx(t)+v(t),$$

where $v(t)\in R^q$ – noise in output measurements. Consider observer (13) at $p(\cdot)$ given by (14) [4.7]. Then, in accordance, one can propose the following adaptation algorithm:

$$\left.\begin{array}{l}\dot{\hat{\gamma}}=l_1\|C\hat{x}(t)-y(t)\|^2-[l_2\|C\hat{x}(t)-y(t)\|+l_3]\overline{\gamma}(t)\\ \hat{\gamma}(t_0)=\hat{\gamma}_0\end{array}\right\},$$

where $l_1,\,l_2,\,l_3>0$. The equation for the error has the form

$$\dot{e}(t)=\overline{A}e(t)-P^{-1}C^T\hat{\gamma}(t)[C\hat{x}(t)-y(t)]-d(x(t),\,u(t),\,\sigma(t),t).$$

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 11 , November 2019

VI.CONCLUSION AND FUTURE WORK

The above algorithms make it possible to increase the degree of observability of controlled objects and qualitative indicators of the processes of managing complex systems.

REFERENCES

[1]. Kuntsevich V.M. Control in the face of uncertainty: guaranteed results in control and identification tasks. – Kiev: NaukovaDumka, 2006. - 264 p.
[2]. Emelyanov S.V., Korovin S.K. State observers for indefinite systems. Math modeling. Problems and results. - M.: Science, 2003.
[3]. Bobtsov A.A., Nikiforov V.O., Pyrkin A.A., Slita O.V., Ushakov A.V. Adaptive and robust control methods for nonlinear objects in instrumentation: a textbook for higher educational institutions. - St. Petersburg: NRU ITMO, 2013. - 277 p.
[4]. Afanasyev V.N. Dynamic systems with incomplete information. Algorithmic design. –M.: KomKniga, 2007. -216 p.
[5]. Emelyanov S.V., Korovin S.K. Stabilization of indefinite dynamic objects with continuous time. On Sat «New methods for managing complex systems», –M.: Nauka, 2004.
[6]. Polyak B.T. Robust stability and control. –M.: Nauka, 2002. –303 p.
[7]. Afanasyev V.N. Managing undefined dynamic objects. –M.: Fizmatlit, 2008. – 208 p.
[8]. Zhou K., Doyle J.C., Glover K. Robust and Optimal Control. – Upper Saddle River, NJ: Prentice Hall, 1996. – 586 p.
[9]. Emelyanov S.V., Korovin S.K. New types of feedbacks: Management in conditions of uncertainty. –M.: Nauka, 1997. - 352 p.
[10]. Filtering and stochastic control in dynamic systems. / Ed. K. T. Leondes Per. from English, –M.: Mir, 1980. - 407 p.
[11]. Ogarkov M.A. Methods of statistical estimation of random process parameters. –M.: Energoatomizdat, 1990. - 208 p.
[12]. Poznyak A.S. Fundamentals of robust control (H∞-theory). –M.: MIPT Publishing House, 1991. - 128 p.
[13]. Pilishkin V.N. Robust control algorithms in intelligent systems // Vestnik MSTU. Series instrumentation. - 1998. - No. 1. - S. 23–28.
[14]. Rustamov G.A. Absolutely robust control systems // Automatic Control and Computer Sciences. - 2013. - V. 47. - No. 5. - P. 227–241.
[15]. Shelenok E.A. Adaptive-robust control system for nonlinear objects of periodic action // Informatics and control systems. - 2012. - No. 4 (34). - S. 128–137.
[16]. Robust Control Toolbox™ Getting Started Guide / G. Balas, R. Chiang, A. Packard, M. Safonov. - Natick, MA: Math Works, Inc., 2005-2010. - 135 p.
[17]. Smith R.S., Dahleh M. (eds.) The Modeling of Uncertainty in Control Systems / Lecture Notes in Control and Information Sciences. V. 192. London, U.K.: Springer-Verlag, 1994.
[18]. Sokolov V.F. Estimating Performance of the Robust Control System under Unknown Upper Disturbance Boundaries and Measurement Noise // Autom. Remote Control. 2010. V. 71. No. 9. P. 1741 ‑ 1757.
[19]. Yusupbekov N.R., Igamberdiev H.Z., Mamirov U.F. Algorithms of sustainable estimation of unknown input signals in control systems // Journal of Multiple-Valued Logic and Soft Computing, Volume 33, Issue 1-2, 2019, Pages 1-10.