ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 3, March 2019

Basic characteristics of the theory of shock and vibration

Alexander Shulemovich.

PhD, Independent researcher, New York, USA, email: a49shulex@gmail.com

ABSTRACT: The Timoshenko theory of shock, unifying the Hertz theory and effect of vibrations, was introduced for impact of solid sphere on a beam with hinged ends. The integral equation has been numerically solved by Timoshenko for two specific problems with incomplete analysis for the second problem. The purpose of this paper is to clarify the peculiarity of the interaction between striking sphere and a beam and to determine the basic characteristics of the transverse impact of a mass on a beam such as domain of modes of vibrations and frequencies necessary for analysis, ratio of mass of a sphere and dynamic mass of a beam, necessary for appearance of a beam vibration, and supplementary collisions. The presented analysis has shown that vibrations of a beam can exist only if a mass of striking sphere is less then dynamic mass of a beam. The completed numerical solution of second problem, considered by S. P Timoshenko, demonstrated that this case is actually quasi – static loading of a beam with three accompanied collisions. The revealed basic characteristics of shock and vibrations are universal since the range of validity of the Timoshenko equation is related to any elastic body.

KEY WORDS: shock; impact; modes of vibrations; collisions; mass; stiffness.

I. INTRODUCTION

The state of the art, devoted to the shock and vibration, is presented, for example, in [1 – 5]. The purpose of this paper is, in addition to well – known results, to demonstrate that vibration of a beam, excited by impact of a striking mass M, is possible only if M is less than dynamic mass m of a beam. The analysis of this phenomenon also has shown the limited domain of frequencies participating in vibration and collisions. First consider the values and ratio of mass M and mass m. For approximate evaluation of dynamic masses of beams with hinged ends consider the simplified model shown in Fig. 1.

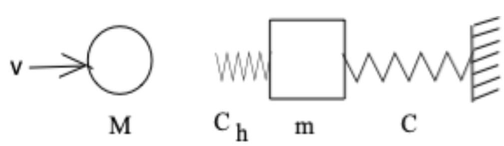

Fig. 1. Schematic model of a striking mass and a dynamic mass of a beam.

Here M – mass of solid sphere, m – dynamic mass of a beam, C – dynamic stiffness of a beam, v – velocity of a mass, C_h – contact stiffness.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 3, March 2019

Three following problems are under consideration for the model depicted in Fig.1: a) M < m, b) M = m, c) M > m. For the qualitative estimate the dynamic mass of a beam can be determined as $m = C_s \omega^{-2}$, $\omega = \pi^2 l^{-2} (lEIm_b^{-1})^{0.5}$ is first natural frequency and $\omega^2 = \pi^4 l^{-3} lEIm_b^{-1}$ [1- 5], $C_s = 48\ l^{-3} EI$ – static stiffness of a beam [6], m_b – mass of a beam, l – length of a beam, E – modulus of elasticity, I – moment of inertia of area. Finally, $m = C_s\omega^{-2} = 48\pi^{-4} m_b = 0.4927 m_b$ and corresponding volume $V_b = 0.4927Al$. A – area of cross section of a beam. Since the material properties of steel sphere and beam are the same $Mm^{-1} = VsV_b^{-1}$, volume of a sphere with radius r is $Vs = 4\pi r^3 3^{-1}$ and $Mm^{-1} = VsV_b^{-1} = 4.188\,(0.4927Al)^{-1} = 8.5\, r^3 (Al)^{-1}$ (1)

Notes: In integral equation for numerical analysis S. P. Timoshenko used $m = 0.5m_b$. The ratios Mm^{-1} for two specific problems, solved by S. P. Timoshenko, are presented below. First problem: l = 15.35 cm, A = 1cm^2, r = 1cm; second problem: l = 30.7cm, A = 1cm, r = 2cm. So, for first problem $Mm^{-1} = 0.554$ and for second problem $Mm^{-1} = 2.215$. The influence of value Mm^{-1} on characteristics of shock and vibrations will be discussed hereafter.

II. DOMAIN OF FREQUENCIES EXCITED BY AN IMPACT ON A BEAM WITH HINGED ENDS

The integral equation introduced by S. P. Timoshenko is following:

$$v\,t - M^{-1}\int_0^t dt_1 \int_0^{t_1} P\, d\,t = kP^{2/3} + 2\, m_b^{-1} \sum_{i=1,3,5\ldots}^{\infty} \omega_i^{-1} \int_0^t P \sin \omega_i (t - t_1)\, dt_1 \quad (2)$$

This equation is applicable for analysis of shock and vibration of any elastic body with corresponding dynamic mass. The following data are necessary for numerical analysis of Eq. (2): v, M, m and set of modes of vibration of the elastic body, excited by impulse. There is no way for direct determination of such modes, however, some preliminary information can be obtained. It is reasonable assumed that shape of acting force is semi - wave of sine, $P = P_0 \sin \lambda t$ and P_0 depends only on velocity v. Consider dynamic deflection at the middle of a beam

$$2\, P_0\, m_b^{-1} \sum_{i=1,3,5\ldots}^{\infty} \omega_i^{-1} \int_0^t \sin \lambda\, t \sin \omega_i (t - t_1)\, dt_1 = 2\, P_0\, m_b^{-1} \sum_{i=1,3,5\ldots}^{\infty} N_1^{-1} \sin (\lambda - \omega_i)\, t - N_2^{-1} \sin (\lambda + \omega_i)\, t\,, \quad (3)$$

where $N_1 = 2\,\omega_i(\lambda - \omega_i)$, $N_2 = 2\,\omega_i(\lambda + \omega_i)$.

After substitution $\omega_i t = \beta_i \lambda t$, $\beta_i = \omega_i \lambda^{-1}$ the dynamic deflection (3) is as follows:

$$P_0 \lambda^{-2} m_b^{-1} \sum_{i=1,3,5\ldots}^{\infty} Q_i^{-1} [(1 + \beta_i) \cos \omega_i\, t + (1 - \beta_i) \cos \omega_i\, t] = P_0 \lambda^{-2} m_b^{-1} \sum_{i=1,3\,5,\ldots}^{\infty} Q_i^{-1} \cos \beta_i \lambda t \quad (4)$$

Here $Q_i = \beta_i (1 - \beta_i^{-2})$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 3, March 2019

The further analysis is aimed on determination of modes in the course of impulse and posterior modes of vibration after the end of impulse, i.e. accordingly for $\lambda t = 0.5\,\pi$ and $\lambda t = \pi$. Since the total dynamic deflection relative to the dynamic reflection of first mode of vibration, utterly determines all excited, symmetrical modes, the characteristic designated as $R_i\,R_1^{-1}$ is appropriate for numerical analysis.

Here $R_i = \sum_{i=1,3,5...}^{\infty} Q_i^{-1} \cos \beta_i \lambda t$, $R_1 = Q_1^{-1} \cos \beta_1 \lambda t$.

Results of numerical analysis are presented in Table 1for $\lambda t = 0.5\,\pi$ and in Table 2 for $\lambda t = \pi$.

Table 1. Modes in the course of impulse

mode No	$R_i R_1^{-1}, \lambda t = 0.5\,\pi$		
	$\beta_i = 0.125$	$\beta_i = 0.25$	$\beta_i = 0.5$
1	1	1	1
3	0.324	0.296	0
5	0.183	0.138	0

Table 2. Modes after the end of impulse

mode No	$R_i R_1^{-1}, \lambda t = \pi$		
	$\beta_i = 0.125$	$\beta_i = 0.25$	$\beta_i = 0.5$
1	1	1	0
3	0.158	0	0
5	0	0	0

Notes: a) the data presented in Tables 1 and 2 are applicable for evaluation of any elastic body' number of excited modes of vibration; b) the number of excited modes in the posterior vibrations are less than in the course of impulse; c) for large λ and, hence, very small β_i the amplitudes of excited modes become very small and practically negligible. Also, there is the upper limit for high frequencies of beams excited by transient forces for beams [7]; d) the lack of modes of vibration for shock with $\lambda t = \pi$ and $\beta_i = 0.5$ (Table 2) is indication that this case actually represents the quasi – static case.

III. ANALYSIS AND EVALUATION OF BASIC CHARACTERISTICS

There is no sufficient data concerning the physical qualities of shock and vibration, however the basic characteristics can be established with acceptable reliability and accuracy. Three problems with M < m, M = m and M > m are used for evaluation of basic characteristics.

THE FIRST PROBLEM with M < m was numerically solved by S. P. Timoshenko [1]. The graph exhibits the shape of force as semi – wave of sine, recoil of striking sphere with velocity lesser than initial one, one collision and posterior vibration of a beam. It is evident that some part of energy is transmitted for vibration of a beam. The number of modes, used for numerical analysis, is unknown. Apparently only first mode was used for numerical analysis.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 3, March 2019

The solution of SECOND PROBLEM with M = m is sat for indirect interpretation with reference to the experimental observations of Newton [2]. The collision of a sphere with mass M = m is resulted in instantaneously transfer the whole of energy of incoming sphere to motionless mass m, the stop dead of mass M and vibration of mass m. After a have of period of vibration mass m comes back and collides with sphere, transmitting the whole of energy back to the sphere with mass M, and stops dead, compelling the mass M to move in opposite direction with initial velocity.

THE THIRD PROBLEM with M > m was numerically solved by S. P. Timoshenko [1]. The number of modes, used for numerical solution, is unknown. The graph of solution shows 2 collisions, increasing sagging of a beam together with displacement of a sphere and, therefore, the explicit indication that numerical solution was not completed. The comprehensive numerical solution of this problem was presented in [8] with graphical results shown in Figs. 4 – 7. This problem was considered in the horizontal plane.

Although the Hertz' theory is valid only for the relatively small velocities of striking mass (stress in contact should not exceed the limit of elasticity), the role of contact force in presented analysis was considered as insignificant factor with influence only on the shape of impulse. The results of analysis are discussed below with comments for purpose of evaluation of basic characteristics of shock and the posterior vibrations.

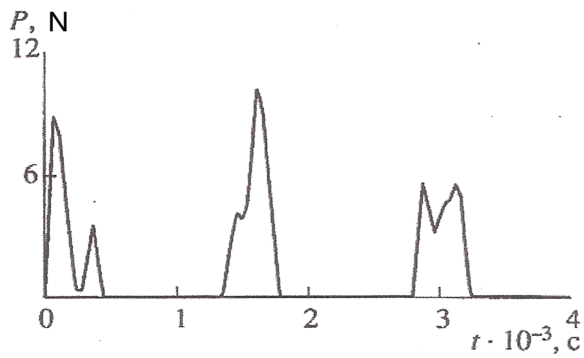

Fig. 2. Contact force P as function of time, c – sec

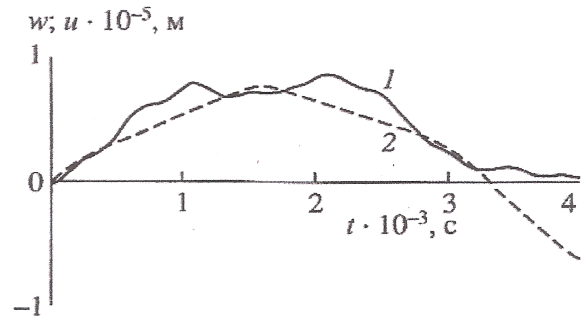

Fig. 3. Sagging w of the beam (1) and displacement of the sphere u (2) as function of time

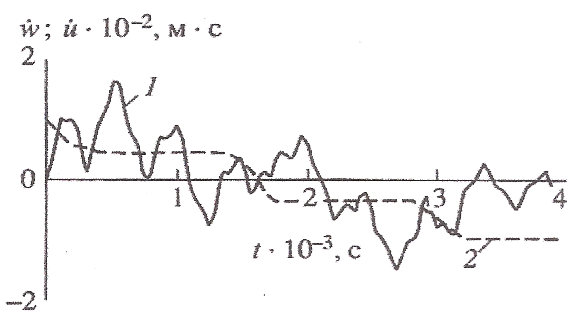

Fig. 4. Velocity of the middle of the beam (1) and velocity of the sphere (2) as function of time

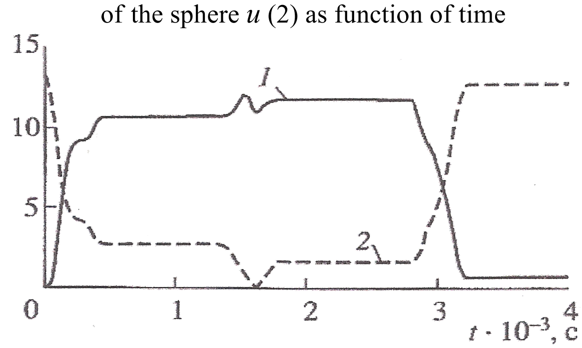

Fig. 5. Energy A_w of the beam (1) and energy A_u of the sphere (2) as function of time

In contrast to the solution, revealed by S. P. Timoshenko, the presented, comprehensive solution demonstrates three collisions instead of two, almost synchronous motion of a middle of a beam and a sphere, absence of posterior vibration of a beam and recoil of a sphere with initial velocity in opposite direction (Figs. 3 and 4). As can be seen the joint motion of a sphere and a beam has all attributes of the quasi – static deformation of a beam under slowly applied force P with passing collisions. The change of velocities (Fig. 4) and interchange of the beam and the sphere

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 6, Issue 3, March 2019

obtained from analysis of three problems call for general summary of results. In the light of these facts the linear approximations (Figs. 6 -9) are introduced for determination of basic characteristics of a beams with hinged ends.

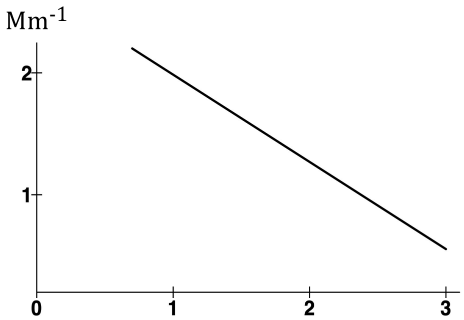

Fig. 6. Modes of vibration in the course of impulse

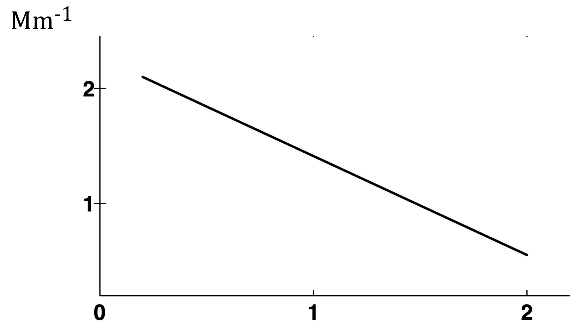

Fig. 7. Modes of vibration after the end of impulse

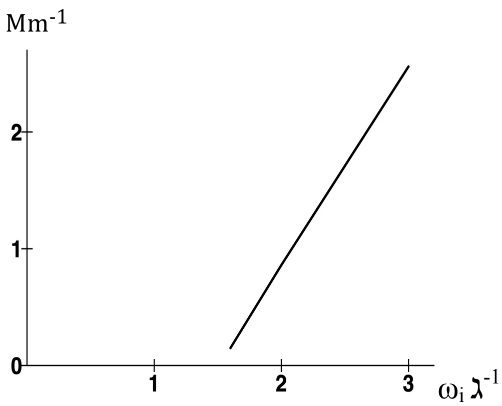

Fig. 8. Ratio of $\omega_i \lambda^{-1}$ as function of Mm^{-1}

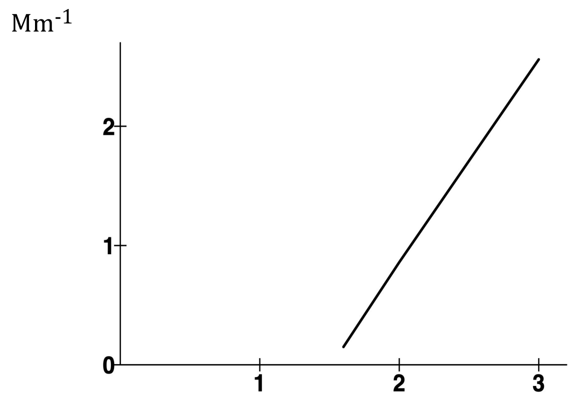

Fig. 9. Number of collisions as function of Mm^{-}

IV. CONCLUSION

The basic characteristics of shock and vibration were derived based on analysis of a beam's deflections. By heuristic consideration it was assumed that only shape and maximum magnitude of force P depend on velocity of a solid sphere and "Hertz stiffness". The utmost important is the duration of an acting impulse. The influence of the semi – sinusoidal impulse's amplitude was excluded using the relative deflections of a beam. The analysis of three typical problems demonstrated that ratio of the sphere' mass M with respect to the dynamic mass of a beam m is the definitive for basic characteristics of shock and vibrations. All basic characteristics shown on diagrams in Figs, 6 - 9 demonstrate the functional dependence upon the ratio of masses M and m. The number of modes, necessary for numerical analysis, is indirect indication of appearance of vibrations of a beam. The posterior vibrations after shock exist only if the number of modes participating in the shock is three or bigger. The problem with the only one mode

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 6, Issue 3, March 2019

in the course of shock is actually the quasi – static accompanied by passing collisions. The number of collisions is just opposite. The results of this investigation demonstrate the evidence that impact of a solid sphere on a beam with hinged ends can excite very limited spectrum of frequencies and limited number of collisions. The impact of a solid mass on a beam with elastic ends can result in large number of collisions.

REFERENCES

1. Timoshenko S. P. "Vibration Problems in Engineering", 3d ed., p. 413,
D. Van Nostrand Company, Inc., Princeton, N. J., 1955.
2. Timoshenko S. P., Young D. H. "Engineering Mechanics", McGraw – Hill Book Company, New York,1940.
3. De Silva, C. W. "Vibration and Shock Hand book", CRC, 2005. ISBN 0 – 8493 – 1580 – 8.
4. Peirsol, A. G. and Paez L. T. "Shock and Vibration, Handbook", sixth ed. McGraw Hill Co. Inc., 2010, ISBN: 9780071508173.
5. Lalanne Christian, "Mechanical Vibration and Shock Analysis". Vol. 2, WILEY, May 12, 2014.
6. Timoshenko S. P. "Strength of Materials", Part 1, Elementary Theory and
Problems, 3d ed.,D. Van Nostrand Company Inc., Princeton, N. J., 1955.
7. Shulemovich A. "Advanced theory of vibration of uniform beams", Int. Journal of Engineering & Technology (UAE), 7(1), pp. 70 – 73, 2018. www.sciencepubco.com/index.php/IJET.
8. Gartsman S. D., A. M. Shulemovich "Repeat Collisions in S. P. Timoshenko's Theory of Shock", Problems of Mechanical Engineering and Reliability", Russian Academy of Science, #5, 2001, (in Russian).