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The Spectral Density of Measurement Errors with the Periodic Nature

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ABSTRACT: The article discusses the issue of determining the density of measurement errors, which is of a periodic nature, which makes it possible to determine the hidden periodicity in the measurement results.

KEYWORDS: **s**pectral density, random function, correlation function, cross section of a random function, distribution, probability, normalized spectral density, autocorrelation, periodicity, theodolite, horizontal circle.

I.INTRODUCTION

The spectral density of a random function $x(t)$ is called $g(\omega)$, which is associated with the correlation function $K_x(\tau)$by the reciprocal cosine of the Fourier transform, i.e.

$$g(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}K_x(\tau)cos\omega\tau d\tau, \tag{1}$$

where $K_x(\tau)$ is the correlation function of the ergodic random function

$$K_x(\tau)=\frac{1}{T}\int_0^T[x(t)-m_x][x(t+\tau)-m_x]dt. \tag{2}$$

In practice, the argument t geodetic measurements may be not only time, but other parameters, for example, coordinates x, y, angle, temperature, etc. Each implementation of a random function is its transformation into a normal one - non-random. For some fixed value of t_j, the random function $x(t)$ becomes the usual value of $x(t_j)$, which is called the cross section of the random function. With an increase in the number of n sections, such a system more and more accurately characterizes a random function.

A random process is called stationary in the narrow sense of the word, if the probability density of the Kth order does not depend on the shift of all observation points t_1, t_2, ... $t_k$to set the axis of the argument by the same value F, i.e.:

$$f_N(x_1,t_1,x_2,t_2,\dots,x_N,t_N)=f_N(x_1,t_1+\tau,x_2,t_2+\tau,\dots,x_N,t_N+\tau). \tag{3}$$

Here $f\,(x_1, t_1, x_2,t_2, \dots x_N, t_N)$ – $N$is the dimensional probability density;

N is any integer; t - any shift in time.

In this case, all statistical characteristics do not depend on the origin of time. One-dimensional probability density does not depend on time. The two-dimensional probability density depends only on one parameter $\tau= t_2 - t_1$ and at $\tau = const$ does not change for any time values t_1 and t_2

$$f_2(x_1,t_1,x_2,t_2)=f_2(x_1,t_1+\tau,x_2,t_2+\tau). \tag{4}$$

Since the correlation function is determined through a two-dimensional distribution function, it depends on only one argument τ

$$K_x(t_1,t_2)=K_x(t_1,t_1+\tau)=K_x(\tau), \tag{5}$$

(5) a wide-scale stationary process.

The main properties of the correlation function of the stationary process:

$$K_x(t,t')=K_x(\tau);\quad K_x(0)=D_x>0;$$
$$|K_x(\tau)|\le K_x(0);\ K_x(-\tau)=K_x(\tau), \tag{6}$$

where D_x is the variance.

Attitude

$$r(\tau)=\frac{K(\tau)}{K(0)} \tag{7}$$

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is called the normalized correlation function and when *τ = 0, r (0) = 1*.

The correlation function in turn is expressed in terms of the spectral density using the formula

$$K_x(\tau) = \int_0^{\infty} g(\omega) cos\omega\tau d\tau. \quad (8)$$

In the practice of research, instead of the spectral density, the normalized spectral density is used:

$$G(\omega) = \frac{g(\omega)}{D_x} \quad (9)$$

or

$$G(\omega) = \int_{-\infty}^{\infty} r(\tau) cos\omega\tau d\tau. \quad (10)$$

The spectral density (normalized spectral density) describes the distribution of dispersions of a stationary random function with a continuously varying frequency.

Expression (8) is also called the Fourier integral, which in its general form shows the expansion in a Fourier series of even cosine harmonics.

The spectral density estimate (10) is calculated by the formula

$$\bar{G}(\omega) = \frac{2}{N} \sum_{\tau=0}^{N-\tau} \bar{r}(\tau) cos\omega_x \tau, \quad (11)$$

where

$$\omega_0 = \frac{\pi}{N}; \quad \omega_K = K\omega_0; \quad K \neq 0 \quad K = (\overline{1, N}).$$

If a graph of the spectral density frequency ω_i will correspond to a peak, it shows that in the process there is a hidden frequency whose period is respectively $\frac{2N}{i} = T$.

Using formula (10), we define the normalized spectral densities for the functions [1]

$$r_x(\tau) = e^{-\alpha|\tau|}; \quad (12)$$

$$r_x(\tau) = e^{-\alpha|\tau|} cos\omega_0\tau; \quad (13)$$

$$r_x(\tau) = e^{-\alpha|\tau|}(cos\omega_0\tau + |\gamma| sin\omega_0\tau), \quad (14)$$

i.e.:

$$G_1(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-\alpha\tau} cos\omega\tau d\tau = \frac{2\alpha}{\pi(\alpha^2 + \omega^2)}; \quad (15)$$

$$G_2(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-\alpha\tau} cos\omega_0\tau cos\omega\tau d\tau = \frac{1}{\pi}\left[\frac{\alpha}{\alpha^2 + (\omega + \omega_0)^2} + \frac{\alpha}{\alpha^2 + (\omega + \omega_0)^2}\right]. \quad (16)$$

$$G_3(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-\alpha\tau} (cos\omega_0\tau + \gamma sin\omega_0\tau) cos\omega\tau d\tau =$$

$$= \frac{1}{\pi}\left[\frac{\alpha}{\alpha^2 + (\omega + \omega_0)^2} + \frac{\alpha}{\alpha^2 + (\omega + \omega_0)^2} + \frac{\gamma(\omega_0 + \omega)}{\alpha^2 + (\omega_0 + \omega)^2} + \frac{\gamma(\omega_0 - \omega)}{\alpha^2 + (\omega_0 - \omega)^2}\right] \quad (17)$$

For example, we will show the definition of the spectral density of the autocorrelation function of the total errors of the diameters of the horizontal circle of the theodolite, which is given [2]

$$\bar{r}(\tau) = e^{-0.042\tau} cos 0.1250\tau$$

$$\frac{2\pi}{T} = \omega = 7°12'$$

Using the formula (16), we have

$$G_{x_{cp}}(\omega) = \frac{0{,}0134}{0{,}0018 + (\omega - 0{,}1250)^2} + \frac{0{,}0134}{0{,}0018 + (\omega + 0{,}1250)^2}, \quad (18)$$

The graph of the spectral density, built by shown in Figure 1.

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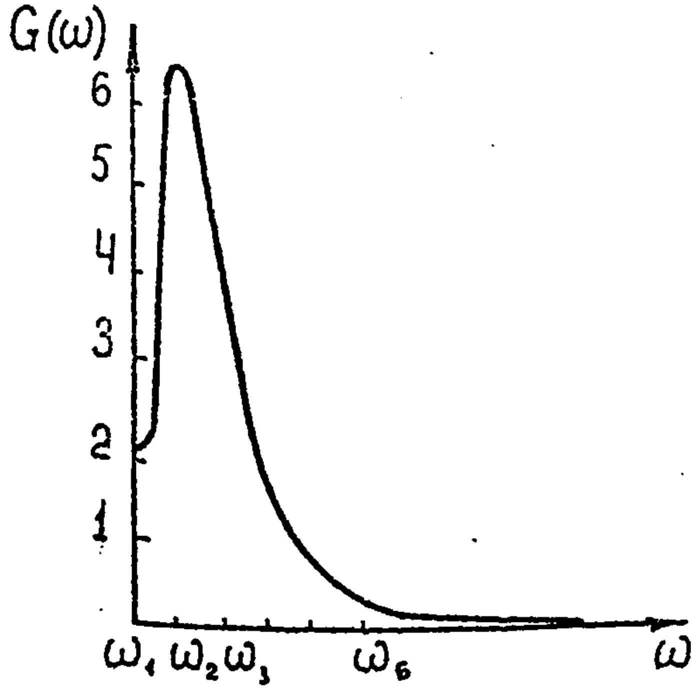

Figure 1.

As you can see, the peak in the spectral densities corresponds to i = 2. According to the formula $\frac{2N}{i} = T$, we find that $T = N = 60$ or $T = 180°$. Therefore, the total errors of the diameter of the horizontal circle of theodolite have a hidden periodicity, the period of which is 180°.

REFERENCES

1. Livshits N.A., Pugachev V.N., Probabilistic analysis of automatic control systems. M., Soviet Radio, 1963, v.1.
2. Tashpulatov S.A. Approximation of the autocorrelation function. K., Engineering Geodesy, no. 27, 1984, pp.87-89.