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About Some Results Related to Minimal Pre-connected Space , Maximal Pre-connected Space

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ABSTRACT : In this paper , we introduce a new spaces are said to be minimal pre- connected space and Maximal pre-connected space by using the set pre- open , where we studied identified its properties and we can define the sets maximal pre- open (reps. minimal pre- open) sets .We find the relation between them and with set Pre- open (resp. maximal open , minimal open) sets . Where every maximal pre- open (resp. minimal pre - open) sets is pre- open , but the converse is not true in general and every maximal pre- open (resp. minimal pre- open) sets is maximal open (resp. minimal open) sets , but the converse is not true in general . Also , we can define the spaces minimal pre- connected (resp. Maximal Pre- connected) spaces and we find the relation between them and with the spaces connected (resp. pre- connected , minimal connected , maximal connected) space .Where every connected space is minimal pre-connected (resp. maximal pre- connected) space also every connected space is minimal connected (resp. maximal connected) space , but the converse of them is not true in general .But we find every pre- connected is connected space .But not conversely also every pre - connected space is minimal connected (resp. maximal - connected) space .But not conversely in general also we find every minimal connected (resp. maximal connected) space is minimal pre- connected (resp. maximal pre - connected) space .But not conversely in general .

KEYWORDS: Minimal pre-connected space , Maximal pre-connected space , Minimal connected space, Maximal connected space , connected space , pre-connected space ..

Mathematics subject classification: 54 XX

I. INTRODUCTION

In [1] introduced set of class pre-open byNavalgi , G , B . In [2] Gemignani , M.C. The definition was introduced that connected space and gave relation about the connected space . In [3] AL-Maleki , N.J. The definition was introduced that pre-connected and proved that every pre-connected space is aconnected spaceand the converse is not true in general . In [4] F.Nakaoka and N. oda . gave some application of minimal open sets . In [5] F. Nakaoka and N. Oda , introducedsome application of Maximal open sets . In [6] K. FadhilRadhi introduced anew calssB*c-open set and proved some application about that . In [7] Reszard Engel- king introduced properties about open and closed sets also in [8] Relly , I and Gansrter , M. , showed the relation between open sets and Pre- open sets , where proved every open set is a Pre- open set , but the converse is not true in general . In [9] A. S. Mashhour , M. E. Abd- El- Monsef and S. N. ElDeeb , showed that properties Pre-open sets and Pre- closed sets , where he proved A is a Pre- open set if and only if A^c is a Pre- closed set .

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II. MAIN RESULTS

Definition (1.1) : [1]A subset A of a topological space (X , τ) is called a pre-open set if $A \subseteq \overset{\circ}{\overline{A}}$.
The complement of pre-open set is called a pre-closed set . The set A is named pre-clopen if it is pre-open and pre-closed . The set of all pre-open subsets of X is limited by po(x) . The set of all pre-closed subset of X is limited by pc(X) .

Definition (1 .2) : Let X be a space of topology . A proper nonempty pre-open subset U of X is called to be :
i) A pre-open of minimal set if any pre-open sets that is included in U is ϕ or U .
ii) A pre-open of Maximal set if any pre-open sets that contains U is X or U .

Definition (1.3) : [2]A space of topology (X , τ) is called to be disconnected space if X may be expressed asthe union of two disjoint open nonempty sub set of X otherwise , X is said to be connected space .

Definition (1 . 4) : Let X be a topological space . A proper nonempty open subset U of X is said to be :
i) A minimal open set [4] if any open set which is contained in U is ∅ or U .
ii) A maximal open set [5] if any open set which is contains U is X or U .

Definition (1.5) : A space of topology (X , τ) is called :
i) Minimal connected space if X is not a union of two nonempty disjoint minimal open sets .
ii) Maximal connected space if X is not a union of two nonempty disjoint Maximal open sets .

Definition (1.6) : [3] A space of topology (X , τ) is called pre-connected space if X is not a union of two nonempty disjoint pre-open set .

Definition (1.7) :A space of topology (X , τ) is called :
i) Minimal pre-connected space if X is not a union of two nonempty disjoint Minimal pre-open set .
ii) Maximal pre-connected space if X is not a union of two nonempty disjoint Maximal pre-open set .

Remark (1.8) : Let X be A space of topology and $A \subseteq X$. then :
i) A is an open if and only if A^C is a closed [7]
ii) A is a maximal open if and only if A^C is a minimal closed [4]
iii) A is a pre-open if and only if A^C is pre-closed [9]

Lemma (1 . 9) : Let X be A space of topology and $A \subseteq X$. then :
A is a pre-open of Maximal if and only if A^C is pre-closed set of minimal .

Proof: Let A be a pre-open of maximal . Then A is a pre-open , then A^C is a pre-closed .
Let V be pre-closed such that $V \subseteq A^C$, then $A \subseteq V^C$. Since A is a Maximal pre-open , then
$V^C = A$ or $V^C = X$, then $V = A^C$ or $V = \phi$, hence A^C is a minimal pre-closed

Conversely Let A^C be a minimal pre-closed , then A^C is a pre-closed , then A is a pre-open .
Let W be is a pre-open such that $A \subseteq W$, then $W^C \subseteq A^C$. Since A^C minimal pre-closed , then $W^C = A^C$ or $W^C = \phi$, then W = A or W = X .
hence A is a Maximal pre-open .

Remark (1.10) :i) Every minimal open (resp. maximal open) is open . [6]
ii) Every minimal pre- open (resp. maximal pre- open) is Pre-open .
iii) Every minimal closed (resp. maximal closed) is closed .
The converse of above remark is generally not right .

Example (1.11) : Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$
$Po(x) = \{X, \phi, \{1\}, \{3\}, \{1,2\}, \{2,3\}\}$
i) A = { 1 } is an open set , but not maximal open and B = { 1 , 2 } is open set , but not minimal open .

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ii) A = { 1 } is a pre-open set , but not maximal pre- open and B = { 1 , 2 } is a pre-open set, but not minimal pre- open.
iii) A = { 3 } is closed set , but not maximal closed and B = { 1 , 3 } is closed , yet not minimal closed .

Theorem (1.12) : Every connected space is minimal connected space .

Proof : Let (X , τ) be a connected space yet it is not minimal connected space so X = A ∪ B such that A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B minimal open sets then by remark (1.10) (i) we get A and B are open , therefore X is not connected space , that is a contradiction . Then X is minimal connected space .

Remark (1.13) : The theorem converse (1.12) is generally not right .

Example (1.14) : Let X = { 1 , 2 , 3 } , $\tau = \{ X, \phi, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$
minimal open = $\{\{1\},\{2\}\}$. It is clear that X is a minimal connected space yet it is not connected space because X = { 1 } ∪ { 2 , 3 } .

Theorem (1.15) : Every connected space is maximal connected space .

Proof : Let (X , τ) be a connected space yet it is not maximal connected space so X = A ∪ B such that A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B maximal open sets then by remark (1.10) (i) we get A and B are open , therefore X is not connected space that is a contradiction . Then X is maximal connected space . The Theorem converse (1.15) is generally not right .

Example (1.16) : Let X = { 1 , 2 , 3 } , $\tau = \{ X, \phi, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$, maximal open = $\{\{1,2\},\{2,3\}\}$
It is clear that X is a maximal connected space but it is not connected space .because X = { 1 } ∪ { 2 , 3 }

Theorem (1 .17) : Every pre-connected is minimal pre -connected space .

Proof : Let (X , τ) be a pre -connected space yet it is not minimal pre- connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B minimal pre-open sets then by remark (1.10) (ii) we get A and B are pre-open . Therefore X is not pre-connected space . That is a contradiction . Then X is minimal pre-connected space .

Remake (1.18) : Then converse of Theorem (1.17) is generally not right .

Example (1.19) : Let X = { 1 2 , 3 } , $\tau = \{X, \phi, \{1,2\}\}$, Po(x) = $\{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$
minimal pre-open = $\{\{1\},\{2\}\}$.It is clear that X is a minimal pre- connected space yet it is not pre-connected space because X = { 2 } ∪ { 1 , 3 }.

Theorem (1.20) : Every pre-connected is maximal pre- connected space .

Proof : Let (X , τ) be a pre-connected space but it is not maximal pre- connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B maximal pre-open sets then by remark (1.10) (ii) we get A and B are pre-open . Therefore , X is not pre-connected space . That is a contradiction . Then X is maximal pre-connected space .

Remark (1.21) : The Theorem converse (1 .20) is generally not right

Example (1.22) : Let X = { 1 , 2 , 3 } , $\tau = \{X, \phi, \{1,2\}\}$,Po(x) = $\{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$
maximal pre-open = $\{\{1,2\},\{1,3\},\{2,3\}\}$.It is clear that X is a maximal pre-connected space yet it is not pre-connected space . because X = { 2 } ∪ { 1 , 3 }.

Remark (1.23) : [3] Every Pre-connected space is a connected space [3] . But the converse is generally not right .

Example (1.24) : Let X = { 1 , 2 , 3 } , $\tau = \{X, \phi, \{1,2\}\}$,Po(x) = $\{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$
We sum up X is connected space but it is not pre-connected space because X = { 2 } ∪ { 1 , 3 } .

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Vol. 6, Issue 7 , July 2019

Theorem (1.25) : Every pre-connected space is minimal connected space .

Proof : By Remark (1.23) and by Theorem (1.12)

Remark (1.26) : The Theorem converse (1.25) is generally not right .

Example (1.27) Let X = { 1 , 2 , :3 } , $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\{2,3\}\}$

minimalopen = $\{\{1\},\{2\}\}$,Po(x) = $\{X, \phi, \{1\}, \{3\}, \{1,2\}, \{2,3\}\}$ .It is clear that X is a minimal connected space yet it is not pre-connected space . because X = { 1 } ∪ { 2 , 3 }

Theorem (1 . 28) : Every pre-connected space is maximal connected space .

Proof : By Theorem (1.23) and by Theorem (1.15)

Remark (1.29) : The Theorem converse (1.28) is generally not right .

Example (1.30) : Let X = { 1 , 2 , 3 } , $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$, maximal open = { 1 , 2 } , { 2 , 3 } pre-open = $\{X, \phi, \{1\}, \{2\}, \{1,2\}, \{2,3\}\}$.It is clear that X is a maximal connected space yet it is not pre-connected space because X = { 1 } ∪ { 2 , 3 }.

Theorem (1 . 31) : Every open set is a pre – open set , [8] .

Theorem (1.32) :A space of topology X and A ⊆ X . Then :
i) If A is a minimal pre- open , then A is a minimal open .
ii) If A is a maximal pre - open , then A is a maximal open .

Proof :(i) Let A be minimal pre-open and suppose that A not minimal open , then there exists open set U in X as U ⊆ A and U ≠ ϕ , U ≠ A , then there exists pre-open set U in X as U ⊆ A and U ≠ ϕ , U ≠ A , then A not minimal pre-open which is contradiction . Therefore A is a minimal open in X .
(ii) Let A be maximal pre -open and suppose that A not maximal open , then there exists open set U in X . As A ⊆ U and U ≠ X . U ≠ A , then there exists Pre-open set U in X such that A ⊆ U and U ≠ X , U ≠ A , then A not maximal pre- open which is contradiction . Hence A is a maximal open in X .

Theorem (1.33) :A space of topology X . And A ⊆ X . Then :
i) If A is aminimal pre-open (resp. maximal pre-open) , then A isanopen .
ii) If A is aminimal pre-closed (resp. maximal pre-closed) , then A is a closed .

Proof :(i) Follows from the (1.32) (i) , (ii) and Remark (1.10) , (iii) This diagram showing the relationships among kinds of minimal open (resp. maximal open)

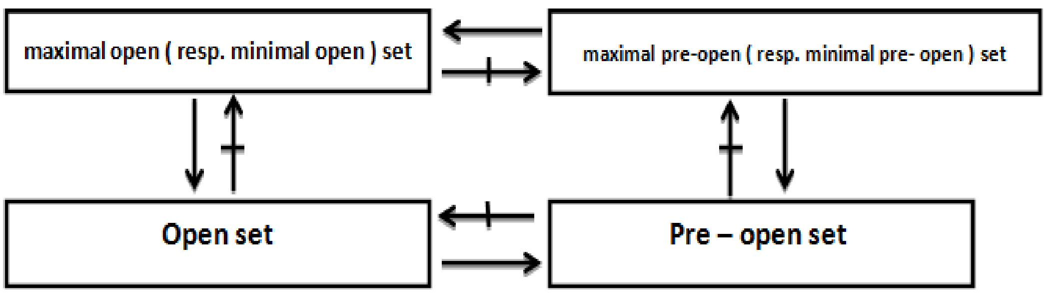

Diagram. 1

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Theorem (1.34) : Every connected space is maximal pre-connected space

Proof : Let (X , τ) be a connected space yet it is not maximal pre-connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B maximal pre-open set then by Theorem (1.33) (i) we get A and B are open , therefore X is not connected space , that is a contradiction . Then X is maximal pre-connected space .

Remark (1.35) : The Theorem converse (1 .34) is generally not right .

Example (1.36) : Let X = { 1 , 2 , 3 } , $\tau = \{X, \phi, \{1\}, \{2.3\}\}$, Po(x) = {X , ϕ ,{1},{2},{3},{1,2}, {1,3}, {2,3}} maximal pre-open = {{ 1, 2 },{ 1 , 3 },{ 2 ,3 }}.It is clear that X is maximal pre-connected space but it is not connected space because X = { 1 } ∪ { 2 , 3 }.

Theorem (1.37) : Every connected space is a minimal pre-connected space .

Proof : Let (X , τ) be a connected space yet it is not minimal pre-connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B minimal pre-open set then by Theorem (1.33) (i) we get A and B are open , therefore X is not connected space , that is a contradiction .Then X is minimal pre-connected space .

Remark (1.38) : The theorem converse (1 .37) is generally not right .

Example (1.39) : Let X = { 1 , 2 , 3 } , τ = { X , ϕ , { 1 } , { 2 , 3 } }, Po(x) = { X , ϕ , { 1 } , { 2 } , { 3 } , { 1 , 2 } , { 1 , 3 } , { 2 , 3 }},minimal pre-open = { { 1 } , { 2 } , { 3 }}.We sum up X is minimal pre-connected space but it is not connected space because X = { 1 } ∪ { 2 , 3 }

Theorem (1.40) : Every minimal connected space is minimal pre-connected space .

Proof : Let (X , τ) be a minimal connected space yet it is not minimal pre-connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B minimal pre-open set then by Theorem (1.32) (i) we get A and B are minimal open , therefore X is not minimal connected space . that is a contradiction . Then X is minimal pre-connected space .

Remark (1.41) : The Theorem converse (1 .40) is generally not right .

Example (1.42) : Let X = { 1 , 2 , 3 } , τ = { X , ϕ , { 1 } , { 2 , 3 } } , Po(x) = { X , ϕ , { 1 } , { 2 } , { 3 } , { 1 , 2 } , { 1 , 3 } , { 2 , 3 }},minimal pre-open = { { 1 } , { 2 } , { 3 } }.It is clear that X is minimal pre-connected space yet it is not minimal connected space because X = { 1 } ∪ { 2 , 3 }

Theorem (1.43) : Every maximal connected space is maximal pre-connected space .

Proof : Let (X , τ) be a maximal connected space yet it is not maximal pre-connected space so X = A ∪ B as A ≠ ϕ , B ≠ ϕ , A ∩ B = ϕ and A , B minimal pre-open set then by Theorem (1.32) (i) we get A and B are minimal open , therefore X is not minimal connected space . that is a contradiction . Then X is maximal pre- connected space .

Remark (1.44) : The theorem converse (2 .43) is generally not right .

Example (1.45) : Let X = { 1 , 2 , 3 } , τ = { X , ϕ , { 1 } , { 2 , 3 }}, Po(x) = { X , ϕ , { 1 } , { 2 } , { 3 } , { 1 , 2 } , { 1 , 3 } , { 2 , 3 }},maximal pre- open = {{ 1 , 2 } , { 1 , 3 } , { 2 , 3 }}.It is clear that X is maximal pre-connected space yet it is not maximal connected space because X = { 1 } ∪ { 2 , 3 }.

Remark (1.46): This diagram showing , the relationships among kinds of minimal connected (resp. maximal connected) space

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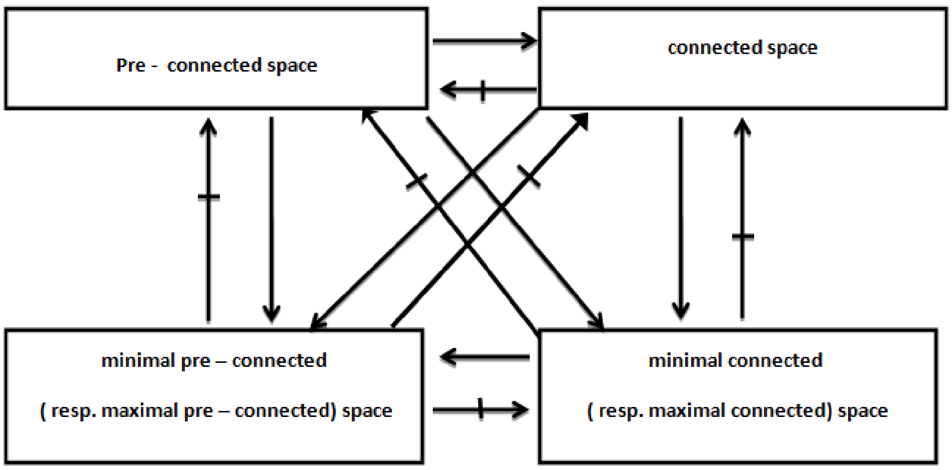

Diagram. 2

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