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Fixed point Theorems on Fuzzy Soft normed space

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ABSTRACT: In this paper, we have introduced the definition of fuzzy soft fixed point in fuzzy soft normed space and obtained some new properties of it in the topology . Moreover, we studied this property in fuzzy soft normed space.

KEY WORDS: Fuzzy soft set, Fuzzy soft topology, Fuzzy soft fixed point

I.INTRODUCTION

In 2002 , Maji et.al gave a new concept called fuzzy soft set , After the rontier work of Maji, many investigator have extended this concept in various branches of mathematics and Kharal and Ahmad in [1] introduced new theories like new properties of fuzzy soft set and then in [8] defined the concept of mapping on fuzzy soft classes and studies of fuzzy soft in topological introduced by Tanay and Kandemir [9].Mahanta and Das [10] continued studies. we essentially concerned in theory of fuzzy soft normed spaces and their generalization . In this paper we have studied fixed point property in fuzzy soft normed space

II. PRELIMINARIES

Definition (2.1) [5] : Let A be a subset of E . A pair (F ,A) is called a fuzzy soft set over (X, E) ,if F: $A \to I^X$ is a mapping from A into I^X .The collection of all fuzzy soft sets over (X , E) is denoted by F(X , E) .

Definition(2.2) [5] :For tow fuzzy soft sets (F ,A) and (G,B) in **F**(U, E) we say that (F, A) $\subseteq$ (G , B) if A $\subseteq$ B and F(e) $\leq$ G(e) .

Definition (2.3) [5]: Two fuzzy soft set (F, A) and (G , B) in **F**(U , E) are equal if $F \subseteq G$ and $G \subseteq F$.

Definition(2.4) [5]: The different between two fuzzy soft set (F,A) and (G,B) in **F**(U , E) is a fuzzy soft set ($F/G, E$) (say) defined by (F/G) (e)= $F(e)/G(e)$ for each $e \in E$.

Definition (2.5) [5]: The complement of a fuzzy soft set (F ,E) is a fuzzy soft set(F^c, E) defined by F^c(e)=$1/F(e)$ for each $e \in E$.

Definition (2.6) [1] : Let (F , A) and (G , B) be two fuzzy soft sets in **F**(U , E) with $A \cap B \neq \Phi$, then a) their intersection ($F \cap G , C$) is a fuzzy soft set , where $C = A \cap B$ and
$(F \cap G)e = F(e) \cap G(e)$ for each $e \in C$ and
b)their union $(F \cup G, C)$ is a fuzzy soft set , where
$C = A \cup B$ and $(F \cup G)e = F(e) \cup G(e)$ for each $e \in C$

Definition (2.7) [7]:A fuzzy soft topology τ on $F \in \boldsymbol{F}(U, A)$ is collection of fuzzy soft subsets of F satisfying :

1. $\Phi , F \in \tau$ (this means that E is fuzzy soft subset of F , that is 1(e) $\leq$ F(e) , that is 1$\leq F(e)(x)$
2. If F_1 , $F_2 \in \tau$ then $F_1 \cap F_2 \in \tau$.
3. If $F_\alpha \in \tau$ for all $\alpha \in \Lambda$, with Λ an index set , then $\cup_{\alpha \in \Lambda} F_\alpha \in \tau$.
4. If τ is a fuzzy soft topology on F then the pair (F, τ) is called a fuzzy soft topological space .

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Definition (2.8) [4],[6]: Let e be any element in a set $A \subseteq E$. A fuzzy soft F over A is called a fuzzy soft element if F(e') is a null fuzzy set for each $e' \in A - \{e\}$. We denote it by (F^e,A) or simply by F^e
A fuzzy soft element F^e is said to be in fuzzy soft set (G, B)if (F^e , A) $\subseteq (G , B)$.
That is , A $\subseteq$ B and $F^e(e') \le G(e')$ for each $e' \in A$, that is $F^e(e) \le G(e')$ for each $e' \in A$. We write it as $F^e \in G$.

Remark(2.9) [2]: Note that if F is a fuzzy soft set in **F**(U, E) and $F^e \in F$ then
F = $\{\cup_{F^e \in F} F^e : e \in E \}$.

Proposition (2.10) [2] : Let F_1, F_2 *be two* fuzzy soft sets over (U , E) and e ∈ E
The following hold .
i)Φ is an empty fuzzy soft element of every fuzzy soft set .
ii) If F is a fuzzy soft such that $F \ne \Phi$, then F contains at least one non empty fuzzy soft element .
iii) If $F^e \in F_1 \cup F_2$ then F^e is a fuzzy soft element of F_1 *or* F_2.
iv) $F^e \in F_1 \cap F_2$ if and only if F^e is a fuzzy soft element of F_1 *and* F_2 .
v)If $F^e \in F_2/F_2$ *then* F^e is a fuzzy soft element of F_1 but not necessarily a fuzzy soft element of F_2.

Proposition (2.11)[2]: Let F_1, F_2be two fuzzy soft sets over E . Then $F_1 \subseteq F_2$ if and only if $F^e \in F_1$ implies $F^e \in F_2$.

Definition(2.12)[4] : A fuzzy soft topological space (F , τ) is said to be a fuzzy soft Hausdorff space if for distinct fuzzy soft element F^e , $F^{e'}$ of F , there exists disjoint fuzzy soft open sets (F_1 , A) and F_2 , A) such that $F^e \in F_1$ and $F^{e'} \in F_2$.

Definition(2.13)[3]: The Cartesian product of two fuzzy soft sets (F , A) and (G , B) is defined as a fuzzy soft set (H , C) = (F, A) × (G , B) , where C = A×B and H : C→**F**(U , E) is defined by

H (e , e') = F (e) × G (e') for all (e , e') ∈ C ,

where F (e)× G (e') = $\{ \frac{x}{\min\{F(e')(x), G(e')(x)\}} : x \in U\}$.

Definition(2.14) [2] : Let F ,G be fuzzy soft sets in **F**(U , E) . A fuzzy soft relation T : $F \to G$ if the following conditions are satisfied .
C1 for each fuzzy soft element F^e ∈ F , there exist only one fuzzy soft element G^e ∈ G such that $F^e T\ G^e$ which will be denoted as T (F^e) = G^e.
C2 for each fuzzy soft empty element $F^e \in F, T(F^e)$ is empty fuzzy soft element of G .

Definition (2.15) [2] : Let (F , τ) be a fuzzy soft topological space and K $\subseteq$ F . A fuzzy soft open cover for K is a collection of fuzzy soft open sets $\{V_i\}_{i \in I} \subseteq \tau$ whose union contain K .

Definition(2.16) [2] : A fuzzy soft topological space (F ,τ) is compact if for each fuzzy soft open cover $\{V_i\}_{i \in I}$ of K there exist $i_1, i_2, \dots, i_k \in I , k \in N$ such that $K \subseteq \cup_{i=1}^{k} V_{i_n}$.

Definition(2.17) [2] : Let $(F , \tau), (G , \upsilon)$ be fuzzy soft topological spaces and T : F → G
A soft

Proposition(2.18) [2]: Let (K , τ) be a fuzzy soft compact topological space and T $K \to K$ a fuzzy soft continuous mapping . Then T(K) is a fuzzy soft compact set in (K , τ) .

Definition(2.19) [2]: Let F ∈ **F**(U,A) BE A fuzzy soft set and T: $F \to F$ a fuzzy soft mapping . A fuzzy soft element $F^e \in F$ is called a fixed point of T if T $(F^e) = F^e$.

Example(2.20) [2] : If T:$F \to F$ is defined as an identity map , then each fuzzy soft element of F is affixed point .

Proposition(2.21) [2] : Let (F ,τ) be A fuzzy soft compact topological space anf { $F_n : n \in N$} a countable family of fuzzy soft subsets of F satisfying :

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1) $F_n \neq \emptyset$ for each $n \in N$.
2) F_n is fuzzy soft closed for each $n \in N$,
3) $F_{n+1} \subseteq F_n$ for each $n \in N$. Then $\cap_{n \in N} F_n \neq \Phi$.

Proposition(2.22)[2]: Let (F, τ) be a fuzzy soft topological space and T:$F \to F$ a fuzzy soft mapping such that for each non empty fuzzy soft element $F^e \in F$, $T(F^e)$ is a non empty fuzzy soft element of F . If $\cap_{n \in N} T^n(F)$ contains only one nonempty fuzzy soft element $F^e \in$ F , then F^e is a unique fixed point of T .

Proposition(2.23)[2] :Let (F, τ) be a fuzzy soft Hausdorff topological space. Then every Fuzzy soft compact set in F is Fuzzy soft closed in F .

III. MAIN RESULTS

Definition(3.1): Let $(X, E, \|.\|)$ be a fuzzy soft normed space and $f : X \to X$ be a function from a set X to itself. We call a point $x \in X$ is called a fixed point of f if $f(x) = x$**.**

Example (3.2) [2]: If $f: F \to F$ is defined as an identitiy function , then each fuzzy soft element of F is a fixed point .

Definition(3.3), : Let $(X, E, \|.\|)$ be a fuzzy soft normed space and The function $f: X \to X$ is said to be contraction on X, if there exist $0 < k < 1$, $t > 0$ such that $E(f(x), f(y), t) \leq kE(x, y, t)$ for all $x, y \in X$.

Theorem(3.4) : Let $(X, E, \|.\|)$ be a Fuzzy Soft Banach space and let $f: X \to X$ be a contraction function , then f has a unique fixed point on X.

Theorem(3.5) : let $f: X \to X$ be a contraction on Fuzzy Soft banach space X, let Y be a closed subset of X . Then the unique fixed point of f is in Y.

Proof :
Since Y is closed subset of a F.S Banach space .
Then Y is complete .
We can apply the contraction function $f: Y \to Y$, so f has fixed point in Y
Since f has only one fixed point in X , then f has one fixed point in Y.

Theorem(3.6) : If there exist $0 < k < 1$ such that
$E(x, y, kt) \leq E(x, y, t)$ for all $x, y \in X$ and $t > 0$. Then $x = y$.

Theorem(3.7) : Let $(X, M, *)$ be a F.S Banach space and $f: X \to X$ be a contraction $E(f(x), f(y), t) \leq E(x, y, t)$ for all $x, y \in X$
then f has a unique fixed point

Theorem(3.8) : Let A be a nonempty , closed , convex subset of a fuzzy soft normed space X with $f : A \to A$ nonexpansive and $f(A)$ a subset of a compact set of A . Then f has a fixed point .

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