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Sustainable Algorithms of Adaptive Management of Lagged Objects

H.Z. Igamberdiev, N.Z. Kamalov, D.R.Karimov

Professor, Department of Information processing and control systems, Tashkent State Technical University, Uzbekistan
Professor, Joint-stock company "Paxtasanoat ilmiy Markazi" , Uzbekistan
Doctoral Student, Department of Information processing and control systems, Tashkent State Technical University, Uzbekistan

ABSTRACT: Algorithms for the formation of stable algorithms for adaptive control of objects with delay based on the concepts of matrix pseudo-inversion are presented. For stable pseudo-inversion of matrices, the Hermite method is used by means of two-sided multiplication by non-degenerate matrices. The given stable algorithms for the synthesis of the system of adaptive control of objects with delay allow programmed motion according to the desired law of the state vector change over time and ensure the convergence of the motion of the system to a certain neighborhood relative to the reference motion.

KEY WORDS: objects with delay, adaptive control, robust algorithms, pseudo-inversion.

I.INTRODUCTION

To build an adaptive system with a reference model, you need to implement a reference model in the form of some dynamic link. Obviously, for a complex control object, the implementation of the reference model has certain difficulties. There is a need to build such adaptive systems that do not require the construction of a reference model as a separate subsystem. One of the possible ways to build adaptive systems in such a situation is to use control systems with an implicit reference model, making it possible to reduce the requirements for the main control circuit and information about the measured parameters, and it is much easier to design a control system with not completely measured by the state vector of the object. In addition, after adjusting the parameters of the control device, the quality of transients is close to a certain reference transition process determined by an implicit reference model [1-6].

II. FORMULATION OF THE PROBLEM

We will consider a dynamic object with a delay in control, described by the following equation:

$$x_{k+1} = (A_k + a(x_k,\xi_k))\cdot x_k + (B_k + b(x_k,\xi_k))\cdot u_{k-\tau} + f_k;$$
$$y_k = C\cdot x_k, \qquad (1)$$
$$u(s) = \psi(s),\ s\in[-\tau,0],$$

where $x_k - n-$ measured state vector of the object; $y_k - l-$ dimensional vector of output variables; $u_k - m-$ dimensional vector of control actions; $\tau-$ known persistent object lag; $\xi_k -$ vector of constraints of dimensionality of changing object parameters; $A_1 = A[\xi_k]$ and $B_1 = B[\xi_k]-$ non-stationary matrices of the corresponding dimensions; $a(x_k,\xi_k)$ and $b(x_k,\xi_k)-$ some non-linear adds; $\psi(s)-$ initial vector-function; $f_k - n-$ dimensional vector of external disturbances, such that $\|f_k\| \le M_f$, $M_f = const;$ $C-$ constant output matrix.

The equation of the object (1) is rewritten in the following form:

$$x_{k+1} = A_k\cdot x_k + B_k\cdot u_{k-\tau} + \sigma;\ u_s = \psi_s,\ s\in[-\tau,0];$$
$$y_k = C\cdot x_k, \qquad (2)$$

where $\sigma = a(x_k,\xi_k)\cdot x_k + b(x_k,\xi_k)\cdot u_{k-\tau} + f_k -$ nonlinear part of the object in conjunction with the vector of perturbations.

The control law of the object (2) is given in the form [2,4,6]:

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$$u_k = g_k - D\cdot\tilde{x}_k + B_0^+\cdot\mu_k - B_0^+\cdot K_a\cdot\tilde{x}_k - B_0^+\cdot K_b\cdot u_k\,, \tag{3}$$

where g_k - driving vector; D - constant matrix of linear modal feedbacks; $B_0^+ = (B_0^T B_0)^{-1} B_0^T$ – left pseudo inverse B_0 matrix size $l\times n$; $\mu_k - n$ - dimensional vector of adaptation signals of an object; vector components μ_k are formed by averaging the components of the vector z using filters with small time constants $\tau_i > 0,\; i = 1,...,r$:

$$\tau\cdot\mu_{k+1} + \mu_k = z\,, \tag{4}$$

where $\tau = diag(\tau_1,...,\tau_r)$.

For simplicity, in (4) you can choose the time constants that are the same for all components.

Assuming known linear stationary approximation of the object without delay, having the form $x_{k+1} = A_0\cdot x_k + B_0\cdot u_k$, for which the conditions of controllability and observability are satisfied, we rewrite equation (2) in the form [4,6]:

$$x_{k+1} = (A_0 + K_a)\cdot x_k + (B_0 + K_b)\cdot u_k + \sigma(x_k,k)\,, \tag{5}$$

where $\sigma(x_k,k) = \sigma$ – the residual vector, taking into account the nonlinearity of the object and the action of external disturbances.

III. SOLUTION OF THE TASK

The purpose of adaptive control of an object of the form (1) or (5) is to effect programmatic movement according to the desired law of a change in the state vector over time. Let the software move x_k^M system (5) satisfies the equation implicitly given reference model:

$$x_{k+1}^M = A^M\cdot x_k^M + B^M\cdot g_k,\qquad x_{k_0}^M = x_0^M\,, \tag{6}$$

where $g_k - m-$ dimensional vector of setting effects; $A^M = A_0 - B_0\cdot D;\;\; B^M = B_0$.

It is required to synthesize such a control law u_k, which would ensure the convergence of motion x_k systems (5) to some $\delta_0 -$ neighborhood relative to the reference movement x_k^M. This requires enforcing inequality:

$$\left\|x_k - x_k^M\right\| = \left\|\tilde{e}_k\right\| \le \delta_0\;\;\forall t \ge t_a,$$

where $\tilde{e}_k - n-$ control error vector; t_a - adaptation time.

The equation of state of the source object (5), taking into account the control algorithm (3), is written in the form

$$\begin{aligned} x_{k+1} &= A_0\cdot x_k + K_a\cdot x_k + (B_0 + K_b)\times \\ &\times[g_k - D\cdot\tilde{x}_k + B_0^+\cdot\mu_k - B_0^+\cdot K_a\cdot\tilde{x}_k + B_0^+\cdot K_b\cdot u_k] + \sigma. \end{aligned} \tag{7}$$

Based on (7) and taking into account (6), the control error equation is rewritten as

$$\begin{aligned} \tilde{e}_{k+1} &= A_0\cdot x_k + K_a\cdot x_k + B_0\cdot g_k - B_0\cdot D\cdot\tilde{x}_k - \\ &- B_0\cdot B_0^+\cdot K_a\cdot\tilde{x}_k - A^M\cdot x_k^M - B^M\cdot g_k^M + \sigma + \\ &+ B_0\cdot B_0^+\cdot\mu_k - B_0\cdot B_0^+\cdot K_b\cdot u_k + K_b\cdot u_k. \end{aligned} \tag{8}$$

Select in the expression (8) $e_k = x_k - \tilde{x}_k$ - error of estimation of state variables of the object without delay by the second custom model [4,6,7]:

$$\tilde{x}_{k+1} = A_k\cdot\tilde{x}_k + G\cdot C\cdot e_k - B_k\cdot u_k + \sigma\,.$$

When fulfilling the purpose of identification $\left\|e_k\right\| \le \varepsilon_0,\;\forall t \ge t_M$ will be performed and the purpose of identification $\lim\limits_{k\to k_M}\left\|e_k\right\| = \lim\limits_{k\to k_M}\left\|x_k - \tilde{x}_k\right\| \le \varepsilon_0,\;\forall k \ge k_M$, where $k_M \to k_y,\; k_y -$ object management time.

In this case, expression (8) can be rewritten as

$$\begin{aligned} \tilde{e}_{k+1} &= (A_0 - B_0\cdot D)\cdot x_k + B_0\cdot g_k + B_0\cdot D\cdot e_k - \\ &- A^M\cdot x_k^M - B^M\cdot g_k^M + \sigma + B_0\cdot B_0^+\cdot\mu_k + B_0\cdot B_0^+\times \\ &\times K_a\cdot e_k + (I - B_0\cdot B_0^+)\cdot K_a\cdot x_k + (I - B_0\cdot B_0^+)\cdot K_b\cdot u_k. \end{aligned} \tag{9}$$

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If the conditions of management consistency are fulfilled [8] $(I - B_0 \cdot B_0^+) \cdot (A^M - A_0) = 0$ and $(I - B_0 \cdot B_0^+) \cdot B^M = 0$, then we get the ratio:

$$A_0 - B_0 \cdot D = A^M \text{ and } B_0 = B^M \tag{10}$$

Taking into account relations (10), we rewrite expression (9) in the form:

$$\tilde{e}_{k+1} = A^M \cdot \tilde{e}_k + B_0 \cdot D \cdot e_k + \sigma + B_0 \cdot B_0^+ \cdot \mu_k + B_0 \cdot B_0^+ \cdot K_a \cdot e_k,$$

and the elements of the matrix D are calculated from relations (10).

Entering the designation $\xi_k = B_0 \cdot D \cdot e_k + \sigma + B_0 \cdot B_0^+ \cdot \mu_k + B_0 \cdot B_0^+ \cdot K_a \cdot e_k$, finally get the equation for the control error:

$$\tilde{e}_{k+1} = A^M \cdot \tilde{e}_k + \xi_k.$$

In the above conditions, the consistency of identification

$$(I - L \cdot L^+) \cdot \sigma = 0$$

performed [3,6,7], i.e. asymptotic stability of the estimation process takes place $e_k \to 0$ at $t_k \to \infty$, where $L = P^{-1} C^T$, $P-$ symmetric positive definite matrix.

Let, moreover, the condition of consistency on the adaptation of the species

$$(I - B_0 \cdot B_0^+) \cdot \sigma = 0.$$

In this case, the system is asymptotically stable and the goal of the control is achieved $\lim\limits_{\tau \to 0} \|\tilde{e}_k\| = 0$ [4,7].

In expressions (3) and (7) to form the law of governance u_k and determining the state vector of an object x_k pseudoinverse is present B_0^+. It is clear that the quality of control processes of the synthesized adaptive control system significantly depends on the accuracy of determining the control and state vector in accordance with (3) and (7). In view of this circumstance, it is necessary to use efficient pseudo-inversion algorithms for over determined matrices.

It is known [9–11] that the problem of calculating a pseudoinverse matrix in the general case is unstable with respect to errors in specifying the initial matrix. At the same time, the errors of the source data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of model adequacy to the real process. The effect of rounding errors produced during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed on the basis of known methods of analysis and the balance of accuracy.

In this case, it is advisable to use the Hermite method [12,13], which is economical and has a sufficiently high accuracy. According to this method, the pseudo-inverse matrix is determined based on the expression

$$B_0^+ = B_0^T M_R^- B_0 B_0^T,$$

where

$$M = (B_0 B_0^T)^2,$$

but M_R^- – reflexive g-inverse (generalized inverse) for matrix *M*.

One possible direct construction method g - inverse [12] uses a cast $(m \times n)$ - matrices *N* by two-sided multiplication by non-degenerate matrices to the form

$$R = PNQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{11}$$

Here $rank\, N$, and zero blocks have dimensions $r \times (n-r)$, $(m-r) \times r$ and $(m-r) \times (n-r)$.

If matrices *P* and *Q* in equality (11) are known, then every $(n \times m)$ - matrix $\hat{R}$ block structure

$$\hat{R} = \begin{bmatrix} I_r & U \\ V & W \end{bmatrix}$$

generates g - inverse to the matrix *N* by the formula

$$G = Q\hat{R}P. \tag{12}$$

Thus, the considered method provides for a two-fold reduction to the normal lower case form. In the Hermite algorithm, the matrix M is reduced to the form (12) as follows [11-14]: first, M is reduced to normal lower case form M_1, what can be described by the relation:

$$EM = M_1 , \tag{13}$$

where E – non-degenerate matrix; then the conjugate matrix M_1^T also reduced to normal row form R by a non-degenerate matrix F:

$$FM_1^T = R . \tag{14}$$

The matrices M and R in this case are Hermitian, and R is a diagonal matrix with r units and others - zeros on the main diagonal.
Using this circumstance, one can obtain from (13) and (14) the equality

$$R = EMF^T .$$

According to (12), it follows that

$$M_R^- = F^T RE .$$

IV. CONCLUSION AND FUTURE WORK

The given stable algorithms for the synthesis of the system of adaptive control of objects with a delay allow you to programmatically move according to the desired law of change over time of the state vector and ensure the convergence of the movement of the system to a certain neighborhood with respect to the reference movement.

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