Sustainable Algorithms of Adaptive Management of Lagged Objects

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ABSTRACT: Algorithms for the formation of stable algorithms for adaptive control of objects with delay based on the concepts of matrix pseudo-inversion are presented. For stable pseudo-inversion of matrices, the Hermite method is used by means of two-sided multiplication by non-degenerate matrices. The given stable algorithms for the synthesis of the system of adaptive control of objects with delay allow programmed motion according to the desired law of the state vector change over time and ensure the convergence of the motion of the system to a certain neighborhood relative to the reference motion.

KEY WORDS: objects with delay, adaptive control, robust algorithms, pseudo-inversion.

I. INTRODUCTION

To build an adaptive system with a reference model, you need to implement a reference model in the form of some dynamic link. Obviously, for a complex control object, the implementation of the reference model has certain difficulties. There is a need to build such adaptive systems that do not require the construction of a reference model as a separate subsystem. One of the possible ways to build adaptive systems in such a situation is to use control systems with an implicit reference model, making it possible to reduce the requirements for the main control circuit and information about the measured parameters, and it is much easier to design a control system with not completely measured by the state vector of the object. In addition, after adjusting the parameters of the control device, the quality of transients is close to a certain reference transition process determined by an implicit reference model [1-6].

II. FORMULATION OF THE PROBLEM

We will consider a dynamic object with a delay in control, described by the following equation:

$$x_{k+1} = (A_k + a(x_k, \zeta_k^s)) \cdot x_k + (B_k + b(x_k, \zeta_k^s)) \cdot u_{k-\tau} + f_k;$$

$$y_k = C \cdot x_k,$$  (1)

where $x_k$ - $n$ - measured state vector of the object; $y_k$ - $l$ - dimensional vector of output variables; $u_k$ - $m$ - dimensional vector of control actions; $\tau$ - known persistent object lag; $\zeta_k$ - vector of coefficients of dimensionality of changing object parameters; $A_k = A_{k\|\zeta_k^s\|}$ and $B_k = B_{k\|\zeta_k^s\|}$ - non-stationary matrices of the corresponding dimensions; $a(x_k, \zeta_k^s)$ and $b(x_k, \zeta_k^s)$ - some non-linear adds; $\psi(s)$ - initial vector-function; $f_k$ - $n$ - dimensional vector of external disturbances, such that $\|f_k\| \leq M_f, M_f = const; C$ - constant output matrix.

The equation of the object (1) is rewritten in the following form:

$$x_{k+1} = A_k \cdot x_k + B_k \cdot u_{k-\tau} + \sigma \cdot u_k = \psi(s), s \in [-\tau \Omega],$$

$$y_k = C \cdot x_k,$$  (2)

where $\sigma = a(x_k, \zeta_k^s) \cdot x_k + b(x_k, \zeta_k^s) \cdot u_{k-\tau} + f_k$ - nonlinear part of the object in conjunction with the vector of perturbations.

The control law of the object (2) is given in the form [2,4,6]:

$$u_k = \psi(s).$$
The purpose of adaptive control of an object of the form (1) or (5) is to effect programmatic movement according to the desired law of a change in the state vector over time. Let the software move $x^M_k$ system (5) satisfies the equation implicitly given reference model:

$$x^M_{k+1} = A^M_k \cdot x^M_k + B^M_k \cdot g_k,$$

where $g_k$ - $m$- dimensional vector of settings; $A^M_k = A_0 - B_0 \cdot D$; $B^M_k = B_0$.

It is required to synthesize a control law $u_k$, which would ensure the convergence of motion $x_k$ systems (5) to some $\delta_0$ - neighborhood relative to the reference movement $x^M_k$. This requires enforcing inequality:

$$\left\| x_k - x^M_k \right\| = \left\| \epsilon_k \right\| \leq \delta_0 \forall t \geq t_0,$$

where $\epsilon_k = \text{control error vector}$; $t_0$ - adaptation time.

The equation of state of the source object (5), taking into account the control algorithm (3), is written in the form

$$x_{k+1} = A_0 \cdot x_k + B_0 \cdot x_k + (B_0 + K_0) \times$$

$$\left[ g_k - D \cdot \dot{x}_k + B_0 \cdot \mu_k - B_0 \cdot K_0 \cdot \dot{x}_k + B_0 \cdot K_0 \cdot u_k \right] + \sigma,$$

Based on (7) and taking into account (6), the control error equation is rewritten as

$$\epsilon_{k+1} = A_0 \cdot \epsilon_k + K_0 \cdot \dot{x}_k + B_0 \cdot g_k - B_0 \cdot D \cdot \dot{x}_k -$$

$$- B_0 \cdot B_0 \cdot \dot{x}_k - A^M \cdot x^M_k - B^M \cdot g^M_k + \sigma +$$

$$+ B_0 \cdot \mu_k - B_0 \cdot K_0 \cdot u_k + B_0 \cdot u_k.$$

Select in the expression (8) $\epsilon_k = x_k - \tilde{x}_k$ - error of estimation of state variables of the object without delay by the second custom model [4,6,7]:

$$\tilde{x}_{k+1} = A_0 \cdot \tilde{x}_k + G \cdot C \cdot \epsilon_k - B_0 \cdot u_k + \sigma.$$

When fulfilling the purpose of identification $\left\| \epsilon_k \right\| \leq \epsilon_0$, $\forall t \geq t_M$ will be performed and the purpose of identification $\lim_{k \to M} \left\| \epsilon_k \right\| = \lim_{k \to M} \left\| x_k - \tilde{x}_k \right\| \leq \epsilon_0$, $\forall k \geq k_M$, where $k_M \rightarrow k_\gamma$, $k_\gamma$ - object management time.

In this case, expression (8) can be rewritten as

$$\epsilon_{k+1} = (A_0 - B_0 \cdot D) \cdot x_k + B_0 \cdot g_k + B_0 \cdot D \cdot \epsilon_k -$$

$$- A^M \cdot x^M_k - B^M \cdot g^M_k + \sigma + B_0 \cdot B_0 \cdot \mu_k + B_0 \cdot B_0 \cdot \times$$

$$\times K_0 \cdot \epsilon_k + (I - B_0 \cdot B_0) \cdot K_0 \cdot x_k + (I - B_0 \cdot B_0) \cdot K_0 \cdot u_k.$$

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If the conditions of management consistency are fulfilled \([8]\) \( (I - B_0 \cdot B_0^T) \cdot (A^T - A_0) = 0 \) and \((I - B_0 \cdot B_0^T) \cdot B^T = 0 \), then we get the ratio:

\[
A_0 - B_0 \cdot D = A^T \quad \text{and} \quad B_0 = B^T
\]  
(10)

Taking into account relations (10), we rewrite expression (9) in the form:

\[
\varepsilon_{k+1} = A^T \cdot \varepsilon_k + B_0 \cdot D \cdot e_k + \sigma + B_0 \cdot B_0^T \cdot \mu_k + B_0 \cdot B_0^T \cdot K_u \cdot e_k,
\]

and the elements of the matrix \(D\) are calculated from relations (10).

Entering the designation \( \varepsilon_k = B_0 \cdot D \cdot e_k + \sigma + B_0 \cdot B_0^T \cdot \mu_k + B_0 \cdot B_0^T \cdot K_u \cdot e_k \), finally get the equation for the control error:

\[
\varepsilon_{k+1} = A^T \cdot \varepsilon_k + \varepsilon_k.
\]

In the above conditions, the consistency of identification

\[
(I - L \cdot L^T) \cdot \sigma = 0
\]

performed \([3,6,7]\), i.e. asymptotic stability of the estimation process takes place \( e_k \to 0 \) at \( t_k \to \infty \), where \( L = P^{-1} C^T \), \( P \) – symmetric positive definite matrix.

Let, moreover, the condition of consistency on the adaptation of the species

\[
(I - B_0 \cdot B_0^T) \cdot \sigma = 0.
\]

In this case, the system is asymptotically stable and the goal of the control is achieved \( \lim_{t \to \infty} \|e_k\| = 0 \) \([4,7]\).

In expressions (3) and (7) to form the law of governance \( u_k \) and determining the state vector of an object \( x_k \) pseudoinverse is present \( B_0^T \). It is clear that the quality of control processes of the synthesized adaptive control system significantly depends on the accuracy of determining the control and state vector in accordance with (3) and (7). In view of this circumstance, it is necessary to use efficient pseudo-inversion algorithms for over determined matrices.

It is known \([9–11]\) that the problem of calculating a pseudoinverse matrix in the general case is unstable with respect to errors in specifying the initial matrix. At the same time, the errors of the source data naturally depend on the accuracy of the experimental studies, and the characteristics of the calculated process depend on the degree of model adequacy to the real process. The effect of rounding errors produced during the implementation of the computational procedure on the accuracy of the desired solution can be analyzed on the basis of known methods of analysis and the balance of accuracy.

In this case, it is advisable to use the Hermite method \([12,13]\), which is economical and has a sufficiently high accuracy. According to this method, the pseudo-inverse matrix is determined based on the expression

\[
B_0^T = B_0^T M_R B_0 B_0^T,
\]

where

\[
M = (B_0 B_0^T)^{-1},
\]

but \( M_R \) – reflexive \( g \)-inverse (generalized inverse) for matrix \( M \).

One possible direct construction method \( g \) - inverse \([12]\) uses a cast \((m \times n)\) - matrices \( N \) by two-sided multiplication by non-degenerate matrices to the form

\[
R = P N Q = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.
\]  
(11)

Here \( rank N \), and zero blocks have dimensions \( r \times (n - r) \), \((m - r) \times r\) and \((m - r) \times (n - r)\).

If matrices \( P \) and \( Q \) in equality (11) are known, then every \((n \times m)\) - matrix \( \tilde{R} \) block structure

\[
\tilde{R} = \begin{bmatrix} I_r & U \\ V & W \end{bmatrix}
\]

generates \( g \) - inverse to the matrix \( N \) by the formula

\[
G = Q \tilde{R} P.
\]  
(12)

Thus, the considered method provides for a two-fold reduction to the normal lower case form. In the Hermite algorithm, the matrix \( M \) is reduced to the form (12) as follows \([11-14]\): first, \( M \) is reduced to normal lower case form \( M_j \), what can be described by the relation:
\[ EM = M_1, \quad (13) \]

where \( E \) – non-degenerate matrix; then the conjugate matrix \( M_1^T \) also reduced to normal row form \( R \) by a non-degenerate matrix \( F \):

\[ FM_1^T = R. \quad (14) \]

The matrices \( M \) and \( R \) in this case are Hermitian, and \( R \) is a diagonal matrix with \( r \) units and others - zeros on the main diagonal.

Using this circumstance, one can obtain from (13) and (14) the equality

\[ R = EMF^T. \]

According to (12), it follows that

\[ M_r = F^T RE. \]

**IV. CONCLUSION AND FUTURE WORK**

The given stable algorithms for the synthesis of the system of adaptive control of objects with a delay allow you to programatically move according to the desired law of change over time of the state vector and ensure the convergence of the movement of the system to a certain neighborhood with respect to the reference movement.

**REFERENCES**