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Analysis of the Stability of Control Systems of Technological Objects Based on the Concepts of the Interval Representation of Basic Data

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ABSTRACT: The article proposes a procedure for formalizing mathematical models of control objects with uncertain parameters. The basic definitions are given on stability analysis, root localization and the application of simplified criteria for resistance to polynomials, the coefficients of which can take any value from given intervals. Algorithms for stability analysis of controlled systems with uncertain parameters are proposed.

KEYWORDS: control object, interval uncertainty, stability of polynomials and matrices, instability, crushing of mountain ores, interval algorithm, interval control systems for technological objects.

I. INTRODUCTION

In order to provide linear and nonlinear control systems of continuous technological objects of such properties as high accuracy, stability and reliability, when designing it it is necessary to take into account the possibility of deviation of the real parameters of the control object from the nominal values under the influence of various destabilizing factors. The latter include, for example, vibrations, equipment defects, measurement errors, aging and wear of individual elements of the system, changes in operating modes, fluctuations in temperature, humidity, pressure. Usually, the calculation of the parameters of control systems is performed on the basis of a simplified mathematical model that roughly describes the real physical process and has undefined parameters, for which only the boundaries of the ranges of change are known. Therefore, one of the important problems arising in the design of control systems is the problem of stability analysis in the conditions of uncertainty of parameters, which is one of the key factors that guarantee the applicability of models and reliability of the designed systems. In fact, the results obtained in the theory of stability in the conditions of uncertainty of parameters allow to ensure the dynamic safety of controlled systems at the stage of their design and operation.

II. RELATED WORK

This problem was first considered by Faedo [1] in 1953, which introduced sufficient conditions for the stability of a family of polynomials in a form close to the Routh table. Applying the Routh criterion to the interval polynomial and performing all the operations of interval arithmetic, two Routh tables (lower and upper) are obtained. The positivity of the lower bounds for the first column guarantees the stability of the family of polynomials.

The fundamental results that determine the necessary and sufficient conditions for the asymptotic stability of the interval characteristic polynomial were obtained by Kharitonov [2]. They are contained in the statements of two theorems, called the weak and strong Kharitonov theorem, respectively. The proof of these theorems was constructed by the author on the basis of induction on. It should be noted that in the future a simpler and more obvious proof of a strong theorem was proposed, using an analysis of the properties of interval polynomials in the frequency domain [3].

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An analysis of the scientific and technical literature of recent years concerning research on the development of interval methods for analyzing the stability or instability of control systems for continuous technological objects, indicates the achievement of significant theoretical and practical results in this area.

At the same time, the literature does not sufficiently assess the capabilities of interval methods in the problems of analysis and study of the asymptotic stability of interval control systems for continuous technological objects. Algorithms for the analysis of robust stability of interval control systems also require their development. This is due to the fact that no constructive methodology has been developed to create analytical and computational methods for studying the stability and instability of control systems for continuous technological objects under conditions of interval-parametric uncertainty.

II. FORMULATION OF THE PROBLEM

In the theory of automatic control, two main types of mathematical models of objects are widely used - models in physical input-output variables and models in state variables. Parametric uncertainty can be linked to both of these kinds of models in various ways. These methods have one and the same physical meaning, one or another of its mathematical interpretations allows the use of various methods for solving problems and, as a result of this, to obtain results that differ in practical orientation.

The greatest variety of methods for introducing parameter uncertainty exists for models in state variables. Most often [4] the following object is investigated:

$$\dot{x}(t) = (A_0 + \Delta A)\cdot x(t) + (B_0 + \Delta B)\cdot u(t),\qquad (1)$$
$$y(t) = (C_0 + \Delta C)\cdot x(t),$$

where $x - n$ - dimensional state vector, $u - r$ - dimensional control vector, $y - r_1$ - dimensional vector of measured variables, t - independent variable (time), A_0, B_0, C_0 - numerical matrices matched with vectors of x, u, y sizes, characterizing the nominal (calculated) mode of the object, $\Delta A, \Delta B, \Delta C$ - matrices, matching in size with matrices A_0, B_0, C_0 and containing indefinite elements.

One way to describe the uncertainty of object parameters is to introduce the dependence of matrices A, B, C on a vector or scalar parameter q, limited by two-sided inequality $q_{\min} \le q \le q_{\max}$ [5]:

$$\dot{x}(t) = A(q)\cdot x(t) + B(q)\cdot u(t),\qquad (2)$$
$$y(t) = C(q)\cdot x(t).$$

Dimension τ of vector parameter q is equal to the number of independent indefinite parameters in matrices A, B, C. If in this case the nominal mode q_0 is set, then an intermediate model between (1) and (2) can be obtained [6]

$$\dot{x}(t) = (A_0 + \Delta A(q_1))\cdot x(t) + (B_0 + \Delta B(q_2))\cdot u(t),\qquad (3)$$
$$y(t) = (C_0 + \Delta C(q_3))\cdot x(t),$$

where $q^T = \left\| q_1^T \vdots q_2^T \vdots q_3^T \right\|$, $q_{1\min} \le q_1 \le q_{1\max}$, $q_{2\min} \le q_2 \le q_{m2ax}$, $q_{3\min} \le q_3 \le q_{3\max}$, $q_1 - n_1$ - dimensional vector, $q_2 - n_2$ - dimensional vector, $q_3 - n_3$ - dimensional vector. If the matrices $\Delta A, \Delta B, \Delta C$ depend on q_1, q_2, q_3 linearly, then (3) can be represented as

$$\dot{x}(t) = \left(A_0 + \sum_{i=1}^{n_1} A_i \cdot q_1^{(i)}\right)\cdot x(t) + \left(B_0 + \sum_{i=1}^{n_2} B_i \cdot q_2^{(i)}\right)\cdot u(t),\qquad (4)$$
$$y(t) = \left(C_0 + \sum_{i=1}^{n_3} C_i \cdot q_3^{(i)}\right)\cdot x(t).$$

The most common way to set the uncertainty in solving practical problems is the numerical interval.

Indeed, ignorance of the exact numerical value of the physical parameter q is easier and more natural to describe in the form

$$q \in [\underline{q}, \overline{q}],$$

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where $\underline{q}$ is the lower boundary, $\overline{q}$ is the upper boundary of the interval $\mathbf{q}$ ($\underline{q},\overline{q}$-know numbers).

In this case, a parametric indefinite object has a model (2), in order to emphasize the interval character of the uncertainty of parameters, we introduce the following notation

$$\dot{x}(t) = A(\mathbf{q}) \cdot x(t) + B(\mathbf{q}) \cdot u(t). \qquad (5)$$

If matrices $A(\mathbf{q}),\ B(\mathbf{q})$ depend on different elements of interval vector $\mathbf{q}$, then they are interval matrices

III. SOLUTION OF THE TASK

So, we have the following model, which describes the dynamics of the object in a first approximation

$$\dot{x}(t) = A(\mathbf{q}) \cdot x(t) + b(\mathbf{q}) \cdot u(t), \qquad (6)$$

where $x(t) - n$ is the dimensional state vector, $u(t)$ is the scalar control, $\mathbf{q} = \|\mathbf{q}_1 \vdots \mathbf{q}_2 \vdots \cdots \vdots \mathbf{q}_\tau\| - \tau$ is the dimensional row vector of the indefinite parameters of the object, $A(\mathbf{q}) - n \times n$ is the matrix depending on the uncertain parameters, $b(\mathbf{q}) - n$ is the dimensional vector depending on the uncertain parameters.

Matrix A and vector b can depend both on all $\mathbf{q}_i\ (i = \overline{1,\tau})$ and on separate elements of vector $\mathbf{q}$, and the dependence can be linear or nonlinear. Once again, we note that a record of the form (6) is symbolic; it shows that a lot of linear continuous stationary objects are considered, each of which is obtained with an arbitrary set of parameters $q \in \mathbf{q}$. For vector $\mathbf{q}$, we will use record $\mathbf{q} = \lfloor \underline{q}, \overline{q} \rfloor$, where by $\underline{q}$ we mean a real vector made up of the lower boundaries $\underline{q}_1, \underline{q}_2, \ldots, \underline{q}_n$, and $\overline{q}$ means a real vector made up of the upper borders $\overline{q}_1, \overline{q}_2, \ldots, \overline{q}_r$ of the corresponding given intervals.

Since the implementation of control law $\mathrm{u(t) = k \cdot x(t)}$ can be carried out only with some tolerances on the coefficients, it is proposed to consider it in the form

$$u(t) = \mathbf{k} \cdot x(t), \qquad (7)$$

where

$$k_j \in [\underline{k}_j, \overline{k}_j]\ \ (j = \overline{1,n}) \qquad (8)$$

$\underline{k}_j$ and $\overline{k}_j$ - are the desired numbers.

The task of the synthesis of controllers is to find the vector $\mathbf{k}$ of the control law (7) with constant coefficients and maximum tolerances (8) on them so that for any combination of parameters $q_i\ \ (i = \overline{1,r})$ and $k_j\ \ (j = \overline{1,n})$ the poles of the closed system (6), (7) are located in the desired region Ω of the left half of the complex plane, having the form of a trapezoid with given η^*, ξ^*, φ^*.

The modal control method consists of two main stages:

- wording of the system requirement in the form of a distribution of the roots of a closed system on a complex plane or setting the coefficients of the desired polynomial;
- solving a system of linear equations obtained by equating the coefficients of the characteristic polynomial, written as functions of the parameters of the object and the regulator, to the numerical values of these coefficients obtained in the first stage.

So, we have a model of the object

$$\dot{x}(t) = A \cdot x(t) + b \cdot u(t), \qquad (9)$$

where the designations of the quantities included in it correspond to model (6), but do not contain undefined parameters. The law of control is sought in the form

$$u(t) = k \cdot x(t), \qquad (10)$$

where k is a row vector of size n with real coefficients.

The characteristic equation of the closed system “object (9), controller (10)” has the form

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$$-\frac{k\cdot\det(Es-A)\cdot b}{d(s)}+1=0, \qquad (11)$$

where E is the identity matrix of size n ; $adj(Es-A)$ - matrix attached to $(Es-A)$; $d(s)=\det(Es-A)$ - characteristic polynomial of an object; S - the symbol of the Laplace transform with zero initial conditions (hereinafter also understood as the symbol of differentiation with respect to t).

We bring it to a common denominator and introduce the notation

$$\Delta(S)=-k\cdot adj(Es-A)\cdot b+d(s)=0. \qquad (12)$$

$\Delta(S)$ is nothing more than an analytical expression of the characteristic polynomial of the closed system in question through the parameters of the object and the coefficients of the controller.

The requirements for the system can be formulated either by specifying the desired distribution of the eigenvalues of the matrix of the closed-loop system on the complex plane, or by the method of standard coefficients of the characteristic polynomial. In the case when the system has zeros, preference is given to the first method of specifying requirements since it allows to take into account the influence of these zeros on dynamics to some extent by rational choice of poles. If we denote by $\rho_1,\rho_2,....,\rho_n$ the desired poles (complex conjugate or real numbers), then the polynomial can be written as

$$(S-\rho_1)\cdot(S-\rho_2)....(S-\rho_n)=S^n+\Delta_1\cdot S^{n-1}+\Delta_2\cdot S^{n-2}+...+\Delta_n. \qquad (13)$$

Coefficients $\Delta_y \quad (\gamma=\overline{1,n})$ are easily determined by multiplying the simplest factors on the left side and by grouping the terms at equal powers of S. In numerical specification $\rho_1,\rho_2,....,\rho_n$, values $\Delta_1,\Delta_2,....,\Delta_n$ are also numbers.

At the same time, expression (12) can be written as follows

$$S^n+\Delta_1(a_{ij},b_i,k_j)\cdot S^{n-1}+\Delta_2(a_{ij},b_i,k_j)\cdot S^{n-2}+...+\Delta_n(a_{ij},b_i,k_j)=0 \qquad (14)$$

where the notation $\Delta_y \quad (a_{ij},b_i,k_j) \quad (\gamma=\overline{1,n})$ shows that the coefficients of the polynomial are functions of the parameters of the object and the regulator. The form of these functions is determined by the structure of matrix A and vector b. In the general case, these functions are nonlinear, but always linear in $(i,j=\overline{1,n})$.

Equating the coefficients of the polynomials (13) and (14) for equal powers of S, we obtain a system of algebraic equations

$$\{\Delta_1(a_{ij},b_i,k_j)=\Delta_1,\ \ \Delta_2(a_{ij},b_i,k_j)=\Delta_2,\ ....\Delta_n(a_{ij},b_i,k_j)=\Delta_n \ . \qquad (15)$$

Due to the linearity of functions $\Delta_y(a_{ij},b_i,k_j)$ through k_j, system (2.26) can be written in matrix form $P\cdot k+d=\Delta$, where k - is the transposed vector of the desired controller coefficients (column vector), we omit the transpose sign below to simplify the notation,

$P=\left\|P_{\gamma l}(a_{ij},b_i)\right\| \quad (\gamma,l=\overline{1,n})-n\times n$ is the matrix of the corresponding structure;

$\mathrm{d}=\left\|\mathrm{d}_\gamma(a_{ij})\right\| \quad (\gamma=\overline{1,n})$ is a column vector composed of coefficients of the characteristic polynomial;

$\Delta=\left\|\Delta_\gamma\right\| \quad (\gamma=\overline{1,n})$ is a column vector composed of coefficients of the desired characteristic polynomial of a closed system.

Since, in the case of precisely known object parameters, the quantities $a_{ij},b_i(i,j=\overline{1,n})$ are real numbers, the matrix D and the vector d are numerical. Then system $P\cdot k+d=\Delta$ is easily reduced to

$$D\cdot k=h, \qquad (16)$$

where h is found by the usual rule of subtracting vectors

$$h=\Delta-d. \qquad (17)$$

System (16) has a solution calculated by the formula

$$k=D^{-1}\cdot h \qquad (18)$$

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provided that matrix P is non-degenerate, i.e. its determinant is non-zero

$$\det P \neq 0. \qquad (19)$$

Since there are no restrictions on choice Δ, it follows from [8] that condition (19) is satisfied when pair (A,b) is completely controllable

Thus, if the object does not contain indefinite parameters and is completely controllable, then the problem of synthesizing a controller that provides the desired arbitrary arrangement of the poles of a closed system is reduced to calculating the coefficients of polynomial (13), matrix P, vector d, and then to solving a system of linear algebraic equations (15) by formulas (17), (18).

Since the characteristic polynomial of a closed system coefficients are functions of the parameters of the controller and the object, then with the uncertainty of the latter, the considered coefficients will also be uncertain. If the parameter uncertainty is interval, then according to the interval analysis, polynomial (12) will have numerical intervals as coefficients. Each set of parameters $q \in \mathbf{q}$ (a certain point in the space of the object's parameters) will correspond to a certain characteristic polynomial of a closed system (let's call it point), whose coefficients will be numbers (points) belonging to the corresponding intervals of polynomial (14). Thus, the intervals $\mathbf{q}_i (i=\overline{1,r})$ possible values of the parameters of the object (16) are displayed using functions $\Delta_\gamma(a_{ij}, b_i, k_i)$ of (14) according to the rules of interval analysis in intervals $\Delta_\gamma \quad (\gamma=\overline{1,n})$ of the polynomial

$$\Delta(S) = S^n + \Delta_1 \cdot S^{n-1} + \Delta_2 \cdot S^{n-2} + ...+ \Delta_n \cdot \qquad (20)$$

It is clear that each point characteristic polynomial of a closed system corresponds to a well-defined distribution of roots $p_1, p_2, ..., p_n$, and that when the set of parameters is changed, the numbers $p_i (i=\overline{1,n})$ change. It is appropriate to recall here that this effect is used to construct the root hodograph when one parameter in the object changes in a given interval. If there are more than one such parameter in the object, and they change inconsistently with each other, then the numbers $p_i (i=\overline{1,n})$ occupy some areas on the complex plane.

The main idea of solving the modal control problem for objects with indeterminate parameters using interval mathematics is based on the property of monotonicity for the inclusion of interval polynomials [11]. The essence of this property for this particular case is that if the set of roots of some interval polynomial $\Delta(S)_\Omega$ occupies a certain region Ω of the complex region, then any other interval polynomial $\Delta(S)_\Omega$ of the same order as whose coefficients satisfy the inclusions

$$\Delta_1 \subseteq \Delta_{1\Omega}, \quad \Delta_2 \subseteq \Delta_{2\Omega}, ..., \Delta_n \subseteq \Delta_{n_\Omega} \qquad (21)$$

has a root location ω such that

$$\omega \subseteq \Omega \qquad (22)$$

Sign $\subseteq$ means the set-theoretic non-strict inclusion [9,10].

In relation to the numerical intervals from (21), it is interpreted as follows:

$$\underline{\Delta}_{i\Omega} \leq \underline{\Delta}_i \leq \overline{\Delta}_i \leq \overline{\Delta}_{i\Omega}, \quad (i=\overline{1,n})$$

The monotonicity of the inclusion of interval polynomials suggests that for the case of uncertain parameters, the modal control problem can be solved in two stages:

- an interval polynomial $\Delta(S)_\Omega$ is constructed according to the desired region Ω of the location of the roots of the closed system;

- vector $\mathbf{k}$ is calculated so that polynomial (20) satisfies (21).

The last stage, as already mentioned, is not always solvable. However, the first stage causes certain difficulties. Within the interval approach, it would be convenient, at first glance, to specify the localization regions in the form of intervals separately for the real and separately for the imaginary parts of the roots of the polynomial, and then, using formula (13), find its coefficients.

Consider the ATS, consisting of an object described by formulas (6) - (7).

It is required to determine whether the ATS remains stable for any set of values of the object and controller parameters from given intervals (8). In solving this problem, we will adhere to the following classical methodology, applying it to interval matrices. Substituting (7) into (6), a homogeneous vector-matrix equation is formed

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$$\dot{x}(t) = \mathbf{G}x(t), \tag{22}$$

where the square-sized n interval matrix $\mathbf{G} = [\underline{G}, \overline{G}]$ is calculated by the formula

$$\mathbf{G} = \mathbf{A} + \mathbf{BK} \tag{23}$$

and is called a closed system matrix. The stability (22) can be judged by the coefficients of the polynomial of the n st degree

$$\Delta(s) = \det(Es - \mathbf{G}), \tag{24}$$

where s is a complex variable, E is a unit $n \times n$ matrix. The polynomial (24) will be called the interval characteristic polynomial of the closed system.

Thus, the problem of stability analysis is solved in three stages:

1 stage - the formation of a matrix $\mathbf{G} = [\underline{G}, \overline{G}]$ of a closed system,

2 st stage - the calculation of the coefficients interval characteristic polynomial closed system,

3 st stage - the analysis of the stability of interval characteristic polynomial closed system.

Consider the 1st stage of solving the problem. We introduce the following notation for interval matrices: $\mathbf{Q} = \|\mathbf{q}_{ij}\|$, $\underline{Q} = \|\underline{q}_{ij}\|$, $\overline{Q} = \|\overline{q}_{ij}\|$, $(i = \overline{1.n},\ j = \overline{1,r})$. The set of numerical matrices $\{G^D\} = \{G \mid G = A + BK\}$ is denoted by $D(\underline{A}, \overline{A}, \underline{B}, \overline{B}, \underline{K}, \overline{K})$, with $a_{ij} = \underline{a}_{ij}$ or $a_{ij} = \overline{a}_{ij}$, $b_{i\gamma} = \underline{b}_{i\gamma}$ or $b_{i\gamma} = \overline{b}_{i\gamma}$, $k_{\gamma j} = \underline{k}_{\gamma j}$ or $k_{j\gamma} = \overline{k}_{j\gamma}$, $(i, j = \overline{1,n},\ \gamma = \overline{1,r})$. The coefficients of matrix G^D are denoted by g^D_{ij}, $(i, j = \overline{1,n})$. Matrices $\underline{G}$ and $\overline{G}$ are composed of coefficients of matrices belonging to $D(\underline{A}, \overline{A}, \underline{B}, \overline{B}, \underline{K}, \overline{K})$, with $\underline{g}_{ij} = \min_{G^D} g^D_{ij}, \overline{g}_{ij} = \max_{G^D} g^D_{ij}, (i, j = \overline{1,n})$. Indeed, it follows from the structure of matrix G that each function g_{ij} has the form $g_{ij} = a_{ij} + \sum_{\gamma=1}^{r} b_{i\gamma} k_{\gamma i} \quad (i, j = \overline{1,n})$, that is, does not contain the same interval variables.

The above relations outline the following method of obtaining matrix $\mathbf{G}$, bypassing interval arithmetic: you need to calculate G for all possible combinations of boundaries a_{ij}, $b_{i\gamma}$, $k_{\gamma i}$ and choose the minimum and maximum values for each coefficient g_{ij}.

If $\mathbf{A}$, $\mathbf{B}$ K contain Ψ interval elements, then 2^{Ψ} matrices G^D are subject to calculation.

The second stage of solving the problem is to calculate the intervals of the values of the interval characteristic polynomial closed system coefficients. Let be

$$\Delta^D(s) = s^n + \Delta^D_1 s^{n-1} + \ldots + \Delta^D_{n-1} s + \Delta^D_n$$

is the characteristic polynomial of matrix $G^D \in D(\underline{G}, \overline{G})$, where

$$D(\underline{G}, \overline{G}) = \{G^D \mid g_{ij} = \underline{g}_{ij} \quad \text{or} \quad g_{ij} = \overline{g}_{ij} \quad (i, j = \overline{1,n})\ \}$$

$$\Delta(s) = s^n + \Delta_1 s^{n-1} + \ldots + \Delta_{n-1} s + \Delta_n$$

is the characteristic polynomial of matrix $G \in \mathbf{G}$,

$$\underline{\Delta}^D_i = \min_{G^D} \Delta^D_i,\ \overline{\Delta}^D_i = \max_{G^D} \Delta^D_i \quad (i = \overline{1,n}),\ \underline{\Delta}_i = \min_{G} \Delta_i,\ \overline{\Delta}_i = \max_{G} \Delta_i \quad (i = \overline{1,n}) .$$

Then according to the result [7]:

$$\underline{\Delta}_i = \underline{\Delta}^D_i,\ \overline{\Delta}_i = \overline{\Delta}^D_i .$$

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IV. CONCLUSION

Thus, in order to calculate the interval characteristic polynomial closed system by matrix $\mathbf{G}$, it suffices to calculate the characteristic polynomials for all matrices from the set $D(\underline{G},\overline{G})$, and choose the minimum and maximum values for each coefficient $\Delta_i \ (i=\overline{1,n})$.

At the third stage of solving the problem, the stability criteria for interval polynomials given in [9] are applied.

In the first two stages, calculations are performed using interval-arithmetic operations. The last stage implements the necessary and sufficient stability criterion for interval polynomials. Note that even with a small number of uncertain parameters in the automatic control system, the solution of the stability analysis problem without the use of interval mathematics requires significant computational costs.

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