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Influence of MHD oscillatory slip flow for Williamson fluid through inclined channel with varying temperature and concentration

Ahmed A .Hussein Al-Aridhee , Qassim Ali Shaker

Department of mathematic Science, College Of computer Science and InformationTechnology,University of Al-Qadisiyah , Diwaneyah – Iraq

Department of mathematic Science,College Of computer Science and InformationTechnology, University of Al-Qadisiyah , Diwaneyah – Iraq.

ABSTRACT:The main theme of the present examined the influence of heat transfer on magneto hydrodynamics (MHD) for the oscillatory flow of Williamson fluid with constant viscosity model for kind of geometries "Poiseuille flow flow" through a porous medium inclined channel. The momentum equation for the problem, is a non-linear differential equations, has been found by using "perturbation technique" and intend to calculate the solution for the small number of Weissenberg ($We \ll 1$) to get clear forms for the velocity field by assisting the (MATHEMATICA) program to obtain the numerical results and illustrations. The physical features of Darcy number, magnetic parameter, Grashof number and radiation parameter are discussed simultaneously through presenting graphical discussion. Investigated through graphs the variation of a velocity profile for various pertinent parameters. While the velocity behaves strangely under the influence of the Brownian motion parameter and local nanoparticle Grashof number effect. On the basis of this study, it is found that the velocity directly with Grashof number, Darcy number, radiation parameter, and reverse variation with magnetic parameter and frequency of the oscillation and discussed the solving problems through graphs.

KEY WORDS: Williamson Fluid, constant Viscosity, Heat Transfer, (MHD), inclined channel.

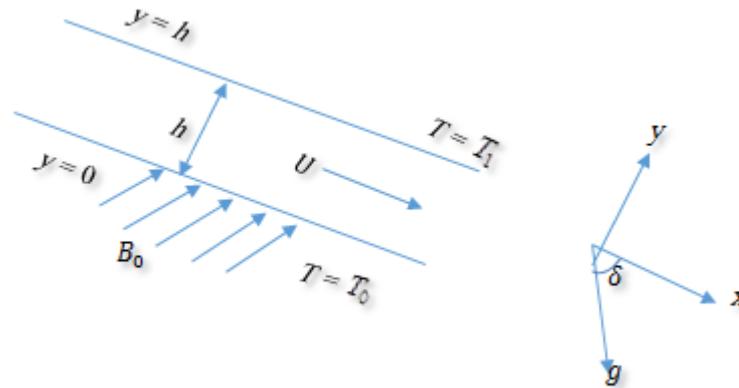
1.INTRODUCTION

Central porosity is a matter containing a number of small holes distributed throughout the matter. A porous medium flows through the fluid infiltration and water infiltration into the river beds. The movement of groundwater, water, and oils are some important examples of flows through porous means. The oil tank often contains a sedimentary structure such as limestone and sandstone in which the oil is contained. Another example of flow through a porous medium is leakage under the dam which is very important. Examples: of natural porosity such as sand ash, wood, filtering, human lung, bitterness and yellow stones, oil production engineering and many other processes. In [1] show the exact solutions for fourth kinds of flows between two parallel plates. [2] studied the influence of inclined magnetic field between two infinite parallel plates, [3] discussed the laminar flow between parallel plates under the action of the transverse magnetic field and heat transfer. [4] discussed the two kinds of geometries Poiseuille flow and Couette flow of Carreau fluid with pressure dependent viscosity in a variable porous medium. Viscosity is one of the most important specifications for fluids, [6] studied the variable viscosity through a porous medium and used the homotopy analysis method to solve the problem. [7] studied the related of the variable viscosity through a porous medium by using generalized Darcy's law, to solve the problem he using the perturbation technique. [8] Influence of heat transfer on magneto hydrodynamics oscillatory flow for Williamson fluid through a porous medium. [9] studied the variable viscosity of Jeffrey fluid in an asymmetric channel. In most systems, channels or ducts used are sloped. This fact stimulated scientists to explore the associated flows in a slanted channel (see Refs [10-13]).

II. MATHEMATICAL FORMULATION

Let us consider the flow of a Williamson fluid in a channel of width h under the effects of electrically applied magnetic field and radioactive heat transfer as depicted in (Fig.1). Supposed that the fluid has very small electromagnetic force

produced and the electrical conductivity is small. We are considering Cartesian coordinate system such that, $(u(y), 0, 0)$ is a velocity vector in which u is the x -component of velocity and y is perpendicular to the x -axis.



III. BASIC EQUATIONS

The basic equations governing for Williamson fluid are given by:

The continuity equation is given by: $\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$ (1)

The momentum equations are:

In the x - direction:

$$\rho \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} + \rho g \beta_T (T - T_0) \sin(\xi) + \rho g \beta_C (C - C_2) \sin(\xi) - \sigma B_0^2 \sin^2(\xi) \bar{u} - \frac{\mu_0}{k} \bar{u} + \rho g \sin(\delta)$$
 (2)

In the y - direction:

$$\rho \left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{yy}}{\partial \bar{y}} - \frac{\mu_0}{k} \bar{v} + \rho g \cos(\delta)$$
 (3)

The temperature equation is given by:

$$\frac{\partial T}{\partial \bar{t}} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{\rho C_p} \frac{\partial q}{\partial \bar{y}} + \frac{Q_H}{\rho C_p} (T - T_0)$$
 (4)

The concentration equation is given by:

$$\frac{\partial C}{\partial \bar{t}} = D \frac{\partial^2 C}{\partial \bar{y}^2} - K_r^* (C - C_2) + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial \bar{y}^2}$$
 (5)

where \bar{u} is the axial velocity, ρ is the density of the fluid, p is the pressure, σ is the electrical conductivity, B_0 is the strength of the magnetic field, g is the acceleration due to gravity, T is temperature, C is the concentration, C_p is specific heat at constant pressure, q is the radiation heat flux, K is thermal conductivity, Q_H is heat generation, D is the coefficient of mass diffusivity, $(0 \leq \xi \leq \pi)$ is the angle between velocity field and magnetic field strength and K_T is the thermal diffusion ratio. The corresponding boundary conditions are given below:

$$T = T_0, C = C_1 \text{ at } \bar{y} = 0 \text{ and } T = T_1, C = C_2 \text{ at } \bar{y} = h$$
 (6)

The radioactive heat flux [26] is given by:

$$\frac{\partial q}{\partial \bar{y}} = 4\eta^2 (T_0 - T)$$
 (7)

The radiation absorption denoted by η .

The Fundamental Equation:

The fundamental equation for Williamson fluid given by:

$$\mathbf{S} = -\bar{p}\mathbf{I} + \boldsymbol{\tau} \tag{8}$$

$$\bar{\boldsymbol{\tau}} = [\mu_\infty + (\mu_0 - \mu_\infty)(1 + \Gamma\dot{\gamma})^{-1}]A_1 \tag{9}$$

where \bar{p} is the pressure, \mathbf{I} is the unit tensor, $\bar{\boldsymbol{\tau}}$ is the extra stress tensor, Γ is the time constant, μ_∞ and μ_0 are the infinite shear rate viscosity and zero shear rate viscosity, then $\dot{\gamma}$ is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi} \tag{10}$$

Here Π is the second invariant strain tensor. We consider the fundamental Eq. (9), the case for which $\Gamma\dot{\gamma} < 1$, and $\mu_\infty = 0$. We can write the component of extra stress tensor according to follows as:

$$\bar{\boldsymbol{\tau}} = \mu_0[(1 + \Gamma\dot{\gamma})]A_1 \tag{11}$$

where μ_0 is the zero shear rate viscosity and $\dot{\gamma}$ is the strain. The Rivlin-Ericksen tensors are given by:

$$A_1 = \nabla\bar{V} + (\nabla\bar{V})^T \tag{12}$$

The governing equations of the motion, we may introduce the non-dimensional conditions are as follows:

$$\left. \begin{aligned} x &= \frac{\bar{x}}{h}, y = \frac{\bar{y}}{h}, u = \frac{\bar{u}}{U}, \theta = \frac{T-T_0}{T_1-T_0}, p = \frac{\bar{p}h}{\mu U}, Pe = \frac{\rho h U c_p}{K}, R = \frac{4\eta^2 h^2}{K} \\ K_r &= \frac{h K_r^*}{U}, We = \frac{\Gamma U}{h}, \tau_{xx} = \frac{h}{\mu_0 U} \bar{\tau}_{xx}, \tau_{xy} = \frac{h}{\mu_0 U} \bar{\tau}_{xy}, \dot{\gamma} = \frac{h}{U} \bar{\dot{\gamma}}, \Phi = \frac{C-C_2}{C_1-C_2} \\ t &= \frac{\bar{t}U}{h}, Re = \frac{\rho h U}{\mu}, Da = \frac{k}{h^2}, Gr = \frac{\rho g \beta_T h^2 (T-T_0)}{\mu U}, S_r = \frac{DK_T(T_1-T_0)}{UT_m h(C_1-C_2)} \\ Fr &= \frac{U}{gh}, M^2 = \frac{\sigma B_0^2 h^2}{\mu}, S_c = \frac{U h}{D}, Q = \frac{Q_H h^2}{K}, Gc = \frac{\rho g \beta_C h^2 (T-T_0)}{\mu U} \end{aligned} \right\} \tag{13}$$

where (U) is the mean flow velocity, (Da) is Darcy number, (Re) is Reynolds number, (M) is magnetic parameter, (Pe) is the Peclet number, (R) is the radiation parameter. (S_c) is the Schmidt number, (S_r) is the Soret number, (Q) is the heat generation parameter, (T_m) is the mean temperature, (Gr) is Thermal Grashof number and (G_c) is Solutal Grashof number. Substituting (7) and (13) into equations (1), (2), (4), (5), (6) we have the following of non-dimensional equations:

Finally, we get

$$Re \frac{\partial u}{\partial t} = -\frac{dp}{dx} + \frac{\partial}{\partial y} \left[\frac{\partial u}{\partial y} + we \left(\frac{\partial u}{\partial y} \right)^2 \right] + Gr\theta \sin(\xi) + Gc\Phi \sin(\xi) - \left(M_1^2 + \frac{1}{Da} \right) u + \frac{Re}{Fr} \sin(\delta) \tag{14}$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (R + Q)\theta \tag{15}$$

$$\frac{\partial \Phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \Phi}{\partial y^2} - K_r \Phi + S_r \frac{\partial^2 \theta}{\partial y^2} \tag{16}$$

where $M_1 = M \sin(\xi)$.

Now, we solve the temperature Eq. (28) with boundary conditions

$$\theta(0) = 0, \theta(1) = 1 \tag{17}$$

Let

$$\theta(y, t) = \theta_0(y) e^{i\omega t} \tag{18}$$

the frequency of the oscillation denoted by ω .

Substituting the Eq.(31) into the Eq.(28), we have

$$Pe \frac{\partial}{\partial t} (\theta_0(y) e^{i\omega t}) = \frac{\partial^2}{\partial y^2} (\theta_0(y) e^{i\omega t}) + (R + Q)(\theta_0(y) e^{i\omega t})$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + (R + Q - i\omega Pe)\theta_0 = 0 \tag{19}$$

The solution of Eq.(32), is:

$$\theta_0(y) = \csc(A) \sin(iAy) \tag{20}$$

where $A = \sqrt{R + Q - i\omega Pe}$

Hence

$$\theta(y, t) = \csc(A) \sin(iAy) e^{i\omega t} \tag{21}$$

Now, we discuss the solution of the concentration Eq.(29) with boundary conditions:

$$\Phi(0) = 0, \Phi(1) = 1 \tag{22}$$

Let

$$\Phi(y, t) = \Phi_0(y)e^{i\omega t} \tag{23}$$

Substituting the equations (36) and (31) into the Eq.(29), we have

$$\frac{\partial}{\partial t}(\Phi_0(y)e^{i\omega t}) = \frac{1}{Sc} \frac{\partial^2}{\partial y^2}(\Phi_0(y)e^{i\omega t}) - Kr(\Phi_0(y)e^{i\omega t}) + Sr \frac{\partial^2}{\partial y^2}(\theta_0(y)e^{i\omega t}) \tag{24}$$

$$\frac{\partial^2 \Phi_0}{\partial y^2} - Sc(Kr + i\omega)\Phi_0 + Sc Sr \frac{\partial^2 \theta_0}{\partial y^2} = 0 \tag{25}$$

The solution of Eq.(38), is:

$$\Phi_0(y) = e^{\sqrt{B}y} \left(\frac{e^{\sqrt{B}(A+B+A(SrSc))}}{(A+B)(-1+e^{2\sqrt{B}})} \right) + e^{-\sqrt{B}y} \left(-\frac{e^{\sqrt{B}(A+B+A(SrSc))}}{(A+B)(-1+e^{2\sqrt{B}})} \right) - \frac{A(SrSc)Csc[\sqrt{A}]Sin[\sqrt{A}y]}{A+B} \tag{26}$$

where $B = \sqrt{Sc(Kr + i\omega)}$.

Hence

$$\Phi(y, t) = ((e^{\sqrt{B}y} - e^{-\sqrt{B}y}) \left(\frac{e^{\sqrt{B}(A+B+A(SrSc))}}{(A+B)(-1+e^{2\sqrt{B}})} \right) - \frac{A(SrSc)Csc[\sqrt{A}]Sin[\sqrt{A}y]}{A+B}) e^{i\omega t} \tag{27}$$

V. SOLUTION OF THE PROBLEM

We suppose that the rigid flakes are at $y = 0$ and $y = h$ are at rest. Therefore:

$$\bar{u} = 0 \text{ at } \bar{y} = 0, \text{ and } \bar{u} = 0 \text{ at } \bar{y} = h.$$

The non-dimensional boundary conditions are as follows:

$$u = 0 \text{ at } y = 0, u = 0 \text{ at } y = 1 \tag{28}$$

To solve the momentum equation (27), let

$$-\frac{dp}{dx} = \lambda e^{i\omega t} \tag{29}$$

$$u(y, t) = u_0(y)e^{i\omega t} \tag{30}$$

where λ is a real constant.

Equation (27) is a non-linear differential equation and it is hard to find an exact solution, so will be used the perturbation technique to find the problem solution, as follows:

$$u = u_0 + Weu_1 + O(We^2) \tag{31}$$

By substituting equations (43) and (44) into Eq. (27) with boundary conditions (41).

$$Re \frac{\partial}{\partial t}(u_0 + Weu_1) = -\frac{dp}{dx} + \frac{\partial^2}{\partial y^2}(u_0 + Weu_1) + 2We \left[\frac{\partial}{\partial y}(u_0 + Weu_1) \right] \left[\frac{\partial^2}{\partial y^2}(u_0 + Weu_1) \right] + Gr\theta + Gc\Phi + \frac{Re}{Fr} \sin(\delta) - (M_1^2 + \frac{1}{Da})(u_0 + Weu_1) \tag{45}$$

$$\begin{aligned} \Rightarrow Re \frac{\partial u_0}{\partial t} + ReWe \frac{\partial u_1}{\partial t} = -\frac{dp}{dx} + Gr\theta \sin(\xi) + Gc\Phi \sin(\xi) + \frac{Re}{Fr} \sin(\delta) - (M_1^2 + \frac{1}{Da})u_0 - (M_1^2 + \frac{1}{Da})Weu_1 + \frac{\partial^2 u_0}{\partial y^2} + \\ We \frac{\partial^2 u_1}{\partial y^2} + 2We \left(\frac{\partial u_0}{\partial y} \cdot \frac{\partial^2 u_0}{\partial y^2} \right) + 2We^2 \left(\frac{\partial u_0}{\partial y} \cdot \frac{\partial^2 u_1}{\partial y^2} \right) + 2We^3 \frac{\partial u_0}{\partial y} + 2We^2 \left(\frac{\partial u_1}{\partial y} \cdot \frac{\partial^2 u_0}{\partial y^2} \right) + 2We^3 \left(\frac{\partial u_1}{\partial y} \cdot \frac{\partial^2 u_1}{\partial y^2} \right) + 2We^4 \frac{\partial u_1}{\partial y} + \\ 2We^3 \frac{\partial^2 u_0}{\partial y^2} + 2We^4 \frac{\partial^2 u_1}{\partial y^2} \end{aligned} \tag{46}$$

Now, let

$$u_j(y, t) = u_{0j}(y)e^{i\omega t}, j = 0,1,2 \tag{32}$$

with boundary conditions (41), then equating the like powers of (We), we obtain:

i - Zeroth-order system (We⁰)

$$\frac{\partial^2 u_{00}}{\partial y^2} - \left(M_1^2 + Rei\omega + \frac{1}{Da} \right) u_{00} = -(\lambda + Gr\theta_0 + Gc\Phi_0 + \frac{Re}{Fr} \sin(\delta)) \tag{33}$$

The associated boundary conditions are:

$$u_{00}(0) = u_{00}(1) = 0 \tag{34}$$

ii – First - order system (We¹)

$$\frac{\partial^2 u_{01}}{\partial y^2} - \left(M_1^2 + Re i \omega + \frac{1}{Da} \right) u_{01} = -2 \left(\frac{\partial u_{00}}{\partial y} \frac{\partial^2 u_{00}}{\partial y^2} \right) e^{i \omega t} \tag{38}$$

The associated boundary conditions are:

$$u_{01}(0) = u_{01}(1) = 0 \tag{39}$$

The associated boundary conditions are:

$$u_{02}(0) = u_{02}(1) = 0 \tag{40}$$

where $N = \left(M_1^2 + Re i \omega + \frac{1}{Da} \right)$ and $Z = \left(\lambda + Gr \theta_0 + Gc \Phi_0 + \frac{Re}{Fr} \sin(\delta) \right)$.

Finally, the perturbation solutions up to second order for u_0 is given by

$$u_0 = u_{00} + We u_{01} + O(We^2)$$

Therefore, the fluid velocity is given as:

$$u(y, t) = u_0(y) e^{i \omega t} \tag{41}$$

VI. RESULTS AND DISCUSSION

We discuss the influence of MHD oscillatory slip flow for Williamson fluid through inclined channel with varying temperature and concentration in some results through the graphical illustrations. Numerical assessments of analytical results and some of the graphically significant results are presented in fig. (2-11). We use the (MATHEMATICA-11) program to find the numerical results and illustrations. The momentum equation is resolved by using perturbation technique and all the results are discussed graphically. Fig.(2) shows the temperature increases with the increase in R . Fig.(3). show us that with the increasing of ω the temperature θ decreases. Fig.(4) we observed that the influence ω in concentration profile Φ by the increasing ω then Φ decreases. The concentration profile Φ decreases with increase R in Fig. (4). Fig.(5) illustrates the influence Fr on the velocity profiles function u vs. y . It is found by increasing Fr the velocity profile function u decreases. Fig.(6) shows that velocity profiles increase with the increase of the parameters δ . Figures Fig. (7) and Fig. (8) illustrate the influence Gr and Gc on the velocity profiles function u vs. y . It is found by the increasing Gr and Gc the velocity profiles function u increases. Fig.(9) shows that velocity profiles increase with the increase of the parameters Da . Fig. (10) illustrates the influence M on the velocity profiles function u vs. y . It is found by increasing M the velocity profile function u decreases.

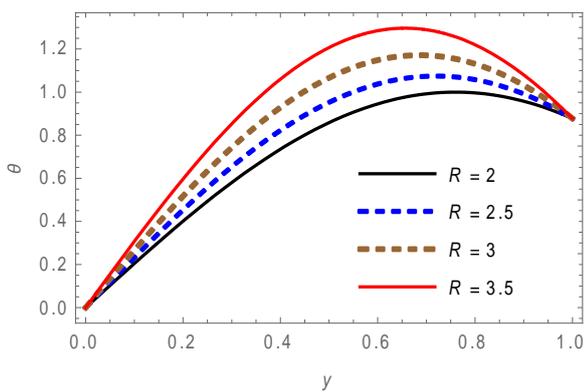


Fig.(2) Influence of R on Temperature for t = 0.5, ω = 1, Q = 2, Pe = 0.7

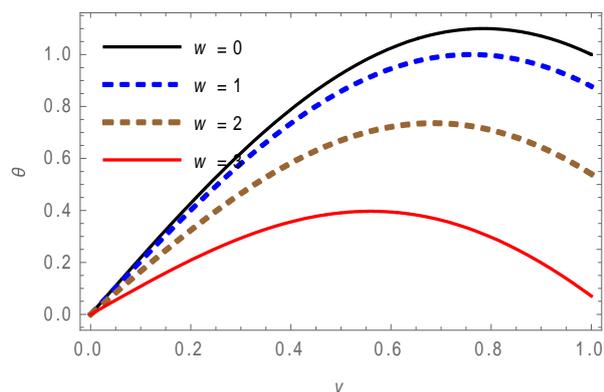


Fig.(3) Influence of ω on Temperature for t = 0.5, R = 2, Q = 2, Pe = 0.7

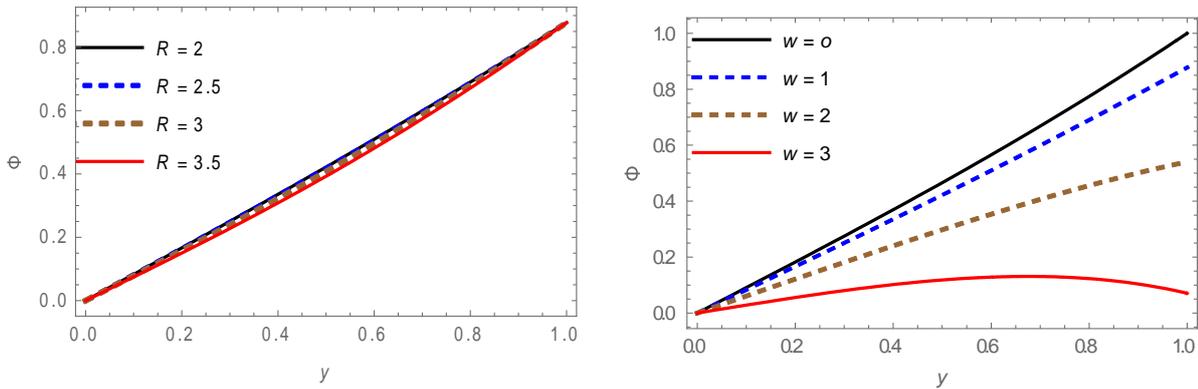


Fig.(4) Influence of R on concentration for $Sr = 0.1, Sc = 0.6, Q = 2, Pe = 0.7, \omega = 1, K_r = 0.5$. Fig(5)Influence of ω on concentration for $Sr = 0.1, R = 2, Q = 2, Pe = 0.7, K_r = 0.5, Sc = 0.6, t = 0.5$.

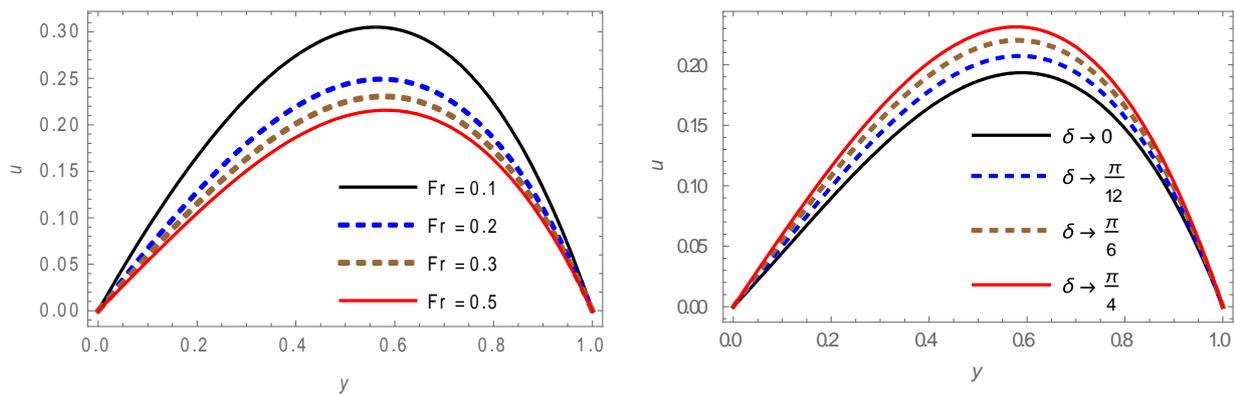


Fig. (6) Velocity profile for Fr with $Gc = 1, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Da = 0.8, Gr = 1, M = 1, K_r = 0.5, \delta = \frac{\pi}{12}, We = 0.05, t = 0.5$.

Fig. (7) Velocity profile for δ with $Gc = 1, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Da = 0.8, Gr = 1, M = 1, K_r = 0.5, Fr = 0.005, We = 0.05, t = 0.5$.

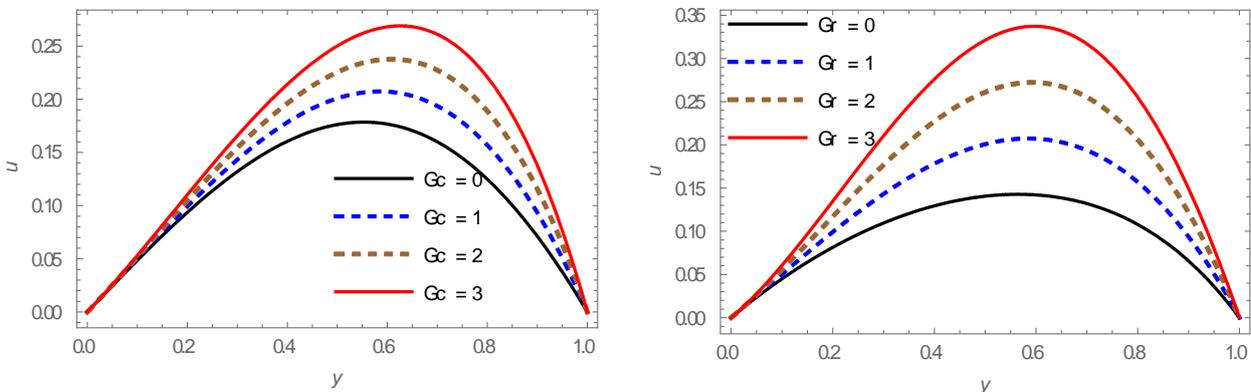


Fig. (8) Velocity profile for Gc with $Fr = 0.005, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Da = 0.8, Gr = 1, M = 1, K_r = 0.5, \delta = \frac{\pi}{12}, We = 0.05, t = 0.5$.

Fig. (9) Velocity profile for Gr with $Gc = 1, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Da = 0.8, \delta = \frac{\pi}{12}, M = 1, K_r = 0.5, Fr = 0.005, We = 0.05, t = 0.5$.

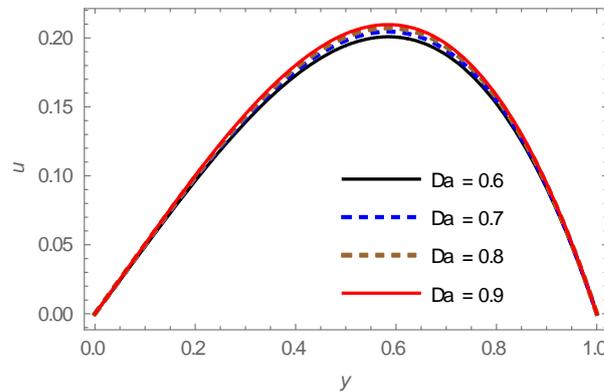


Fig. (10) Velocity profile for Da with $Fr = 0.005, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Gc = 1, Gr = 1, M = 1, K_r = 0.5, \delta = \frac{\pi}{12}, We = 0.05, t = 0.5$.

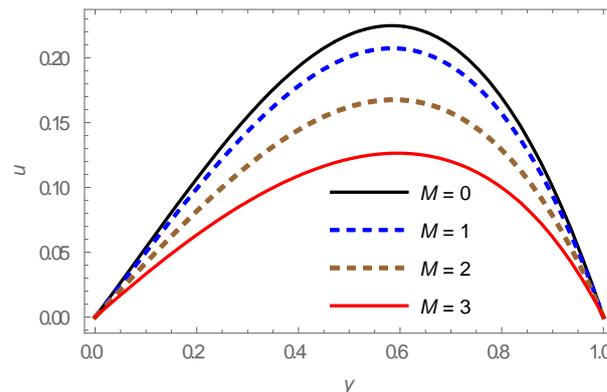


Fig. (10) Velocity profile for M with $Fr = 0.005, R = 2, Pe = 0.7, \xi = \frac{\pi}{4}, \lambda = 1, Q = 2, Sc = 0.6, \omega = 1, Re = 1, Gc = 1, Gr = 1, Da = 1, K_r = 0.5, \delta = \frac{\pi}{12}, We = 0.05, t = 0.5$.

IV. CONCLUSION AND REMARKS

We discuss the influence of heat transfer on MHD oscillatory flow for Williamson fluid with constant viscosity through an inclined channel. The velocity and temperature are found analytical, and use different values to find the results of pertinent parameters, namely for the velocity and temperature. The key point is listed below:

- i. The velocity profiles increase with increasing $Fr, Gc, Gr,$ and Da for the Poiseuille.
- ii. The velocity profiles decrease with increasing magnetic parameter a, δ . for the Poiseuille flow.
- iii. Show that by the increase of R the temperature increases and by the increase of w the temperature decreases.
- iv. Show that by the increase of R the concentration decreases and by the increase of w the concentration decreases.

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