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Construction of Mathematical Models of Technological Systems of Elastically Deformed Nonrigid Shafts Turning

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ABSTRACT: This article presents mathematical models of technological systems for processing the parts of low rigidity in an elastically deformed state in dynamic modes. Functional dependencies for technological systems with various types of elastically deformed state of nonrigid parts are obtained. Mathematical models are developed for controlling the precision of processing of elastically deformed parts under longitudinal-transverse bending. It is found that an increase in rigidity due to tension leads to a decrease in strain of the treated shaft and a decrease in the transient process. Based on simulation results of technological system, a calculation of elastically deformed state of the shaft at turning is proposed. Results of experimental studies of the precision of parts shape at the processing of shafts in an elastically deformed state are presented.

KEYWORDS: Technological system, mathematical model, turning, nonrigid shaft, elastic deformation, accuracy.

I. INTRODUCTION

In modern mechanical engineering, the problem of increasing the precision of machining of low rigidity parts that are widely used in machines and mechanisms is especially acute; this is associated with modern trends to reduce metal consumption, with wide use of parts of functional specific purpose. Such details constitute a significant share of the product range in precision engineering, instrumentation, aviation and space industries. Traditional methods to improve the accuracy of machining of nonrigid parts, based on the introduction of multi-pass processing, the reduction of the intensity of cutting regimes, the application of lunettes, the introduction of additional passes and manual finishing, lead to a significant decrease in productivity and, in many cases, do not give the desired result. This predetermines a special interest in the search for new effective ways to control the accuracy of the technological system (TS) with nonrigid elements [1].

The problem of control the technological systems for the machining of small rigidity parts that provide the required accuracy and surface quality is hampered by the fact that during machining the part itself, the tool and the machine nodes, being in relative motion, represent a complex dynamic system whose behavior in advance it is practically impossible to determine without targeted and theoretical researches. The most expedient direction to solve the problem is the control of technological systems of machining of nonrigid parts in an elastically deformed state on the basis of scientifically grounded technological methods of influencing the workpiece. When forming a mathematical description of the control object - the elastic line of a nonrigid shaft - in steady-state modes, it is necessary to take into account the fixing methods, loading conditions. The model obtained in this case should be simple enough for further use in precision control problems, but at the same time it should correspond to a priori information about the mechanisms, links and parameters of the phenomena. At the same time, factors that exert a dominant influence on the precision indexes of the machining should be singled out [2, 3].

Mathematical description of elastic line based on the studying of detail and tool real interaction is constructed taking into consideration the generally system principles developing at the present time [4]. At that developed mathematical models must meet the community requirements that mean they must provide possibility to include them to given modeling system and CAD. On these positions the method of set problem realization will be standard in some relations, but in cases of coincidence with set purposes the results known researches in this field will be used. Digression from the generally accepted way is to give it some of the following qualities: universality, accuracy and economy.

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II. CONSTRUCTION OF MATHEMATICAL MODELS OF NONRIGID SHAFTS TURNING TECHNOLOGICAL SYSTEMS

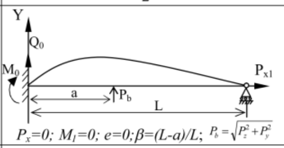
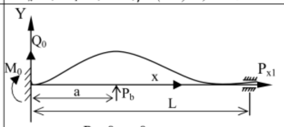
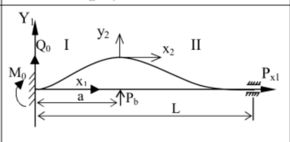
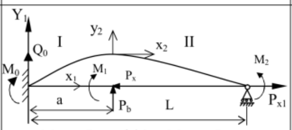
In researches and at the construction of MM in the set modes next assumptions are accepted: a technological process is considered continuous and steady in the flow of treatment of one detail; speed of rotation, serve, cutting ability of tool in the process of treatment of one detail accepted by unchanging; initial conditions are determined by the moment of touch the tool of detail and formation of pull in TS; TS is limited to the area a "head stock - nonrigid shaft - poppet head "; inflexibility and resilient moving of TS are taken into account only in radial direction; as the guided data-out of exactness deviations of form of billow are accepted from cylindrical form; the size of rejection of form is accepted by proportional to bending in the corresponding sections of non-rigid workpiece.

At a TS control of non-rigid shafts turning, based on the change of their elastic deformed state, as regulative influences the separate are used or combination of the managed power influences: central and eccentric tension, eccentrically compression; management by the additional power influences sent to indemnification of force factors from the process of cutting; by flexion moments on supports; control of flexural-torsional force deformation [5-8].

The most important aspect of such research is the possibility to observe transient processes in the cycle of machining and the accompanying dynamic phenomena, which makes it possible to detect the conditions for eliminating or reducing their negative influence on the quality of the treated surface.

The mathematical model (MM) of TS in the set modes is here formed as functional dependence, reflecting influence of the regulative and revolting affecting size of resilient deformations of detail in the examined section. On the basis of laws of mechanics of the deformed solid [8,9] functional dependences are got for TS with the different types of the elastic deformed state of non-rigid details that is presented in a Table 1.

Table 1. Conditions for loading parts at the management of an elastically deformed state

№	Method of fixing. Conditions of loading. Type of detail elastic line.	Boundary conditions. Conditions of equilibrium. Compatibility conditions for deformations.	Bending functions.	Initial parameters Q_0 and M_0
1	2	3	4	5
1	 $P_x=0;\ M_1=0;\ e=0;\beta=(L-a)/L;\ P_b=\sqrt{P_z^2+P_y^2}$	$X=0;\ Y(0)=0;\ Y'(0)=0;$ $EIY''(0)=M_0;\ EIY'''(0)=Q_0;$ $X=L;\ Y(L)=0;$ $\alpha=\sqrt{\frac{P_{x1}}{EI}}$	$y(x)=\frac{Q_0}{P_{x1}}(sh\alpha x-dx)+\frac{M_0}{P_{x1}}(ch\alpha x-1)+f(x)$ $0<x<a;\ f(x)=0;\ 0<x<L;$ $f(x)=P_b/P_{x1}[(x-a)-sh\alpha(x-a)]$	$Q_0=P_b\frac{(\beta\alpha L\cdot ch\alpha L-sh\beta\alpha L)}{(\alpha L ch\alpha L-sh\alpha L)};$ $M_0=P_b\frac{(\beta sh\alpha L-sh\alpha\beta L)}{(\alpha L ch\alpha L-sh\alpha L)}L;$
2	 $P_Z=0;\ e=0$	$X=0;\ Y(0)=0;\ Y'(0)=0;$ $X=L;\ Y(L)=0;\ Y'(L)=0;$ $Y(X)=a_1[1+cos(2\pi X/L)]$ $Dw/da_1=0$	$Y(X)=\frac{P_bL^3[1-\cos(\frac{2\pi x}{L})][1-\cos(\frac{2\pi x}{L})]}{\pi^2(8\pi^2\cdot EI+2P_{x1}L^2)}$	
3		$X_1=0;\ Y_1(0)=0;\ Y_1'(0)=0;$ $EIY_1''(0)=M_0;\ EIY_1'''(0)=Q_0;$ $X_2=0;Y_2(0)=0;Y_2'(0)=Y_1'(a);$ $EIY_2''(0)=EIY_1''(a);$ $EIY_2'''(0)=EIY_1'''(a)+P_b;$ $\alpha=\sqrt{P_{x1}/EI}$	$0<X_1<a$ $Y_1(X_1)=\frac{Q_0}{\alpha P_{x1}}(sh\alpha x_1-\alpha x_1)+\frac{M_0}{P_{x1}}(ch\alpha x_1-1)$ $0<X_2<L$ $Y_2(X_2)=\frac{Q_0ch\alpha a+M_0\alpha sha\alpha+P_{изг}}{\alpha P_{x1}}(sh\alpha x_2-\alpha x_2)+$ $+\frac{Q_0sha+M_0\alpha ch\alpha a}{\alpha P_{x1}}(ch\alpha x_2-1)$	$Y_1(a)=-Y_2(L-a);\ Y'_2(L-a)=0$ $Q_0=\frac{P_{изг}[(ch\alpha(L-a)+ch\alpha a(L-a)\cdot sh\alpha a-1)+}{[(1-ch(L-a)-ch\alpha L+ch\alpha a)+\alpha(L-a)ch\alpha(L-a)+}$ $\frac{+\alpha(L-a)(ch\alpha(L-a)\cdot sh\alpha a-sh\alpha L)]}{+\alpha a(sh\alpha L-sh\alpha a)]}$ $M_0=\frac{P_{изг}[(\alpha(L-a)(ch\alpha L-ch\alpha a-ch\alpha(L-a)+\alpha a(1-ch\alpha)(L-a)+}{\alpha[(1-ch\alpha(L-a)-ch\alpha L+ch\alpha a)+\alpha(L-a)ch\alpha(L-a)+}$ $\frac{+sh\alpha(L-a)ch\alpha a-sh\alpha L+sh\alpha a]}{+\alpha a(sh\alpha L-sh\alpha a)]}$
4	 $M_1=P_x\cdot d/2;\ M_2=P_{x1}\cdot e$	$X_1=0;\ Y_1(0)=0;\ Y_1'(0)=0;$ $X_2=0;Y_2(0)=0;Y_2'(0)=Y_1'(a);$ $EIY_1''(0)=M_0;\ EIY_1'''(0)=Q_0;$ $EIY_2''(0)=EIY_1''(a)+M_1;$ $EIY_2'''(0)=EIY_1'''(a)+P_b;$ $\alpha_1=\sqrt{P_{x1}-P_x/EI}\ \ \alpha_2=\sqrt{P_{x1}/EI}$	$0<X_1<a$ $Y_1(X_1)=\left[\frac{Q_0\cdot\alpha_1}{(P_{x1}-P_x)\alpha_1}\right](sh\alpha_1x_1-\alpha_1x_1)+\left[\frac{M_0}{(P_{x1}-P_x)}\right](ch\alpha_1x_1-1);$ $0<X_2<L$ $Y_2(X_2)=\frac{Q_0ch\alpha_1a+M_0\alpha_1sh\alpha_1a+P_{изг}}{\alpha_2P_{x1}}(sh\alpha_2x_2-\alpha_2x_2)+$ $+\frac{sh\alpha_1a(Q_0/\alpha_1)+M_0\alpha ch\alpha_1a+M_1}{P_{x1}}(ch\alpha_2x_2-1)$	$Y_1(a)=-Y_2(L-a)$ $\sum M(L)=Q_0L+M_0+P_X\cdot\frac{\alpha}{2}+P_b(L-a)-P_{X1}\cdot e=0$ $Q_0=\frac{\alpha_1\alpha_2P_{x1}(ch\alpha_1a-1)[M_2-M_1-P_b(L-a)]+\alpha_1(P_{x1}-P_x)\cdot}{\alpha_1\alpha_2LP_{x1}(ch\alpha_1a-1)+\alpha_1\alpha_2L(P_{x1}-P_x)ch\alpha_1a[ch\alpha_2(L-a)-1]-}$ $\frac{\cdot[(M_2-M_1-P_b(L-a))\alpha_1sh\alpha_1a+P_b][(sh\alpha_2(L-a)-\alpha_2(L-a]+}{-\alpha_2P_{x1}(sh\alpha_1a-\alpha_1a)-\alpha_1(P_{x1}-P_x)(ch\alpha_1a-\alpha_1)sh\alpha_1a\cdot}$ $\frac{+\alpha_1\alpha_2(P_{x1}-P_x)[(ch\alpha_2(L-a)-1][(M_2-M_1-P_b(L-a))ch\alpha_1a+M_1]}{\cdot[ch\alpha_2(L-a)-\alpha_2(L-a)]-\alpha_2(P_{x1}-P_x)sh\alpha_1a[ch\alpha_2(L-a)-1]}$

Fixing of detail at the appendix of central stretching force can be carried out in the collet chuck. Such method of fixing is interpreted as a hard sealing-off with possibility of the axial moving (Table 1, lines 2 and 3). With the purpose of minimization of the resilient bending may be also management by the corner of turn of detail in the place of fixing, by the appendix of eccentric tension [9,10]. This variety of fixing can be also presented by movable joint support (Table 1, line 4). The fundamental difference of the indicated chart is excitation of the guided moment in the point of fixing of detail due to eccentric tension. Thus appendix of one guided force factor - eccentric tension gives an opportunity of

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creation of two force factors in any section of detail set in advance, in particular, in the zone of treatment: longitudinal force P_{x1} and bending moment $M_2 = P_{x1} \cdot e$, of counteractive to cutting forces, that means the purposeful elastic deformed state of shaft.

The question of choice of concrete type of model can be decided only after a final structural decision about a method and construction of fixing model, self-reactance authentication and analysis of closeness of the results got on a simulation model with experimental data.

The following designations are accepted in the Table 1: *e* - eccentricity at eccentric tension; M_1 - the moment created by the axial component of the cutting force P_x; X_1, X_2, X_3 - current coordinates for each of the sections; *a* - the distance between the tool and the shafts fixture at the headstock; Q_0 and M_0 - initial parameters: transversal force and moment in a bonding, accordingly.

One of methods allowing to get MM-description of the shape of the elastic line, depending on the parameters of detail and parameters of treatment (loading efforts), a power method of Ritz is [9,11-13], by means of that is got the function of bending of the stretched-bent shaft with fully-fixed ends (Table 1, line 2).

Another method that allows us to obtain results useful for practical purposes is the construction of a description of the elastic line of a nonrigid part in the longitudinally transverse bending in the form of a system of fourth-order differential equations with constant coefficients [11-13]. In the presence of concentrated forces and moments for each of the sections to which these perturbations break the length of the part, well-grounded the differential equations (central tension - a model 3, Table1)

$$Y_i^{IV} - \alpha^2 \cdot Y_i^{II} = 0 \ , \tag{1}$$

where $\alpha = \sqrt{\dfrac{P_{x1}}{E \cdot I}}$.

$$i \in \{1,2\} . \tag{2}$$

The decision of equation (1) can be written down as

$$Y_i(X_i) = A_i \cdot sh\alpha \cdot X_i + B_i \cdot ch\alpha \cdot X_i + C_i \cdot X_i + D_i \tag{3}$$

and for case Eq. (2)

$$\begin{cases} Y_1(X_1) = A_1 \cdot sh\alpha \cdot X_1 + B_1 \cdot ch\alpha \cdot X_1 + C_1 \cdot X_1 + D_1 \\ Y_2(X_2) = A_2 \cdot sh\alpha \cdot X_2 + B_2 \cdot ch\alpha \cdot X_2 + C_2 \cdot X_2 + D_2 \end{cases} \tag{4}$$

Taking into account that $M_1(X_1) = EI \cdot Y_1''$, $P_1(X_1) = EI \cdot Y_1'''$, $M_2(X_2) = EI \cdot Y_2''$, $P_2(X_2) = EI \cdot Y_2'''$ from boundary conditions (column 3, Table 1) for every system of coordinates at X_1=0, bending $Y_1(0)$=0, slope of the elastic bend $Y_1'(0) = 0$ and $EI \cdot Y_1''(0) = M_0$, $EI \cdot Y_1'''(0) = Q_0$ constant coefficients are determined by:

$$A_1 = \frac{Q_0}{RI \cdot \alpha^3}, \ B_1 = \frac{M_0}{RI \cdot \alpha^2}, \ C_1 = -\frac{Q_0}{RI \cdot \alpha^2}, \ D_1 = -\frac{M_0}{RI \cdot \alpha^2}, \tag{5}$$

and equation of bending for segment I with subject to the Eq. (5) and formula for α can be written as

$$Y_1(X_1) = \frac{Q_0}{\alpha \cdot P_{X1}}(sh\alpha \cdot X_1 - \alpha \cdot X_1) + \frac{M_0}{P_{X1}}(ch\alpha \cdot X_1 - 1). \tag{6}$$

Constant coefficients A_2, B_2, C_2 and D_2 are determined from the boundary conditions X_2=0, the equilibrium conditions and compatibility of deformations $Y'_1(a) = Y'_2(0)$, $EI \cdot Y_1''(a) = EI \cdot Y_2''(0)$, $EI \cdot Y_1'''(a) + P_b = EI \cdot Y_2'''(0) = 0$ and equal

$$\begin{gathered} A_2 = \frac{Q_0 \cdot ch\alpha \cdot a + M_0 \cdot sh\alpha \cdot a + P_b}{\alpha^3 \cdot EI}, \ B_2 = \frac{Q_0 \cdot sh\alpha \cdot a + M_0 \cdot \alpha ch\alpha \cdot a}{\alpha^3 \cdot EI} \\ C_2 = \frac{Q_0 \cdot ch\alpha \cdot a + M_0 \cdot \alpha \cdot sh\alpha \cdot a + P_b}{\alpha^3 \cdot EI}, \ D_2 = \frac{Q_0 \cdot sh\alpha \cdot a + M_0 \cdot \alpha \cdot ch\alpha \cdot a}{\alpha^3 \cdot EI} \end{gathered} \tag{7}$$

and equation of bending on segment II can be written as

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$$Y_2(X_2)=\frac{Q_0\cdot ch\alpha\cdot a+M_0\cdot\alpha\cdot sh\alpha\cdot a+P_b}{\alpha\cdot P_{X1}}(sh\alpha\cdot X_2-\alpha\cdot X_2)+ \\ +\frac{Q_0\cdot sh\alpha\cdot a+M_0\cdot\alpha\cdot ch\alpha\cdot a}{\alpha\cdot P_{X1}}(ch\alpha\cdot X_2-1). \quad (8)$$

The equations of bending in sections I and II for the case Eq. (4) are obtained in a similar way and are given in column 4 of Table 1. The values of the initial parameters Q_0 and M_0 are determined by the boundary conditions at the end of the shaft:

$$Y'_2(L-a)=0 \quad (9)$$

and the equation of bending at the end of a tensile-deflected shaft

$$Y_1(a)=-Y_2(L-a)\,. \quad (10)$$

Results of decision of equations (9) and (10) presented in a column 5, Table 1.
At eccentric tension (model 4, $P_x\neq 0$, $e\neq 0$, Table 1) differential equations (1) for each of I and II sections written as:

$$Y_1^{IV}-\alpha_1^2\cdot Y_1^{II}=0\ , \quad (11)$$

$$Y_2^{IV}-\alpha_2^2\cdot Y_2^{II}=0\ , \quad (12)$$

where $\alpha_1=\sqrt{\frac{(P_{X1}-P_X)}{E\cdot I}}$, $\alpha_1=\sqrt{\frac{(P_{X1})}{E\cdot I}}$, decision Eq. (11) and Eq. (12) are written in the form Eq. (3) with allowance for the value α_1 to I section. The substitution of boundary conditions, equilibrium conditions, and compatibility conditions for deformations (column 3, Table1) into equations (11) and (12) leads to descriptions of bending presented in column 4 of Table 1.
The initial parameters Q_0 and M_0 are determined by the boundary conditions Eq. (9) and the equilibrium conditions

$$\sum M(L)=Q_0L+M_0+P_X\cdot\frac{\alpha}{2}+P_b(L-a)-P_{X1}\cdot e=0\,, \quad (13)$$

and the obtained values of Q_0 and M_0 are given in column 5 of Table 1.

Fig. 1 shows the results of numerical simulation on a computer of the values of elastic deflections of parts in the machining zone $X=a$; the number of analytical dependence corresponds to the model number in Table. 1. Dependence 5 is obtained experimentally at part fixing in the holder of the headstock and in the collet clamp of the tailstock without the possibility of section rotation at the fixing point (model 3, Table 1).

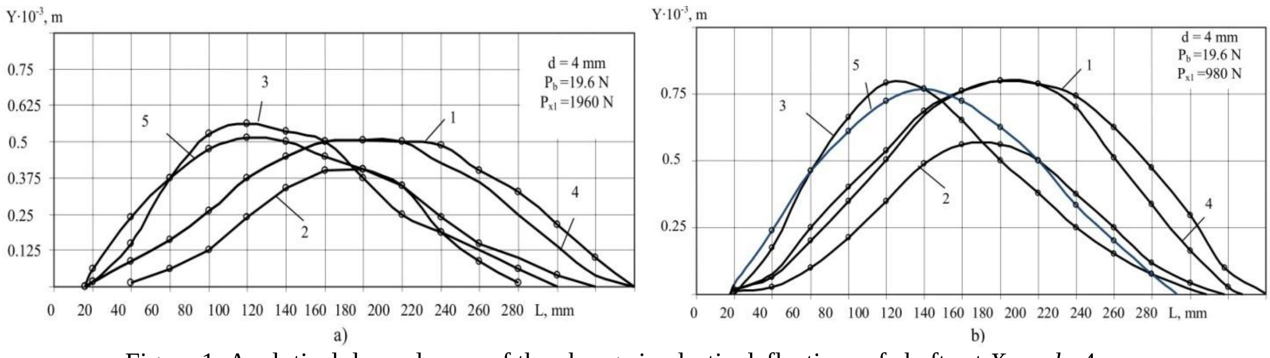

Figure 1. Analytical dependences of the change in elastic deflections of shafts at $X=a$, d= 4 mm:
a) P_{x1}=1960 H; b) P_{x1}=980 H; curve 5 is obtained experimentally

Analysis of results shows that the convergence of results of calculations on a computer of MM 2 with the data of experimental tests on the stand is 3% -12%, and the calculations carried out with MM 4 under the assumption of $P_{X1}\neq 0$, $e=0$ completely correspond to the data obtained with the model 1.

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Mathematical models of various technological processing systems with control of the elastically deformed state in steady-state conditions are received under the assumption that the bending force acting on the workpiece is an external variable that does not depend on the elastic deformations of the system. Such approach is based on neglecting the closure of the elastic system through the cutting process and does not introduce significant errors into the results of analysis of the statistical characteristics of the control object. At the same time, as analysis shows, the construction of an adequate mathematical model of the control object in transient regimes is impossible without taking into account the features of the processes in the processing zone and the closure of the TS through the cutting process.

III. CALCULATION OF THE ELASTICALLY DEFORMED STATE OF THE SHAFT-PULLER AT TURNING ON THE BASIS OF THE RESULTS OF MODELING TECHNOLOGICAL SYSTEM

In order to evaluate the possibilities of the method and establish the theoretical regularities of the behavior of the part in longitudinal-transverse bending [14-16], the equation of the elastic line of a low-rigid shaft was solved on the basis of the calculation scheme in Fig. 2.

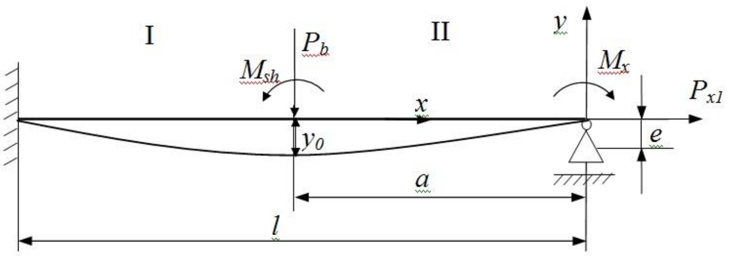

Figure 2. Calculation scheme of stresses and elastic line of the shaft by stretching:e =0.001– eccentricity of stretching force putting; M_{sh}– cutting forces moment; M_x=P_{x1}·e=34.822·0.001=0.034822 Nm –tensile stress moment

Description of low rigged shaft elastic line by longitudinal-transverse bend can be represented in the form of fourth order differential equations with the constant coefficients [12,14]. The equation (1) can be written as:

$$y_i^4 - k^2 y_i'' = 0 \ . \qquad (14)$$

This equation gives the total elastic line equation for stretching and carrying an arbitrary transverse loading beam

$$y = y_0 + y_0' kx + y_0''(1-\cos kx) + y_0'''(kx - \sin kx) + f(x) \,,$$

where $y_0 = -\dfrac{P_y \cdot l}{48EI}$, y_0', y_0'', y_0''' − accordingly deflection, rotary angle, the second and the third derivates in the coordinates beginning; $k = 3...102$ − coefficient determining the details fixing method; *f(x)* – function of transverse loading influence [14].

Elastic line equations on segments I and II (calculation scheme Fig. 2) are

$$\left.\begin{aligned} y_I &= y_0' kx + y_0''(1-\cos kx) + y_0'''(kx - \sin kx) \\ y_{II} &= y_0' kx + y_0''(1-\cos kx) + y_0'''(kx - \sin kx) + f(x) \end{aligned}\right\} . \qquad (15)$$

The initial parameters are determined by the following:

$$y_0' = -\frac{P_y}{kP_{x1}}\left\{\frac{[kl(\alpha - 1 + \cos\alpha kl) - \sin kl](1-\cos kl)}{kl\cdot\cos kl - \sin kl} - (1-\cos\alpha kl)\right\} +$$
$$+\frac{M}{P_{x_1}}\left[\frac{(kl\sin kl + \cos kl - 1)(1-\cos kl)}{kl\cos kl - \sin kl} + \sin kl\right],$$

$$y_0''' = \frac{P_y}{kP_{x1}}\left[\frac{kl(\alpha - 1 + \cos\alpha kl) - \sin\alpha kl}{kl\cdot\cos kl - \sin kl}\right] - \frac{M}{P_{x_1}}\left[\frac{kl\sin kl + \cos kl - 1}{kl\cos kl - \sin kl}\right],$$

where $\alpha = \dfrac{l-a}{l}$; l – detail length; a – coordinate of application of the transverse loading.

Taking into account that the stretching moment was put at the coordinates beginning it is necessary to determine the initial parameter y_0''. We find it by differentiation of Eq. (15)

$$y_{II}'' = -y_0'''k^2 \cos kx - y_0'''' k^2 \sin kx \quad . \tag{16}$$

After the multiplication of Eq.(16) to bending and stretching rigidity we will get the equation of bending moment on segment I, taking into account

$$EJ_x = \frac{M_b}{y_2''}; \quad EF = \frac{M_b \cos\alpha}{y_2''}; \quad \text{then}$$

$$EI \cdot y_2'' = M_b(x); \; EF \cdot y_2'' = M_b(x) \cdot \cos\alpha . \tag{17}$$

If by $x=0$, $M_I(0)=M$, so from Eq.(17) is [8]

$$y_0'' = -\frac{P_y \cdot y_2}{EI} + \frac{P_{x_1} \cdot e}{EF},$$

where $y_2 = r_{sh} \cdot \phi_t$.

Take into account the transverse load influence function

$$f(x) = -\frac{P_y(x-a)}{P_{x1}} + \frac{P_y}{kP_{x1}} \sin k(x-a) \quad .$$

Finally, deflection equations on the segments will be

$$\left. \begin{aligned} y_I(x) = & -\frac{P_y}{P_{x1}}\left[A(1-\cos kl) - (1-\cos\alpha kl)\right]x + \frac{M}{P_{x1}}[B(1-\cos kl) - \sin kl]kx - \\ & -\frac{M}{P_{x1}}(1-\cos kx) + \left(\frac{P_y \cdot A}{P_{x1}} - \frac{M \cdot B}{P_{x1}}\right)(kx - \sin kx), \\ y_{II}(x) = & \; y_I(x) - \frac{P_y}{P_{x1}}(x-a) + \frac{P_y}{k \cdot P_{x1}} \sin k(x-a). \end{aligned} \right\}$$

where $A = \frac{kl(\alpha - 1 + \cos\alpha kl) - \sin\alpha kl}{kl\cos kl - \sin kl}$; $\quad B = \frac{kl \sin kl + \cos kl - 1}{kl \cos kl - \sin kl}$.

On the base of the system numerical variables [14,17-19] and calculation scheme, the deflections and puller shaft turning accuracy were calculated and the results are represented in Table 2 and on Fig. 3.

Table 2. The results of calculation of deflections and shaft turning accuracy.

T, s	y_0 , μm	y_I , μm	y_{II} , μm	y_{def} , μm
0	0	-97.22	32300	32300
0.1	0.1	0.018	-6.38	-6.38
0.2	0.2	0.0585	-7.71	-7.71
0.3	0.3	-0.06295	-3.67	-3.67
0.4	0.4	0.09134	-8.8	-8.8
0.5	0.5	0.00263	-5.853	-5.853
0.6	0.6	0.4888	-7.392	-7.392
0.7	0.7	0.0628	-7.856	-7.856
0.8	0.8	0.04888	-7.392	-7.392
0.9	0.9	-0.1133	-1.994	-1.994
1	1	-0.21978	1.5483	1.5483

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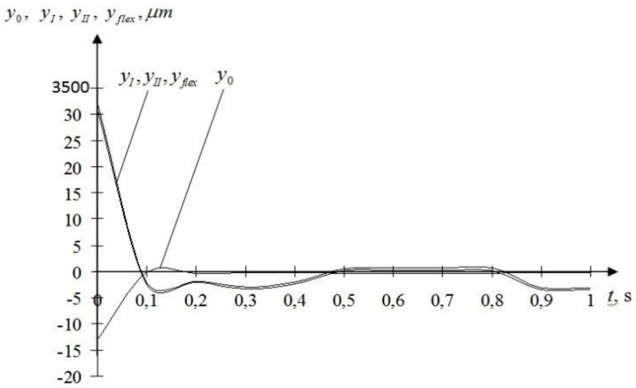
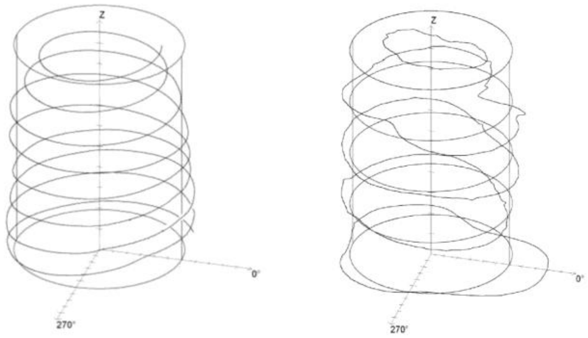


Figure 3. The behavior of accuracy changes for the technological process of shaft turning.

IV. EXPERIMENTAL RESEARCHES OF THE SURFACE ACCURACY OF THE "SHAFT- PULLER" OF THE COTTON PICKING MACHINE APPARATUS AT AN ELASTICALLY DEFORMED STATE TURNED

To perform empirical studies on the influence of machining using control method of elastically deformed state and at standard turning on the geometric accuracy of the form of surface, a roundness measuring machine is used to estimate the error of the surface form. The Taylor-Hobson device was used to construct circular plots of cross-sections of workpiece shafts, used as base surfaces.

The measurements were made on the following deviations of the shape of the manufactured part: 1) removing the deviation from the cylindrical shape. Visualization of measurements is shown in Fig. 4; 2) removing of the roundness deviation of the shape of experimental samples of shafts-pullers in five sections.



a) b)

Figure 4. Cylindricity deviation of the shaft-puller: a) turning in the elastically deformed state; b) at normal turning

Analysis of results on the roundness of the part is carried out in five sections. Visualization of the process of measuring the error from the roundness of the surface form is shown in the form of circular plots in Fig. 5.

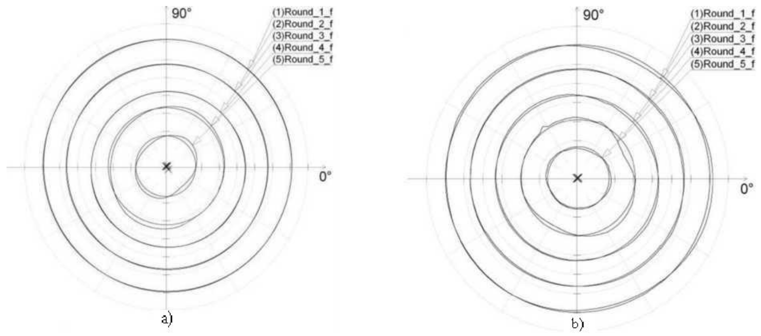


Figure 5. Roundness deviation of the shaft-puller: a) turning of the shaft with tensile force; b) machining without tension, at normal turning

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Analysis of circular plots shows that roundness deviation of the base surface decreases by 1.3 ... 2 times, depending on physical-mechanical properties of material, its initial profile and longitudinal-transverse load on the workpiece shaft.

VI. CONCLUSION

As a result of analysis of control methods of technological systems of nonrigid shafts turning, it has been revealed that the factors that should be taken into consideration are: relatively low rigidity of the part and the metalworking equipment, as well as the inherent elastic deformations of nonrigid details, leading to deviations in surface form and deterioration of the quality of the machined shafts surfaces.

From the analysis of the developed mathematical models and analytically constructed dependences of the changes in the elastic deflections of parts in the control of technological systems for processing parts in an elastically deformed state, using the application of central and eccentric tensile forces as well as longitudinal and transverse bending, it follows that the theoretically specified accuracy of machining can be achieved by controlling two parameters, the value of elastic deflections may be reduced by 15 times, and be equal to $(3\text{-}5)\cdot 10^{-3}$ mm and is practically stabilized along the entire length.

Developed methods of shaft turning due to eccentric tension by longitudinal force and due to application of bending moments provide an increase in processing accuracy by an order of magnitude in comparison with the previous developments.

Improvement in machining accuracy and reliability of technological process of turning is achieved by constructing and using more accurate mathematical models taking into account the features of the phenomena occurring in the processing zone and elastic deflections in technological system and the use of obtained MM in the development of the optimal control algorithms.

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