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Effect of Fourier Coefficients on Electromagnetic Wave Propagation in Complex Plane

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ABSTRACT: The effect of Fourier coefficients on electromagnetic wave propagation in complex plane has been computed for three dielectric function models for spatial line of propagation by the equality of divergence of electric and magnetic wave fields in a field dependent medium. The computed electromagnetic wave field showed sensitivity to dielectric functions, however, the spatial complex plane are similar for different spatial line of propagation making one to deduce that the dielectric material used is not sensitive to the line of propagation of phase plane. All the three models support variety of characteristics, there are attenuation, lossless or solitary propagation and amplitude amplification. Material fabricated based on such models can give way for experimental realization.

KEYWORDS: Fourier coefficients, Complex Plane, Spatial electromagnetic wave field, Dielectric function.

I.INTRODUCTION

The study of Fourier theory on complex plane, began with first attempt to solve the conjugate pairs of Fourier series using inner analytic functions [1],although in ancient time the use of Fourier series to solve a wave-like equation remained a mystery. A Fourier series is a representation employed to express a periodic function *f(x)* defined in an interval say $(-\pi,\ \pi)$ a linear relation between the sines and cosines of same period [2]. In addition, the Fourier transform is an operation performed using mathematical tools that can express mathematical function of time as function of a frequency, known as its frequency spectrum. Fourier's theorem guarantees that this can always be done. The function of time is called the time domain representation, and the frequency spectrum $F(\omega)$, the frequency domain representation [3], given by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-jwt}\, dt \qquad (1)$$

The inverse Fourier transform $f(t)$, expresses a frequency domain function in the time domain. Each value of the function is usually expressed as a complex number that can be interpreted as a magnitude and a phase component Thus we have

$$f(t) = \int_{-\infty}^{\infty} F(t) e^{jwt}\, d\omega \qquad (2)$$

The Fourier transform is beneficial in differential equations because it can transform them into equation which are easier to solve. Also, many transformations can be made simply by applying predefined formulae to the problems of interest.

A window Fourier transform is a mathematical function that is zero-valued outside some chosen interval. It is used in frequency analysis and signal processing. Some common windows are: rectangular, triangular, Hamming, cosine, Barttlet, functions, [4] etc. Three of the window functions and their frequency spectra are illustrated in Figure 1.

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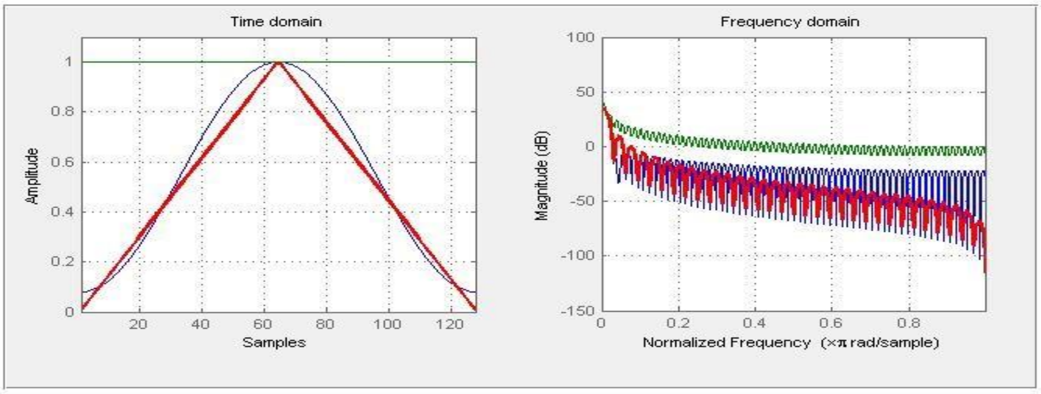

Fig. 1: Some Common Windows: Rectangular, Hamming and Triangular

The Window Fourier Transform (WFT) replaces the Fourier transform's sinusoidal wave by the product of a sinusoid and a window which is localized in time. It takes two arguments: time and frequency. It is defined by:

$$WFT[f(t)] \equiv F(\tau,\omega) = \int_{-\infty}^{\infty} f(t)\omega(t-\tau)e^{-j\omega t}\,dt \qquad (3)$$

The Window Fourier Transform or Short-Time Fourier Transform (STFT) provides a good Time-Frequency representation of signals has a constant time frequency resolution and, is a complete, stable, redundant representation of the signal. But STFT has some limitations due to the nature of the joint time-frequency representation used, an inverse relationship between time domain and frequency domain, and therefore, between their respective signal resolution. This is due to a frequency (Fourier) domain property, whereby a temporal window and its spectrum cannot both be arbitrarily narrow, because if the window (time) narrows its spectrum widens and then, the signal spectrum resolution gets worse [5].

$$S_f(t,\omega) = \int_{-\infty}^{+\infty} f(t)g(t-\tau)^{-j\omega t}\,d\tau \qquad (4)$$

The immediate consequence of this inverse relationship between time and frequency is that, using STFT, there is just one way for improving the identification of spectral detail from a spectrogram, and this is achieved by impairing the temporal resolution. It is usually by quadratics representations because this kind of representation has quadratic (nonlinear) relations with the signal, similar to signal-energy relation, thus showing an important signal feature, the energy distribution. Actually, these quadratic representations have to satisfy some conditions in an energy distribution representation [6].

The purpose of this work is devoted to studying the effect of Fourier coefficients on electromagnetic wave field propagation in complex plane by applying models of field dependent dielectric functions which was not considered before.

This paper is organized as follows. In section II, we briefly introduced the complex representation of Fourier series. In section III, we introduced the fast Fourier transform while the generalized wave-like equation that describe electromagnetic wave field is presented in section 1V. In section V, we presented the numerical solution of the electromagnetic wave equation using the models of field dependent dielectric functions. In section VI, numerical results are presented and discussions are given in section VII. Finally, conclusion is given in section VIII.

II.COMPLEX REPRESENTATION OF A FOURIER'S SERIES

This power mathematical tool solves second order differential equations and partial differential wave equations. Let us consider a periodic function *f(x)* in an interval say $(-\pi,\ \pi)$[7-8] given by

$$f(x) = \sum_{n=-\infty}^{\infty} a_n\, e^{inx} \qquad (5)$$

where, $a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\,dx$ and $a_{-n} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{inx}\,dx$ (6)

Here, a_n *and* a_{-n} are said to be conjugate imaginaries.

Considering a function $f(t)$ which is periodic with a period $T = \frac{2\pi}{\omega}$ then we can write

$$f(t) = \sum_{n=-\infty}^{\infty} a_n\, e^{in\omega t},\ f(t) \text{ being defined in } (-\infty,\ \infty) \qquad (7)$$

where, $\omega = \frac{2\pi}{T}$ and $f(t+T) = f(t)$ (8)

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Now R.H.S of (7) being real, the coefficients of the series on the R.H.S of (7) must be such that no imaginary terms occur. Integrating (7) over 0 to T we have

$$\int_0^{2\pi/\omega} f(t)dt = \int_0^{2\pi/\omega}\left(\sum_{n=-\infty}^{\infty} a_n\, e^{in\omega t}\right) dt = \sum_{n=-\infty}^{\infty} a_n \int_0^{2\pi/\omega} e^{in\omega t}\, dt \quad (9)$$

(Under the assumption that term by term integration is permissible)

$$= 0 \;\; when \;\; n \neq 0 \qquad \because \;\; \int_0^{2\pi/\omega} \frac{1}{in\omega}\left[e^{in\omega t}\right]_0^{2\pi/\omega} = \frac{1}{in\omega}\left(e^{i2n\pi} - 1\right) = 0$$

$$= T \;\; when \;\; n = 0 \qquad \because \;\; \int_0^{2\pi/\omega} dt = \frac{2\pi}{\omega} = T$$

$\therefore$ (9) reduces to $\int_0^T f(t)dt = a_0 T$ given $a_0 = \frac{1}{T}\int_0^T f(t)dt = \overline{f(t)}$ (10)

Where $\overline{f(t)}$ denotes the mean value of $f(t)$.

Now multiplying (7) by $e^{in\omega t}$ and integrating over 0 to T, we have

$\int_0^T f(t)e^{-in\omega t}\, dt = a_n T$ other terms being equal to zero. This gives

$$a_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega t}\, dt \quad (11)$$

Replacing n by $-n$ in (11) we get

$$a_{-n} = \frac{1}{T}\int_0^T f(t)e^{-in\omega t}\, dt \quad (12)$$

From (11) and (12) we conclude that$a_{-n} = \overline{a_n}$.

In order to find the usual real form of the Fourier series, (7) can be expressed as

$f(t) = \sum_{n=-\infty}^{\infty} a_n\, e^{in\omega t} + a_0 + \sum_{n=1}^{\infty} a_n\, e^{in\omega t}$

$= \sum_{n=\infty}^{1} a_{-n}\, e^{-in\omega t} + a_0 + \sum_{n=1}^{\infty} a_n\, e^{in\omega t}$ (Onreplacing n by $-n$ in the first term)

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n e^{in\omega t} + a_{-n} e^{-in\omega t}\right) dt \quad (13)$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n + a_{-n})\cos n\omega t + \sum_{n=1}^{\infty} i(a_n - a_{-n})\sin n\omega t$$

$\therefore e^{in\omega t} = \cos n\omega t + i \sin n\omega t$ and $e^{-in\omega t} = \cos n\omega t - i \sin n\omega t$

If we put $a_n + a_{-n} = \alpha_n,\; i(a_n - a_{-n}) = \beta_n$ and $\alpha_0 = 2a_0$ then we find

$$f(t) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty} \alpha_n \cos n\omega t + \sum_{n=1}^{\infty} \beta_n \sin n\omega t \quad (14)$$

We can thus determine the coefficient α_n and β_n

$$\alpha_n = a_n + a_{-n} = \frac{1}{T}\int_0^T f(t)\{e^{in\omega t} + e^{-in\omega t}\}dt$$

$$= \frac{2}{T}\int_0^T f(t)\cos n\omega t dt \quad (15)$$

and$\beta_n = i(a_n - a_{-n}) = \frac{1}{T}\int_0^T f(t)\, i\{e^{-in\omega t} - e^{in\omega t}\}dt$

$= \frac{1}{T}\int_0^T f(t)\frac{1}{i}\{e^{in\omega t} - e^{-in\omega t}\}dt \because i^2 = -1$

$$= \frac{2}{T}\int_0^T f(t)\sin n\omega t dt \quad (16)$$

The introduction of the term $\frac{\alpha_0}{2}$ in (14) enable (15) to give general term α_n applicable for α_0 as well. In either of the forms, real or complex, the constant term of Fourier series is always equal to the mean value of the function.

III.THE FAST FOURIER TRANSFORM

Fast Fourier Transform (FFT) is a very efficient algorithm to compute Fourier transform. It applies to Discrete Fourier Transform (DFT) and its inverse transforms [9]. DFT is a method that decomposes a sequence of signals into a series of components with different frequency or time intervals. This operation is useful in many fields, but in most cases computing it directly from definition is too slow to be practical. Fast Fourier Transform algorithm can help to reduce DFT computation time by several orders of magnitude without losing the accuracy of the result. This benefit becomes more significant when the number of the components is very large. FFT is considered a huge improvement to make many DFT-based algorithms practical. In Fourier transform spectrometer, signals are often collected by a series of optical or digital channels at the detector. Then FFT is of great importance to quickly achieve the following signal processing and data extraction based on DFT method. By definition, the function Y = fft(x) and y = ifft(x)implement the transform and inverse transform pair given for vectors of length N by:

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$X(k) = \sum_{j=1}^{N} x(j)\omega_N^{(j-1)(k-1)}$ (17)

$x(j) = \left(\frac{1}{N}\right)\sum_{k=1}^{N} X(k)\omega_N^{-(j-1)(k-1)}$ (18)

where, $\omega_N = e^{(-2\pi i)}/_N$ (19)

is an Nth root of unity. By description,Y = fft(X)returns the discrete Fourier transform (DFT) of vector X, computed with a fast Fourier transform (FFT) algorithm. If X is a matrix, fftreturns the Fourier transform of each column of the matrix. If X is a multidimensional array, fftoperates on the first nonsingleton dimension.Y = fft(X, n)returns the n-point DFT. If the length of X is less than n, X is padded with trailing zeros to length n. If the length of X is greater than n, the sequence X is truncated. When X is a matrix, the length of the columns is adjusted in the same manner.Y = fft(X, [], dim) and Y = fft(X, n, dim)applies the FFT operation across the dimension ,dim [10].

IV.GENERALIZED ELECTROMAGNETIC WAVE EQUATION BASED ON SOME ASSUMPTIONS

To arrive at a good results the following assumptions were made:
We shall assume a rectangular symmetry so that Cartesian coordinates x,y,z can be used. We assume that the direction of propagation of the EM waves is the x direction. We assume that the electric and magnetic vectors of the EM waves are in the y and z directions respectively, and that they vary only in the x direction, i.e. $E = E_y(x)j, H = H_z(x)k$, where j and k are unit vectors in y and z directions respectively. We assume that the media are perfect dielectrics and non-magnetic. We assume that the electric and magnetic fields are harmonic in time. We assume that the dielectric properties of the media respond to the spatial component of the electric field only and that it is nonlinear only in the x direction.
Based on the assumptions the wave-like equation is given as:

$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon \frac{\partial^2 E_y}{\partial t^2}$ (20)

As ε is constant, $\varepsilon = \varepsilon(E)$, where E is in fact E_y, therefore (20) is nonlinear wave-like equation and this implies indirectly that ε is a function of x, t since $E = E(x, t)$.In this case the time harmonic assumption for electric fields:

$E(x, t) = W(x)e^{-i\omega t}$ (21)

where, $W(x)$ is y-component of electric field that depends on x-coordinate.From (20) we look for all periodic solution in which the variable x and t are separated in the form [11]

$E(x, t) = X(x).T(t)$ (22)

Then

$$\frac{\partial^2 E}{\partial x^2} = XT'' \ \ and \ \frac{\partial^2 E}{\partial t^2} = X''T$$

Substituting this into (20), we obtain

$$XT'' = \mu_0 \varepsilon X''T \ \ or \ \ \frac{X''}{X} = \frac{T''}{\mu_0 \varepsilon T}$$

Here the function on the left-hand side depends only on x, whereas thefunction on the right-hand side depends only on t. This can happen if both sides of the equation are constant. Hence,

$$\frac{X''}{X} = \frac{T''}{\mu_0 \varepsilon T} = constant = k$$

As k is positive, let put $k = \lambda^2$ to obtain

$$T'' = \mu_0 \varepsilon \lambda^2 T$$

With constant $k = -\lambda^2$, the wave equation splits into two ODEs

$X'' + \lambda^2 = 0$ (23)

$T'' + \mu_0 \varepsilon \lambda^2 T = 0$ (24)

Solution of the first equation gives

$X(x) = C_1 \ cos\lambda x + C_2 \ sin\lambda x$ (24)

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Where C_1 and C_2 are constant of integration. We also consider EM wave propagation in a medium to have a fixed end and so we set the boundary condition to be $(0) = X(L) = 0$. Then $X(0) = C_1 = 0,\ X(L) = C_2\, sin\lambda L$ and $\lambda L = \pi n$ (n is an integer). The eigenvalues are $\lambda_n = \frac{\pi n}{L},\ n = 1, 2, 3, \ldots$ and the corresponding Eigen functions are $X_n(x) = sin\frac{\pi nx}{L}$.

For $\lambda = \lambda_n$ the second equation yields

$T_n = A_n cos(\mu_0 \varepsilon \lambda_n t) + B_n sin(\mu_0 \varepsilon \lambda_n t) = A_n cos\frac{\mu_0 \varepsilon \pi n t}{L} + B_n sin\frac{\mu_0 \varepsilon \pi n t}{L}$(25)

Thus, we can write

$$E_n(x,t) = sin\frac{\pi nx}{L}\left(A_n cos\frac{\mu_0 \varepsilon \pi n t}{L} + B_n sin\frac{\mu_0 \varepsilon \pi n t}{L}\right) \qquad (26)$$

Here n is a positive integer A_n and B_n are arbitrary constants depending on the initial conditions. Now we can combine the general solution of the EM wave equation as a linear combination of the particular solutions

$$E(x,t) = \sum_{n=1}^{\infty} E_n(x,t) = \sum_{n=1}^{\infty} sin\frac{\pi nx}{L}\left(A_n cos\frac{\mu_0 \varepsilon \pi n t}{L} + B_n sin\frac{\mu_0 \varepsilon \pi n t}{L}\right) \qquad (27)$$

Differentiating the series, we find

$$\frac{\partial E(x,t)}{\partial t} = \sum_{n=1}^{\infty} sin\frac{\pi nx}{L}\left(-A_n \frac{\mu_0 \varepsilon \pi n t}{L} sin\frac{\mu_0 \varepsilon \pi n t}{L} + B_n \frac{\mu_0 \varepsilon \pi n t}{L} cos\frac{\mu_0 \varepsilon \pi n t}{L}\right) \qquad (28)$$

Determining the constants A_n and B_n using the initial conditions:

$$E(x,0) = \sum_{n=1}^{\infty} A_n sin\frac{\pi nx}{L} = f(x) \qquad (29)$$

$$\frac{\partial E(x,0)}{\partial t} = \sum_{n=1}^{\infty} B_n \frac{\mu_0 \varepsilon \pi n t}{L} sin\frac{\pi nx}{L} = g(x) \qquad (30)$$

Expanding the functions $f(x)$ and $g(x)$ into the series based on the orthogonal system $\left(sin\frac{\pi nx}{L}\right)$ the formulae for Fourier coefficients become

$$A_n = \frac{2}{L}\int_0^L f(x) sin\frac{\pi nx}{L} dx \qquad (31)$$

$$B_n = \frac{2}{\mu_0 \varepsilon \pi n}\int_0^L g(x) sin\frac{\pi nx}{L} dx \qquad n = 1,2,3, \ldots \qquad (32)$$

Equation (27) is the solution of the electromagnetic wave equation given by the infinite series. The first term $E_1(x,t)$ of the series is called the *fundamental tones*, the other terms $E_n(x,t)$ are called *harmonics*. The *period* and *frequency* of the harmonic are given by;$T_n = \frac{2L}{\mu_0 \varepsilon}$, $\omega_n = \frac{\mu_0 \varepsilon \pi n}{L}$

V.COMPUTATION OF THE ELECTROMAGNETIC WAVE FIELD FOR MODELS OF DIELECTRIC FUNCTION

An appropriate model of field dependent dielectric function is such that the change in dielectric constant due to electromagnetic wave field that propagates through a medium is a typical non-linearity and it vanishes at both boundaries since recombination takes place there [12]. The electric wave field is computed numerically using equation (23), (24) and (27) for the three model of wave-media I, II and II as shown below using fourth order Runge-Kutta scheme implemented in MATLAB software. The spatial Fourier coefficient in complex plane were implemented by symmetric spatial Fast Fourier Transform (FFT) analysis.

Model I $\quad \varepsilon(w) = \varepsilon_0[W + \phi_2 Sin^2(\phi_1 W)]$ (33)

Model II $\quad \varepsilon(W) = \varepsilon_0\left(1 + \alpha W^{\frac{n}{b}}\right)$ (34)

Model III $\quad \varepsilon(W) = \varepsilon_0(1 + \sigma W^2)$ (35)

A substitution of dielectric function model II into equation (27) gives

$$E(x,t) = \sum_{n=1}^{\infty} E_n(x,t) = \sum_{n=1}^{\infty} sin\frac{\pi nx}{L}\left(A_n cos\frac{\mu_0 \varepsilon_0\left(1+\alpha W^{\frac{n}{b}}\right)\pi n t}{L} + B_n sin\frac{\mu_0 \varepsilon_0\left(1+\alpha W^{\frac{n}{b}}\right)\pi n t}{L}\right) \qquad (36)$$

where, $B_n = \frac{2}{\mu_0 \varepsilon_0\left(1+\alpha W^{\frac{n}{b}}\right)\pi n}\int_0^L g(x) sin\frac{\pi nx}{L} dx$ (37)

The fundamental frequency of propagation chosen is $f_0 = 47.7 \times 10^6 Hz$ with vacuum permittivity $\varepsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$ and vacuum permeability$\mu_0 = 4\pi \times 10^{-7} NA^{-2}$. The solution (27) after model substitution are found in the interval of $0 \leq x \leq 200$ with step size of 0.001 for all models. $\phi_1, \phi_2 \alpha$andσ are parameters that play the role of making the unit consistent [$\phi_1 = 100,\ \phi_2 = 300, \alpha = 0.005$ and $\sigma = 0.05$]. The wave frequency $\omega = 2\pi f_0$.

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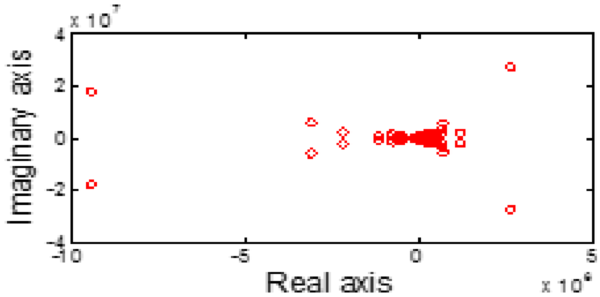
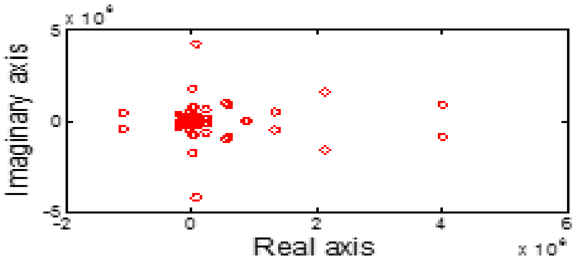
VI.NUMERICAL RESULTS



(a)



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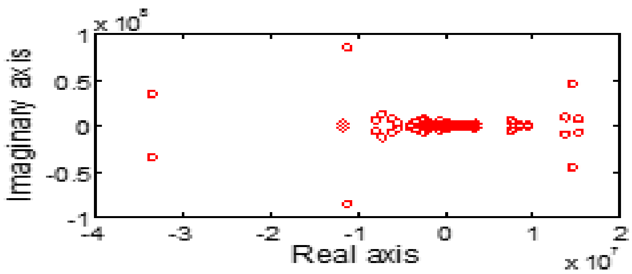

(c)

Figure-1(a-c)

Fourier Coefficients in Complex Plane with Dielectric Function Model I at= $\frac{f_0}{3}$, f_0 *and* $50f_0$.

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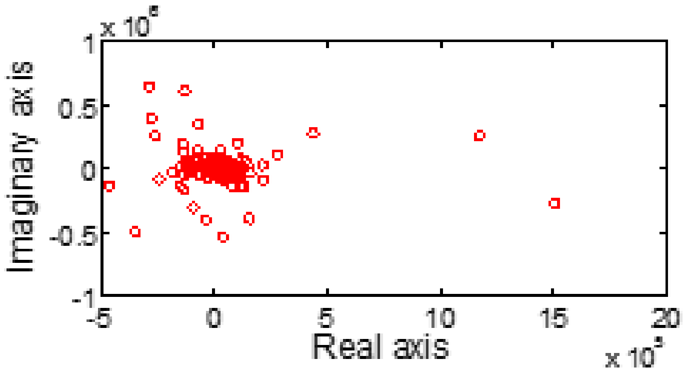

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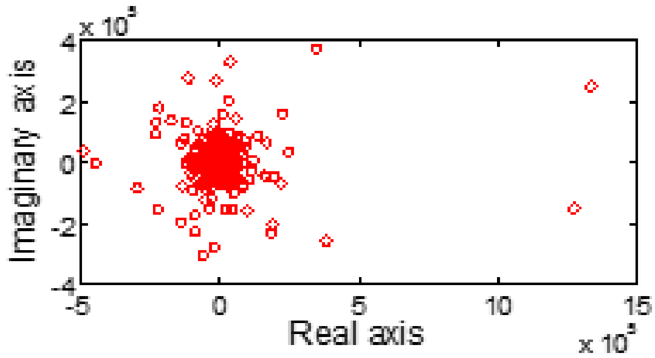

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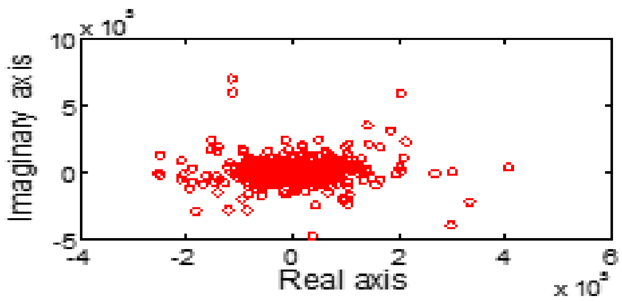

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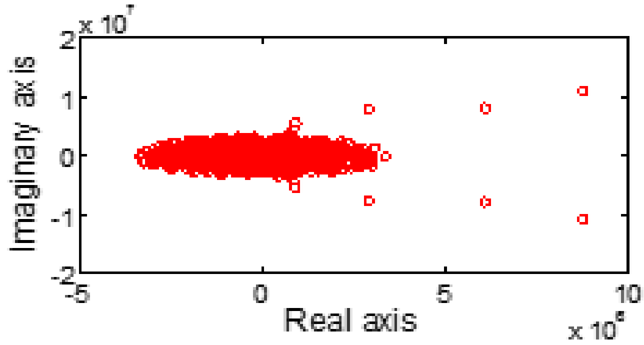
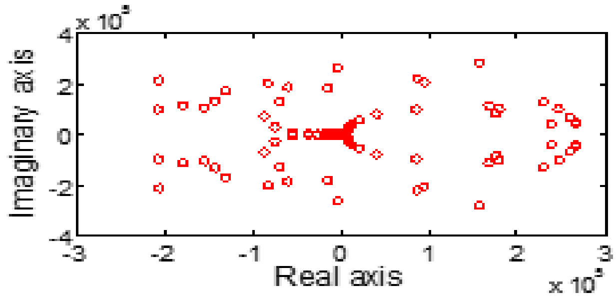


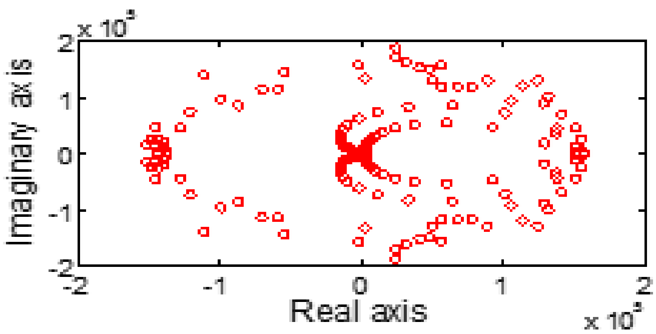
(g)

Figure-2(d-g)

Fourier Coefficients in Complex Plane with Dielectric Function Model IIat $f = \frac{f_0}{3},\ f_0\ , 5f_0\ and\ \ 30f_0$.



(h)



(i)

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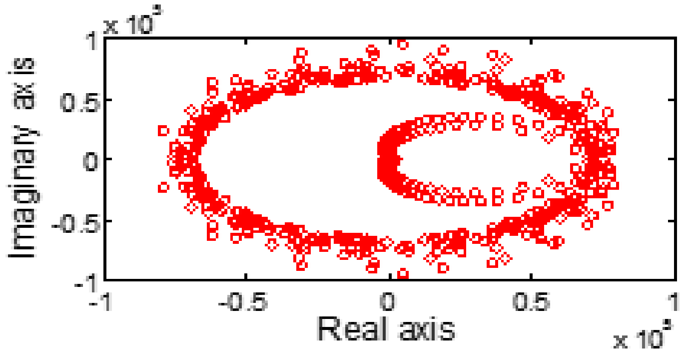

(j)

Figure-3(h-j)
Fourier Coefficients in Complex Plane with Dielectric Function Model III at $f = \frac{f_0}{3},\ 5f_0\ and\ 10f_0$.

Figure-1(a-c) shows the spatial Fourier coefficients in complex plane for dielectric function model I while Figure-2(d-g) is that for dielectric function model II and Figure-3(h-j) is for model III.

VII.DISCUSSION

For dielectric function model I Figure-1 the Fourier coefficients in complex plane containing small circle represents periodic motion of electromagnetic waves propagating at that medium. It can be used to describe how irregular the constant periodic motion of EM waves may be in a particular model occasioned by media property of the material. At phase plane (0, 0), some small circles spread outward in opposite sides of compact point and are in conformity with the line of action. The clear line of compact of the small circles corresponds to the periodic trajectories of the moving EM waves propagation that die down to the lowest on propagating towards the boundary region which is simply an attenuation.

For dielectric function model II Figure-2, the Fourier coefficients in complex plane containing small circle represents periodic motion of electromagnetic waves propagating at that medium. The nature of EM wave propagation in this medium for all frequencies is very interesting. The spatial spectrum reveals the near infinite number of modes that can be propagated in this material. The spectral samples show that odd harmonic of the fundamental wave number k_0, intensifies as there exist many spatial periodic waves distributed at phase plane (0,0).This model is essentially an electromagnetic wave amplificatory. Material model baseon this medium can be used for studying the mode coupling characteristics, for short-distance communication applications, collimated laser, and to transmit/receive light in medical applications. It can be used to transport raw optical power which is then converted to electrical power as alternative to electrical cable. It can also be used to deliver high laser power for some material processing and medical applications.

In model III Figur-3,the computed dielectric function appear in ecliptic-circular formation across the phase plane. The periodic nature of the EM waves in this medium by simulation results circulated outwards with the same amplitude along the direction of spatial line of propagation. The dielectric property of this model material is capable of exhibiting lossless as well as amplitude amplification that are useful as electromagnetic wave guides with much purity.

VII.CONCLUSION

The effect of Fourier coefficient on electromagnetic wave propagation in complex plane has been computed for three dielectric function models. The nature of dielectric function studied differs from one model of dielectric function to another. For each model however, the spatial complex plane are similar for different spatial line of propagation making one to deduce that the dielectric material used is not sensitive to the line of propagation of

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phase plane. In all the three models studied so far support variety of characteristics, there are attenuation, lossless or solitary propagation and amplitude amplification. Material fabricated based on such models can give way for experimental realization. The approach leading to our computations may be improved bytransformation into two-three dimensions in further research work.

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