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Controllability of Nonlinear Fractional Damped Dynamical Systems with Caputo Derivatives

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ABSTRACT: In this paper, controllability of fractional damped dynamical systems with Caputo fractional derivatives is investigated. The solution representations of fractional damped systems have been established by using Laplace transform technique. Sufficient conditions of controllability of nonlinear fractional damped dynamical systems are obtained under the assumption that the corresponding linear system is controllable. An example is provided to illustrate the main results.

KEY WORDS: Controllability, Fractional Damped Dynamical Systems, Caputo Derivatives

I.INTRODUCTION

Controllability is one of the most important issues in mathematical control theory, which means that it is possible to steer a system from an arbitrary initial state to an arbitrary finial state. Recently, the controllability of fractional damped dynamical systems has gained considerable interests, which plays an important role in mathematical description of chemistry, physics and engineering phenomena. Significant headway was made in basic research on controllability results of fractional damped dynamical systems [1-5]. For instance, K. Balachandran et al. [1] considered the controllability of the following nonlinear fractional damped dynamical systems including fractional Caputo derivative

$$\begin{cases} {}^C D^\alpha x(t) - A\,{}^C D^\beta x(t) = Bu(t) + f(t, x(t)),\ t \in J, \\ x(0) = x_0,\ x'(0) = x_0', \end{cases} \tag{1.1}$$

where $0 < \beta \le 1 < \alpha \le 2$, $x \in \mathbf{R}^n$, $u \in L^2(J, \mathbf{R}^m)$, A and B are $n \times n$ and $n \times m$ matrices respectively. Z. Xu and F. Xie [2] studied the controllability of nonlinear fractional damped system with control delay of the following type

$$\begin{cases} {}^C D^\alpha x(t) - A\,{}^C D^\beta x(t) = Bu(t) - Cu(t-\tau) + f(t, x(t), u(t)),\ t \ge 0, \\ x(0) = x_0,\ x'(0) = x_0' \\ u(t) = \varphi(t), -\tau \le t \le 0, \end{cases} \tag{1.2}$$

where $0 < \beta \le 1 < \alpha \le 2$, $x \in \mathbf{R}^n$, $A \in \mathbf{R}^{n\times n}$, $B, C \in \mathbf{R}^{n\times m}$. J. Liu et al. [3] obtained the controllability results of nonlinear higher order fractional damped dynamical system

$$\begin{cases} {}^C D^\alpha x(t) = A\,{}^C D^\beta x(t) + Bu(t) + f(t, x(t), u(t), {}^C D^\alpha x(t), {}^C D^\beta x(t)), \\ x(0) = x_0, x'(0) = x_1, ..., x^p(0) = x_p, \end{cases} \tag{1.3}$$

where $p-1 < \alpha \le p$, $q-1 < \beta \le q$, $q \le p-1$, A is a $n \times n$ matrix and B is a $n \times m$ matrix, $x \in \mathbf{R}^n$, $u \in L^\infty(J, \mathbf{R}^m)$, $t \in [0, T]$, and f is continuous.

Motivated by the above mentioned work, in this paper, controllability of nonlinear fractional dynamical systems of the following type is investigated

$$\begin{cases} {}^C D^\alpha x(t) = A\,{}^C D^\beta x(t) + Bu(t) + f(t, x(t), Gx(t), u(t)),\ t \in J = [0, t_1], \\ x(0) = x_1,\ x'(0) = x_2, \end{cases} \tag{1.4}$$

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where $x \in \mathbf{R}^n$, $u \in L^2(J, \mathbf{R}^m)$, $0 < \beta \le 1 < \alpha \le 2$, A is a $n \times n$ matrix, and B is a $n \times m$ matrix, $f : J \times \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ is a continuous function, Operator G is defined as following

$$Gx(t) = \int_0^t g(t,s)x(s)ds , \tag{1.5}$$

here $g(t,s)$ is continuous function and $(t,s) \in J \times J$.

II.PRELIMINARIES

In this paper, unless otherwise specified, let $\mathbf{R}^n$ be n-dimensional Euclidean space, $\mathbf{R}^+ = [0, \infty)$, ${}^C D^\alpha$ represents Caputo fractional derivative, and

$$E_{\alpha,\beta}(A) = \sum_{k=0}^{\infty} \frac{A^k}{\Gamma(\alpha k + \beta)}$$

represents Mittag-Leffler matrix function, and A^T is the transpose of matrix A.

Definition 2.1 The Caputo fractional derivative of order $\alpha > 0$, $n-1 < \alpha \le n$, is defined as

$${}^C D^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} x^{(n)}(s)ds,$$

where the function $x(t)$ has absolutely continuous derivative up to order n-1 .

Definition 2.2 The fractional integral of order $\alpha > 0$ with the lower limit zero for a function $x \in L^1(J, X)$,is defined a s

$$I^\alpha x(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} x(s)ds,$$

where $\Gamma(\cdot)$ is the Euler gamma function.

Lemma 2.1 ([1]) Linear fractional damped dynamical system

$$\begin{cases} {}^C D^\alpha x(t) = A\, {}^C D^\beta x(t) + Bu(t), \quad t \in J = [0, t_1], \\ x(0) = x_1, \ x'(0) = x_2, \end{cases} \tag{1.6}$$

where $x \in \mathbf{R}^n$, $u \in L^2(J, \mathbf{R}^m)$, $0 < \beta \le 1 < \alpha \le 2$, A is a $n \times n$ matrix, and B is a $n \times m$ matrix, is controllable on J if and only if the Grammian matrix

$$W = \int_0^{t_1} (t_1 - s)^{2\alpha-2} E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta}) BB^T E_{\alpha-\beta,\alpha}(A^T (t_1-s)^{\alpha-\beta}) ds$$

is invertible.

Lemma 2.2 ([6]) Fractional damped dynamical system (1.4) or (1.6) is controllable on J if, for each x_1, x_2, $x_{t_1} \in \mathbf{R}^n$, there exist a control function $u \in L^2(J, \mathbf{R}^m)$ such that the corresponding solution of (1.4) or (1.6) with $x(0) = x_1$ and $x'(0) = x_2$ satisfies $x(t_1) = x_{t_1}$.

Lemma 2.3 Consider the following linear fractional damped dynamical system with Caputo derivatives

$$\begin{cases} {}^C D^\alpha x(t) = A\, {}^C D^\beta x(t) + Bu(t) + f(t), \ t \in J = [0, t_1], \\ x(0) = x_1, \ x'(0) = x_2, \end{cases}$$

where $x \in \mathbf{R}^n$, $u \in L^2(J, \mathbf{R}^m)$, $0 < \beta \le 1 < \alpha \le 2$, A is a $n \times n$ matrix, and B is a $n \times m$ matrix, $f : J \to \mathbf{R}^n$ is a continuous function. Its solution is

$$x(t) = E_{\alpha-\beta}(At^{\alpha-\beta})x_1 + tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2 - t^{\alpha-\beta} E_{\alpha-\beta,\alpha-\beta+1}(At^{\alpha-\beta})Ax_1$$
$$+ \int_0^t (t-s)^{\alpha-1} E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})[Bu(s) + f(s)]ds.$$

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Proof: Take Laplace transform on both side of the equation, we obtain

$$s^{\alpha}X(s) - s^{\alpha-1}x(0) - s^{\alpha-2}x'(0) = As^{\beta}X(s) - As^{\beta-1}x(0) + BU(s) + F(s),$$

then

$$X(s) = \frac{s^{\alpha-1}}{s^{\alpha}I - As^{\beta}}x_1 + \frac{s^{\alpha-2}}{s^{\alpha}I - As^{\beta}}x_2 - \frac{s^{\beta-1}}{s^{\alpha}I - As^{\beta}}Ax_1 + \frac{U(s)}{s^{\alpha}I - As^{\beta}} + \frac{F(s)}{s^{\alpha}I - As^{\beta}}.$$

It is easy to have

$$X(s) = \frac{s^{\alpha-\beta-1}}{s^{\alpha-\beta}I - A}x_1 + \frac{s^{\alpha-\beta-2}}{s^{\alpha-\beta}I - A}x_2 - \frac{s^{-1}}{s^{\alpha-\beta}I - A}Ax_1 + \frac{s^{-\beta}}{s^{\alpha-\beta}I - A}U(s) + \frac{s^{-\beta}}{s^{\alpha-\beta}I - A}F(s),$$

where I is the identity matrix. Taking inverse Laplace transform to both sides of the above equation, we get

$$L^{-1}\{X(s)\}(t)$$

$$= L^{-1}\left\{\frac{s^{\alpha-\beta-1}}{s^{\alpha-\beta}I - A}\right\}(t)x_1 + L^{-1}\left\{\frac{s^{\alpha-\beta-2}}{s^{\alpha-\beta}I - A}\right\}(t)x_2 - L^{-1}\left\{\frac{s^{-1}}{s^{\alpha-\beta}I - A}\right\}(t)Ax_1 + L^{-1}\left\{\frac{s^{-\beta}}{s^{\alpha-\beta}I - A}U(s)\right\}(t)$$

$$+ L^{-1}\left\{\frac{s^{-\beta}}{s^{\alpha-\beta}I - A}F(s)\right\}(t).$$

Thus, the solution of the linear fractional damped dynamical system is

$$x(t) = E_{\alpha-\beta}(At^{\alpha-\beta})x_1 + tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2 - t^{\alpha-\beta}E_{\alpha\text{-}\beta\ \alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1 + t^{\alpha-1}E_{\alpha-\beta,\alpha}(At^{\alpha-\beta}) * [Bu(t) + f(t)].$$

Then

$$x(t) = E_{\alpha-\beta}(At^{\alpha-\beta})x_1 + tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2 - t^{\alpha-\beta}E_{\alpha\text{-}\beta,\ \alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1$$

$$+ \int_0^t (t-s)^{\alpha-1} E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})[Bu(s) + f(s)]ds.$$

III. MAIN RESULTS

Let $X = \{x(t) \mid x(t), {}^{C}D^{\beta}x(t) \in C(J, \mathbf{R}^n)\}$ be a Banach space endowed with the norm

$$\| x \| = \max_{t\in J} |x(t)| + \max_{t\in J} |{}^{C}D^{\beta}x(t)|,$$

where $0 < \beta \le 1$.

Lemma 3.1 Assume that the linear fractional damped dynamical system (1.6) is controllable, and the function f satisfies the following condition:

(H1) there exist a positive constant $M > 0$ such that

$$\| f(t, x, Gx, u) \| \le M, t \in J.$$

(H2) there exist continuous functions $\varphi_i \in C(J, \mathbf{R}^+)$ $i = 1,2,3$ such that

$$\| f(t, x, Gw, u) - f(t, y, Gz, v) \| \le \varphi_1(t) \| x - y \| + \varphi_2(t) \| Gw - Gz \| + \varphi_3(t) \| u - v \|,$$

for $x, y, u, v \in \mathbf{R}^n$.

For convenience, definite

$$n_1 = \max_{t\in J} \| (t_1 - t)^{\alpha-1} B^T E_{\alpha-\beta,\alpha}(A^T(t_1 - t)^{\alpha-\beta}) W^{-1} \|, p = \| y_1 \|$$

$$n_2 = \max_{s\in J} \| (t_1 - s)^{\alpha-1} E_{\alpha-\beta,\alpha}(A(t_1 - s)^{\alpha-\beta}) \|$$

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$$q=\max_{s,t\in J}||(t-s)^{\alpha-1}B^{T}E_{\alpha-\beta,\alpha}(A^{T}(t-s)^{\alpha-\beta})||$$

$$N_1=\max_{t\in J}\|E_{\alpha-\beta}(At^{\alpha-\beta})x_1+tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2-t^{\alpha-\beta}E_{\alpha\text{-}\beta,\ \alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1-x_1\|$$
$$N_2=\max_{t,s\in J}\|(t-s)^{\alpha-\beta-1}E_{\alpha-\beta,\alpha-\beta}\left(A(t-s)^{\alpha-\beta}\right)\|$$

Proof: We define
$x_0(t)=x_1$,
$$x_{n+1}(t)=E_{\alpha-\beta}(At^{\alpha-\beta})x_1+tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2-t^{\alpha-\beta}E_{\alpha\text{-}\beta\ \alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1+\int_0^t(t-s)^{\alpha\text{-}1}E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})$$
$$\times[Bu_n(s)+f(s,x_n(s),(Gx_n)(s),u_n(s))]ds,$$

where
$$u_n(t)=(t_1-t)^{\alpha-1}B^{T}E_{\alpha-\beta,\alpha}(A^{T}(t_1-t)^{\alpha-\beta})W^{-1}$$
$$\times\left[y_1-\int_0^{t_1}(t_1-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta})f(s,x_n(s),(Gx_n)(s),u_n(s))ds\right],$$
and $n=0,1,2,\ldots$ It is obvious that
$$\|u_n(t)\|\le\|(t_1-t)^{\alpha-1}B^{T}E_{\alpha-\beta,\alpha}(A^{T}(t_1-t)^{\alpha-\beta})W^{-1}\|$$
$$\times\left[\|y_1\|+\int_0^{t_1}\|(t_1-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta})\|\|f(s,x_n(s),(Gx_n)(s),u_n(s))\|ds\right]$$
$$\le n_1[p+n_2Mt_1]$$
where $y_1=x_{t_1}-E_{\alpha-\beta}(At_1^{\alpha-\beta})x_1-t_1E_{\alpha-\beta,2}(At_1^{\alpha-\beta})x_2+t_1^{\alpha-\beta}E_{\alpha\text{-}\beta,\ \alpha\text{-}\beta+1}(At_1^{\alpha-\beta})Ax_1$.
After a simple calculation, we get
$$||u_n(t)-u_{n-1}(t)||\le\|(t_1-t)^{\alpha-1}B^{T}E_{\alpha-\beta,\alpha}(A^{T}(t_1-t)^{\alpha-\beta})W^{-1}\|$$
$$\times\int_0^t\|(t_1-s)^{\alpha\text{-}1}E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta})\|\|f(s,x_n(s),(Gx_n)(s),u_n(s))-f(s,x_{n-1}(s),(Gx_{n-1})(s),u_{n-1}(s))\|ds$$
$$\le n_1n_2[\int_0^t\varphi_1(s)\|x_n(s)-x_{n-1}(s)\|+\varphi_2(s)\|(Gx_n)(s)-(Gx_{n-1})(s)\|+\varphi_3(s)\|u_n(t)-u_{n-1}(t)\|ds]$$
Similarly, $\varphi_1(t),\varphi_2(t),\varphi_3(t)$ is continuous on $[0,t_1]$, then there exists $L_1,L_2,L_3>0$ such that
$$||u_n(t)-u_{n-1}(t)||$$
$$\le n_1n_2[\int_0^tL_1\|x_n(s)-x_{n-1}(s)\|+L_2\|(Gx_n)(s)-(Gx_{n-1})(s)\|ds]+n_1n_2L_3t_1\|u_n(t)-u_{n-1}(t)\|$$
Then there exists enough small $t_1>0$ such that
$$||u_n(t)-u_{n-1}(t)\|\le\frac{n_1n_2L_1}{1\text{-}n_1n_2L_3t_1}\int_0^t\|x_n(s)-x_{n-1}(s)\|ds+\frac{n_1n_2L_2}{1\text{-}n_1n_2L_3t_1}\int_0^t\|(Gx_n)(s)-(Gx_{n-1})(s)\|ds$$
and

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$$\begin{aligned}
&\| x_{n+1}(t)-x_n(t)\| \\
&\le \int_0^t \|(t-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})\|\|\|B\|\|\|u_n(s)-u_{n-1}(s)\|ds \\
&+\int_0^t \|(t-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})\|\|\|f(s,x_n(s),(Gx_n)(s),u_n(s))-f(s,x_{n-1}(s),(Gx_{n-1})(s),u_{n-1}(s))\|ds \\
&\le q\|B\|\int_0^t \|u_n(s)-u_{n-1}(s)\|ds+qL_1\int_0^t\|x_n(s)-x_{n-1}(s)\|ds \\
&\quad +qL_2\int_0^t\|(Gx)_n(s)-(Gx)_{n-1}(s)\|ds+qL_3\int_0^t\|u_n(t)-u_{n-1}(t)\|ds \\
&=(q\|B\|+qL_3)\int_0^t\|u_n(s)-u_{n-1}(s)\|ds+qL_1\int_0^t\|x_n(s)-x_{n-1}(s)\|ds+qL_2\int_0^t\|(Gx_n)(s)-(Gx_{n-1})(s)\|ds
\end{aligned}$$

and

$$\begin{aligned}
&\| x_1(t)-x_0(t)\| \\
&\le \|E_{\alpha-\beta}(At^{\alpha-\beta})x_1+tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2-t^{\alpha-\beta}E_{\alpha\text{-}\beta\ \alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1-x_1\|+\int_0^t\|(t-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta})\| \\
&\quad \times[\|B\|\|\|u_0(s)\|+\|f(s,x_0(s),(Gx_0)(s),u_0(s))\|]ds, \\
&\le N_1+q\|B\|(n_1[p+n_2Mt_1])+qMt_1=m_2t_1, m_2>0
\end{aligned}$$

$$\begin{aligned}
&\|u_1(t)-u_0(t)\|\le\|(t_1-t)^{\alpha-1}B^TE_{\alpha-\beta,\alpha}(A^T(t_1-t)^{\alpha-\beta})W^{-1}\| \\
&\times\int_0^t\|(t_1-s)^{\alpha\text{-}1}E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta})\|\|\| f(s,x_1(s),(Gx_1)(s),u_1(s))-f(s,x_0(s),(Gx_0)(s),u_0(s))\|ds \\
&\le 2n_1n_2Mt_1<L_0t_1, L_0>0
\end{aligned}$$

Therefore, it can be obtained according to mathematical induction

$$\| x_{n+1}(t)-x_n(t)\|\le(q\|B\|+qL_3)L_0\frac{t_1^{n+1}}{(n+1)!}+qL_1m_2\frac{t_1^{n+1}}{(n+1)!}+qL_2m_2\frac{t_1^{2n+2}}{(2n+2)!}$$

So the right-hand side of the above inequality can be arbitrarily small by choosing sufficiently large value of n. This implies
that $\{x_n(t)\}$ is a Cauchy sequence in X. Since X is complete, the sequence $\{x_n(t)\}$ converges uniformly to a continuous function $x(t)$. Thus

$$\begin{aligned}
x(t)&=E_{\alpha-\beta}(At^{\alpha-\beta})x_1+tE_{\alpha-\beta,2}(At^{\alpha-\beta})x_2-t^{\alpha-\beta}E_{\alpha\text{-}\beta,\alpha\text{-}\beta+1}(At^{\alpha-\beta})Ax_1+\int_0^t(t-s)^{\alpha\text{-}1}E_{\alpha-\beta,\alpha}(A(t-s)^{\alpha-\beta}) \\
&\qquad\times[Bu(s)+f(s,x(s),(Gx)(s),u(s))]ds, \\
u(t)&=(t_1-t)^{\alpha-1}B^TE_{\alpha-\beta,\alpha}(A^T(t_1-t)^{\alpha-\beta})W^{-1} \\
&\qquad\times\left[y_1-\int_0^{t_1}(t_1-s)^{\alpha-1}E_{\alpha-\beta,\alpha}(A(t_1-s)^{\alpha-\beta})f(s,x(s),(Gx)(s),u(s))ds\right]
\end{aligned}$$

On the other hand,

$$\|{}^{C}D^{\beta}x_{n+1}(t)-{}^{C}D^{\beta}x(t)\|$$
$$=\|\frac{1}{\Gamma(1-\beta)}\int_0^t (t-s)^{-\beta}\left(\int_0^s (s-\tau)^{\alpha-2}E_{\alpha-\beta,\alpha-1}\left(A(s-\tau)^{\alpha-\beta}\right)\right.$$
$$\left.\times\left[B(u_n(\tau)-u(\tau))+\left(f(\tau,x_n(\tau),(Gx)_n(\tau),u_n(\tau))-f(\tau,x(\tau),Gx(\tau),u(\tau))\right)\right]d\tau\right)ds\|$$
$$=\|\int_0^t (t-s)^{\alpha-\beta-1}E_{\alpha-\beta,\alpha-\beta}\left(A(t-s)^{\alpha-\beta}\right)\times\left[B(u_n(s)-u(s))\right.$$
$$\left.+\left(f(s,x_n(s),(Gx_n)(s),u_n(s))-f(s,x(s),Gx(s),u(s))\right)\right]ds\|$$
$$\le N_2\left[\|B\|\|u_n(t)-u(t)\|t_1+L_1t_1\|x_n(t)-x(t)\|+L_2M_4t_1\int_0^t\|x_n(t)-x(t)\|dt+L_3t_1\|u_n(t)-u(t)\|\right]$$
$$=N_2\left[(\|B\|(t_1+L_3t_1)\|u_n(t)-u(t)\|+L_1t_1\|x_n(t)-x(t)\|+L_2M_4t_1\int_0^t\|x_n(t)-x(t)\|dt\right]$$

Obviously, ${}^{C}D^{\beta}x_{n+1}(t)\to{}^{C}D^{\beta}x(t)$ as $n\to\infty$. Thus $x(T)=x_{t_1}$, which means that the control $u(t)$ steers the system (1.4) from the initial state x_0 to the final state x_{t_1} in time t_1 provided that system (1.6) is controllable on J.

IV. EXAMPLE

In this section, an example is presented to illustrate the main results established in the previous section, consider the following nonlinear fractional damped dynamical systems with Caputo derivatives

$$\begin{cases} {}^{C}D^{\frac{3}{2}}x_1(t)=-\frac{3}{4}{}^{C}D^{\frac{1}{2}}x_1(t)+u(t)+\frac{1}{\sqrt{x_1^2(t)+1}}, t\in J=[0,t_1] \\ {}^{C}D^{\frac{3}{2}}x_2(t)=-\frac{1}{2}\,{}^{C}D^{\frac{1}{2}}x_1(t)-\frac{1}{2}\,{}^{C}D^{\frac{1}{2}}x_2(t)+\int_0^t \ln(1+\sin^2(x_2(s)))ds+\cos(u(t)) \\ x(0)=\begin{bmatrix}0\\0\end{bmatrix} \quad x'(0)=\begin{bmatrix}1\\0\end{bmatrix} \end{cases} \tag{4.1}$$

where $x_1=\begin{bmatrix}0\\0\end{bmatrix}$, $x_2=\begin{bmatrix}1\\0\end{bmatrix}, B=\begin{bmatrix}1\\0\end{bmatrix}, A=\begin{bmatrix}\frac{-3}{4} & 0\\ \frac{-1}{2} & \frac{-1}{2}\end{bmatrix}$. By simple matrix calculation based on t_1, one can get the Gramian matrix using Matlab

$$W=\int_0^{t_1}(t_1-s)E_{1,3/2}(A(t_1-s))BB^TE_{1,3/2}(A^T(t_1-s))ds$$

If $|W|>0$, the corresponding linear fractional system of (4.1) is controllable on $[0,t_1]$, and the nonlinear fractional damped dynamical systems (4.1) satisfies the conditions of (H1)-(H2), hence the nonlinear system (4.1) is controllable on $[0,t_1]$.

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