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Universal Criteria for Transition to Turbulence in Industrial Pipelines.

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ABSTRACT: In spite of many years extensive research the transition to turbulence in pipes conveying fluids still remains ambiguous and needs further investigation. The purpose of this paper is to present the semi – empirical method for determination of universal criteria for transition to turbulence in industrial pipes based on few fundamental experiments backed by theoretical considerations. The investigation is presented in two parts. The first part, based on the well – known experimental data, is earmarked for simulation of reciprocal action of vortices with the fluid flow. The results outline the conditions for forming a turbulent velocity profile and relationship between the size of vortices, length between turbulent sparks and intermittency factor. The second part is aimed on consideration of vortex motions generated by Magnus force. The solutions of derived equation, graphically demonstrating the peculiarities of turbulent regime, such as vortex dynamics, intermittency and intrinsic mechanism of breakdown of the larger vortices into smaller ones, were presented for justification that the mathematical model is in compliance with well - known experimental results. The numerical analysis of mathematical model specified the universal laminar - turbulent numbers LT and critical Reynolds numbers for pipes with any diameter.

KEY WORDS: turbulence, transition, intermittency, vortices, velocity profile, drag, Magnus force.

Abbreviations: Re − Reynolds number, LT− critical number of transition to turbulence, F_m – Magnus force, Ma − Mach number.

I. INTRODUCTION

The initial background concerning transition to turbulence (TTT) in pipes conveying fluid was created by outstanding scientists long time ago. The results of their investigations, mainly experimental, are available, in [1 – 3]. Although the experimental results are informative, the theoretical modeling of turbulence and TTT was not successful until now. The changeable properties of fluid in motion, the trajectories of vortices with stochastic component and many other factors hamper theoretical investigations. The reviews of research, related to turbulence in many decades, and direct numerical simulations of the Navier –Stokes equation [4 – 14] concluded that understanding of turbulence is still tentative and not complete. In spite of the singularity of turbulent flow the results of many years research established certain features of turbulent flow such as series of phases (receptive, intermittence and Kolmogorov's microscale), dependences and numbers, introduced by O. Reynolds, L. Prandtl, W. Tollmien, H. Schlichting, Mach and many other scholars for numerical characteristics of turbulent flow. These determined numbers along with experimental results are clear indication that compound of a turbulent flow is the major determined bulk with concomitant, comparatively small stochastic constituent. The purpose of this paper is to introduce the universal numbers for TTT in industrial pipes based on the modeling of vortices motions amenable to mathematical analysis. It is well understood that turbulence is originated by motion of vortices. The established fact is also the disintegration of vortices moving downstream in intermittent phase. The mechanism of vortices disintegration is not known and likely unpredictable. By heuristic considerations, the disintegration of large vortices occurs as a result of vortices clash with piping walls, and this attribute is the indication TTT and a cause of increasing intermittency factor downstream measured by J. Rotta [1, p.152]. So, the swing of vortices equals piping diameter and the disintegration of vortices sizes are preconditions for determination of TTT with alteration in time between laminar or turbulent. Only transversal trajectory of vortices was obtained and graphically demonstrated, since the vortex's motion along a pipeis not important for determination of TTT. The proposed model incorporates the influence of Magnus force on vortex transverse to its velocity. The data pertaining to the proposed model and governing equation are available, for example in [1], with all relevant references and in numerous

ISSN: 2350-0328
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Vol. 5, Issue 11, November 2018

publications devoted to the vortex dynamics, particularly to the effect of the Magnus force. It can be assumed that in steady intermittent phase the turbulent mechanism is independent from disturbances at entrance to a pipe and standard roughness in industrial pipes. The geometrical shape of vortices with washed away outlines cannot be precisely described. It is assumed in the posterior analysis for the modeling purpose that the vortex core is a sphere that rotates and moves as the quasi-rigid body. The vortex rotation is manifestly maintained by parabolic velocity profile of the laminar flow (Fig. 1).

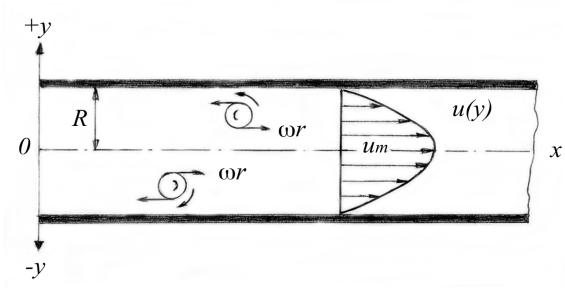

Fig.1 Velocity profile of laminar flow in a pipe and locations of vortices.

Depending on the vortex location, the angular velocity ω is clockwise for y < 0 and counter-clockwise for y > 0. The motion of spinning vortices in the stream of viscous fluid is accompanied by the appearance of transversal Magnus force, which compels the vortices to move across the stream, thus overcoming contrary resistance.

II. SEMI – EMPIRICAL METHOD

A. Simulation of turbulent flow in "intermittent phase".

The distinctive indication of turbulence is a pulsatory flow generated by motion of vortices. Moving across the pipe vortices change in a random manner the laminar velocity profile into the turbulent one. The experiments made, for example by J.C. Rotta, have established this effect of interchange and factors of intermittency. The appearance and forming of the turbulent velocity profile can be theoretically analyzed based on the Navier-Stokes equation. Consider a long, circular pipe with diameter R and coordinate x in the direction of the axis of the pipe. Since the velocity of flow is relatively small, it can be assumed that flow may be taken to be independent on x. When the axial component of velocity u ceases to depend on x, the other velocity component must vanish together with the convective terms parallel to the pipe axis. Thus, instead of the three Navier - Stokes equations in cylindrical coordinates the following equation without any other simplifications can be presented as

$$\partial u/\partial t = -\rho^{-1}\partial p/\partial x + \nu\,(\partial^2 u/\partial y^2 + y^{-1}\partial u/\partial y) \quad (1)$$

with boundary conditions: $u = 0$ at $y = R$ and $y = -R$. In this equation ρ – fluid density, ν – kinematic viscosity, u – velocity of flow, p – pressure, ρ – density of fluid.
For steady laminar flow $\partial u/\partial t = 0$, $\partial p/\partial x = -4\mu R^{-2}u_m$, [1, p.12], viscosity $\mu = \nu\rho$, u_m – maximum velocity on the axis and the solution of Eq. (1) is:

$$u_0(y) = u_m(1 - y^2/R^2) \quad (2)$$

Here $u_0(y)$ – laminar velocity profile, mean velocity of flow $u = 0.5\,u_m$. It is advisable for analysis of turbulent velocity

ISSN: 2350-0328
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Vol. 5, Issue 11, November 2018

profile appearance in a flow to take into consideration two physical factors: the symmetry of the laminar and turbulent velocity profiles and equality of areas of both profiles assuming that flow rate is constant. The symmetry of the turbulent velocity profile is an indication that the vortex is located in the vicinity of the piping axis. The streamlining of vortices is accompanied by increase of the pressure, frontal resistance and the velocity and pressure decrease behind the vortices. For example, the measurements made by O. Flachsband have shown the pressure drop behind the sphere and cylinder in flow (see Ref. in [1], pp. 34, 35). Therefore, the coercion of a vortex on resistance of laminar flow and velocity profile can be presented as

$$\Delta p = 4\mu R^{-2} l u_m (1 - b) \quad (3)$$

Here b – coefficient of pressure and velocity drop behind the vortex, l – length between two cross – sections of a pipe. Hereafter we will define the coefficient b as a ratio W_1/W_0, where W_0 – drag of the laminar flow and W_1 – frontal drag of the vortex in the vicinity of axis.
Accordingly,

$$W_0 = 0.5 \Lambda \rho u^2 l \pi R^2 D^{-1} \quad (4)$$

$$W_1 = 0.5\rho^2 C_w A, \text{ ([1], p. 16)} \quad (5)$$

Here, $D = 2R$, Darcy friction factor $\Lambda = 64\mathrm{Re}^{-1}$, Reynolds number $\mathrm{Re} = Du\nu^{-1}$, the dimensionless coefficient for drag $C_w = f(\mathrm{Res})$, vortex's frontal area $A = \pi r^2$, $d = 2r$, r – vortex's radius. The drag coefficient of spheres plotted against the Reynolds number based on the experimental measurements is shown in Fig. 2, [1, p.17], $\mathrm{Res} = d y' \nu^{-1}$, $y' = u_m$. The measurements performed by J. Nikuradze determined that the fully formed turbulent velocity profile exists already after an inlet length of 25 to 40 diameters [15]. Thereafter, we finally obtain the following empirical relation:

$$b = W_1/W_0 = 0.25 C_w r^2 \mathrm{Re}(Dl)^{-1} \quad (6)$$

The length l is taken as distance between two turbulent profile appearances at the axis of pipe.

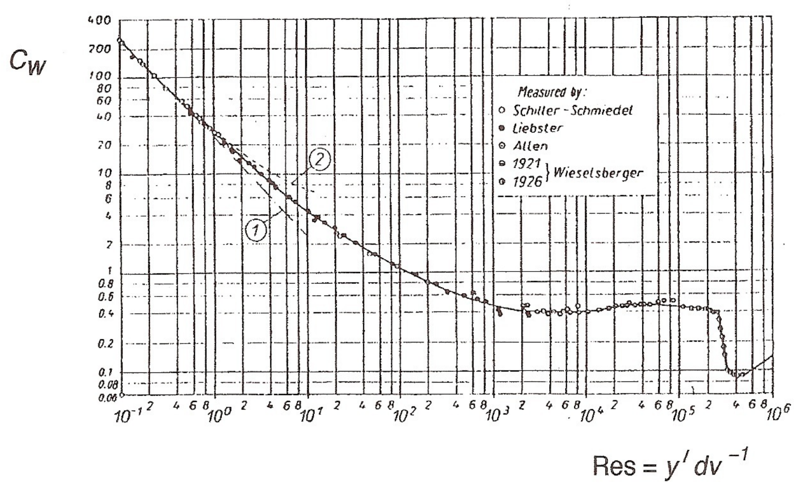

Fig. 2 Drug coefficient for spheres as a function of the Reynolds number. Measurements by C. Wieselsberger, et al. (Ref. in [1], p. 17, Fig. 1. 5).

ISSN: 2350-0328
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The laminar and turbulent velocity profiles after transition are considered on conditions that flow rate and, hence, the areas of both profiles are equal and symmetrical. The "zero function" $f(y)$ was introduced to implement these conditions and change of the laminar velocity profile to turbulent one. For convenience of analysis the origin of coordinates is displaced to $-R$, and, hence, the equation for laminar profile is $u(y) = u_m y^2 R^{-2}$, $u(y) = u_m y^2 R^{-2}$, and "zero function" $f(y)$ is introduced as following:

$$f(y) = a\, u_m \sin \pi y R^{-1} - b\, u_m y R^{-1} \tag{7}$$

The summa of "zero function" is $\int_0^R f(y)dy = 0$. Correspondingly,

$$a\, u_m \int_0^R \sin \pi y R^{-1} dy = a\, u_m R\pi^{-1} \int_0^\pi \sin y dy = 2a u_m R \pi^{-1} = 0.5 b u_m R.$$

Coefficient $a = 0.25\pi b$ and finally $f(y) = 0.7854 b u_m \sin \pi y R^{-1} - b\, u_m y R^{-1}$ (8)

The deduced relationships (6) and (8) provide a chance to graphically demonstrate the formation of turbulent velocity profile in a circular pipe. Let us now consider, for example, the flow of water in a pipe with diameter $D = 0.2$m and Re = 2300. The radius of vortex $r = 0.03$m. In the middle of a pipe $u = 2u_m$, Res = $2rR^{-1}$ Re = 1380 and coefficient for drag $C_w = 0.5$ (Fig.5). The length $l = 36D$. Thereby, $b = 0.18$.

$$f(y) = 0.1414 u_m \sin \pi y R^{-1} - 0.18 u_m y R^{-1} \tag{9}$$

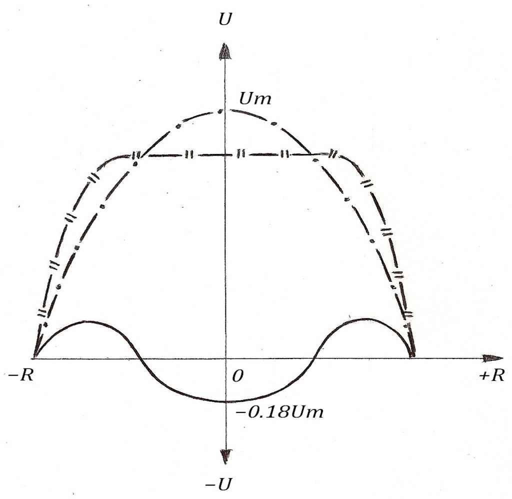

Fig 3. The graph of the regular turbulent velocity profile.

____ "zero function" $f(y)$ - ˙ - laminar velocity profile u_0 — " — turbulent velocity profile u0+ u

ISSN: 2350-0328
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The important conclusion follows from the method dealing with determination of appearance of turbulent velocity profile. Since the shape of turbulent velocity profile for constant flow rate is invariable, the coefficient b is also constant and at first for any Reynolds number the ratio $C_w r^2 l^{-1} \approx$ constant. So, the increase of intermittency factor depends mainly on velocity u. It gives all reasons to state that the pace of vortices decay and, hence, increase of intermittency factor depends on velocity, not on Reynolds numbers. The J. Rotta's experiments [1, p. 452, Fig.16.3] made on a one pipe of small diameter with step by step increased velocity u is the evidence of dependency only on velocity u. Thus, it can be certainly expected that for a pipe with large diameter and Reynolds number, but lesser velocity of flow, the pace of decay of vortices and rate of intermittency factor are sluggish with much longer distance from entrance to the pipe to full turbulence. It will be shown below that the rate of intermittency factor is actually characterized by ratio $u D^{-1}$. Besides that, since $C_w r^2 l^{-1} \approx$ constant on the sufficient distance from the entrance to a pipe the transition to turbulence can be defined within "intermittent phase" with the appropriate size of vortices.

B. Universal numbers for determination of TTT.

The equation of motion of the vortex's center can be presented in the following form:

$$M y^{//} = F_m - F_s \tag{10}$$

Here, M - mass of the vortex's core, F_m - Magnus force, F_s - resistance force,

$y^{//} = d^2y/dt^2$ and t - time...

The Magnus force is defined in accordance with the Kutta - Joukowski's theorem

$$F_m = r\, u_x\, C_r \tag{11}$$

Here, ρ - fluid density, u_x – speed of flow across the pipe, C_r – vortex circulation.

The speed of flow for laminar flow in a circular pipe is described by the Hagen - Poiseuille's law

$$u_x = u_m(1 - y^2 R^{-2}) \tag{12}$$

Here, u_m– largest stream speed at the pipe midst, R – radius of a pipe.

The circulation of the spherical vortex [16]

$$C_r = -\,0.5 u_x A f \tag{13}$$

Here, cross- section area of vortex $A = \pi r^2$, r – radius of vortex core, f – shift coefficient.

Thus

$$F_m M^{-1} = N (R^2 - y^2)^2 \tag{14}$$

Here $M = 4/3 * (\rho \pi r^3)$, $N = -\,0.375\, u_m^2 r^{-1} f (R^2 - y^2)^2$.

So, $N = -\,1.5 \mathrm{Re}^2 f r^{-1} \nu^2 D^{-2} (R^2 - y^2)^{-2}$. Here D represents the swing of a vortex. Moving, for example, from lower wall to upper wall (Fig. 1) the vortex, crossing the pipe axis, changes the direction of rotation. The Magnus force also changes its sign, however, due to the inertia of rotation, the Magnus force changes the sign with some delay. The multiplier $B = (y - y^/) * (\mathrm{I}\, y - a\, y^/\, \mathrm{I})^{-1}$ was introduced to take into account this effect. Here $y^/ = d\,y/d\,t$, a - coefficient of delay. Finally

$$M^{-1} = NB (R^2 - y^2)^2 \tag{15}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 11, November 2018

Moving across the stream the vortex overcomes resistance defined as

$$F_s = 0.5\,\rho_{,} V^2(y)\,AC_w \qquad (16)$$

Here, $V(y) = y'$ - vortex's speed across the stream, C_w – drag coefficient. The theoretical values of C_w were obtained by G. G. Stokes and C. W. Oseen only for Res < 1.0. Here, Res = $y' d\nu^{-1}$, $d = 2\,r$, ν - kinematic viscosity. Since the expected Reynolds numbers Res may significantly exceed Res =1, the vast experimental data for resistance of moving spheres, measured by many researchers was used for further analysis. The graph of the drag coefficient, as a function of Res, presented in Fig. 2. In range of Res between 1.0 and 250 the experimental function $C_w = f$(Res) is in a good compliance with the approximation

$$C_w = 21.66\,Res^{-1} + 6.391\,Res^{-0.417} \qquad (17)$$

and resistance can be presented as

$$F_s M^{-1} = y'\,(k_1 + k_2\,|\,y'\,|^{0.583}) \qquad (18)$$

Here $k_1 \approx 4.061 r^{-2}\nu$, $k_2 = 1.795\, r^{-0.417}\, \nu^{0.417}$.

Upon introducing (10) - (18) the equation for analyzing the vortex's motion becomes

$$y'' = NB(R^2 - y^2)^2 - y'\,(k_1 + k_2\,|\,y'|^{0.583}). \qquad (19)$$

Initial conditions: $t = 0,\ y = -R + r,\ y' = 0$.

The peculiarity of Eq. (19) is the functional attribute of Magnus force to generate the vortex swing depending on the magnitude of Reynolds number and, hence, to define the suitable diameter of a pipe. The coefficients a were obtained on condition that the trajectories of vortices are sinusoidal. As an example, consider the flow of water in a pipe with internal diameter D = 0.2m and $\nu = 1.3 * 10^{-6} m^2$ sec^{-1} at 10^{0}C. The shift coefficient f selected for this example is 0.3. The computations show that, if a = 0, the considered vortex and, hence, all successive vortices will fade and flow downstream remains laminar. For all N < 0 and a > 0 the vortex's motion is sinusoidal across the transverse section. According to the numerous experimental data transition to turbulence in industrial pipes occurs if Reynolds number is equal or exceed 2300. Three cases with r = 0.03m, 0.02m and 0.01m were computed for turbulent flow with Re = 2300 and two cases with Re = 2150, 2000 and r = 0.03m. The swing of the vortex for these two cases is less than piping diameter. The additional case for the pipe with D = 0.3m and Re = 2300 demonstrates that the swing of a vortex is less than piping diameter and TTT can occur only if Re ≥ 2660. The results of computations are presented in Table1.

Table 1 Results of computations

Re	r	N	y_0	k_1 x 10^3	k_2	y_{max}	a
2300	0.01	−106.60	− 0.09	52.80	4.301	± 0.09	4.150
2300	0.02	− 50.29	− 0.08	13.20	1.611	±0.08	1.495
2300	0.03	− 33.53	− 0.07	5.87	0.907	±0.07	0.960
2150	0.03	− 29.30	− 0.07	5.87	0.907	±0.05	0.760
2000	0.03	− 25.35	− 0.07	5.87	0.907	±0.04	0.670
2300 *	0.03	−8.83	− 0.12	5.87	0.907	±0.086	0.906
2660 *	0.03	−15.20	−0.12	5.87	0.907	±0.12	0.960

* Diameter of pipe D = 0.3m

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 11, November 2018

The solutions of Eq. (19) for Re = 2300, 2150, 2000 with r = 0.03 are graphically presented in Fig. 4. It is evident that diminution of the parameter INI below ~ 29.0 and, accordingly, the coefficient a below ~ 0.75 leads to diminution of the vortices swings. Such vortices do not collide with the piping walls and do not disintegrate into smaller vortices. The velocity profile cannot sustain the circulation of vortices with diminished swings and, moving downstream, such vortices gradually cease to exist.

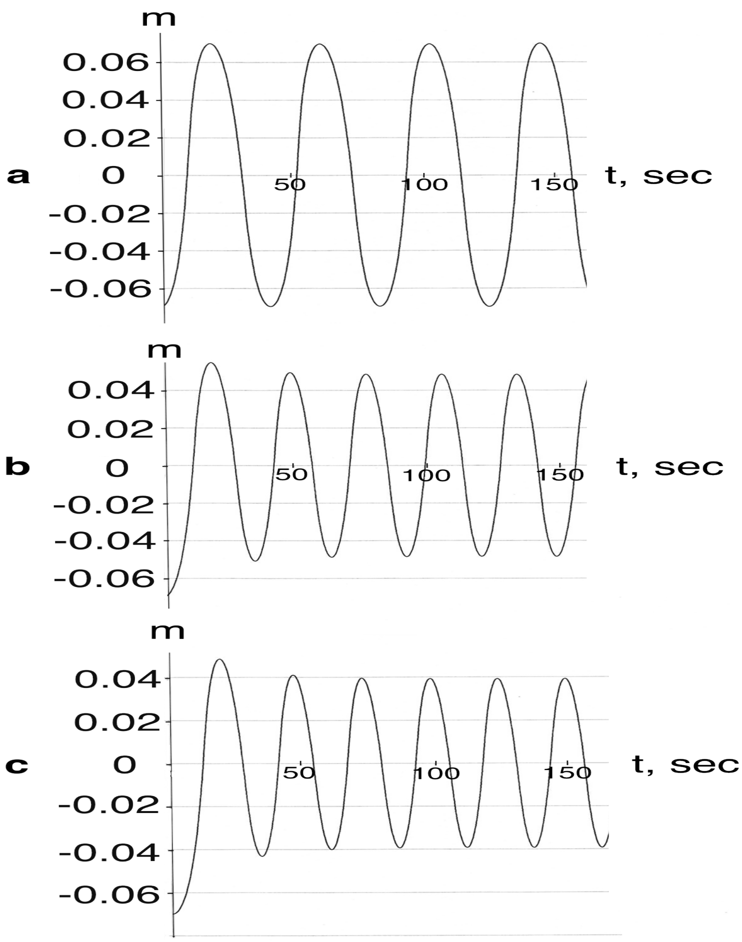

Fig 4. Graphic solutions of Eq. (19) for r = 0.03 m:
a) Re = 2300,
b) Re = 2150,
c) Re = 2000.

These results are in conformity with generally known experimental data. Moving across the stream the vortices excite the periodic change of laminar and turbulent regimes (intermittency) with increasing frequency downstream. The effect of the intermittency in the round pipes was investigated and measured by J. C. Rotta and D. Coles (from [1], p. 452, Fig. 16.3). To introduce this effect coefficient α was modified with an additional multiplier $a\ (1 - |\, sin2pt/t\, |)$ and added to Eq.(19). The period t for Re = 2300 and r = 0.03m was selected based on the time of one cycle (~ 55 sec., Fig. 4a) and experimental data obtained by J. C. Rotta shown in Fig.5.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 11, November 2018

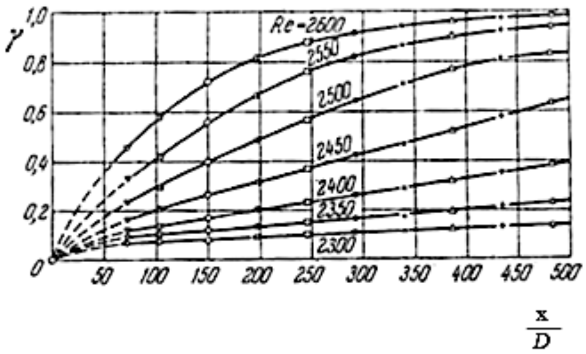

Fig.5. Intermittency factor *g* for pipe flow in the transition range in terms of the axial distance *x* for different Reynolds numbers Re, measured by J. Rotta. γ =1 denotes continuously turbulent, γ = 0 continuously laminar flow ([1], p. 452, Fig. 16. 3).

The intermittency factor *g* was taken equals ~ 0.1 (close to the pipe entrance) and, accordingly, $t \approx 550$ sec. The periods *t* for Re = 2300 and diameters *r* = 0.02m and 0.01m were taken proportional to diameter 0.03m, i.e. for diameter *r* = 0.02m, $t \approx 366$ sec and proportionally for *r* = 0.01m, period $t \approx 180$ sec. The trajectories of the vortices centers in Fig. 6 demonstrate the effect of the intermittency downstream with increasing γ. The vortex model also reveals the essential feature of transition to turbulence. The said experiments performed by J. C. Rotta and D. Coles show that increase of ratio $D/0.5u_m$ with the same Reynolds number leads to increase of the time of transition to complete turbulence and the need to increase the diameter of pipe 0.3m and Re = 2300 (one from the bottom in Table 1). The result shows that the vortex does not reach and does not clash with the piping walls, y_{max} = 0.0862m. Obviously, very little curvature of the velocity profile cannot maintain the vortices rotation and, moving downstream, the vortices gradually disintegrate and flow remains laminar. To provide the vortex's clash with the piping walls Re must be at least 2660. These two cases are the evidence that critical Reynolds number Re_c depends on the ratio $\lambda = 0.5u_{m/D.}$

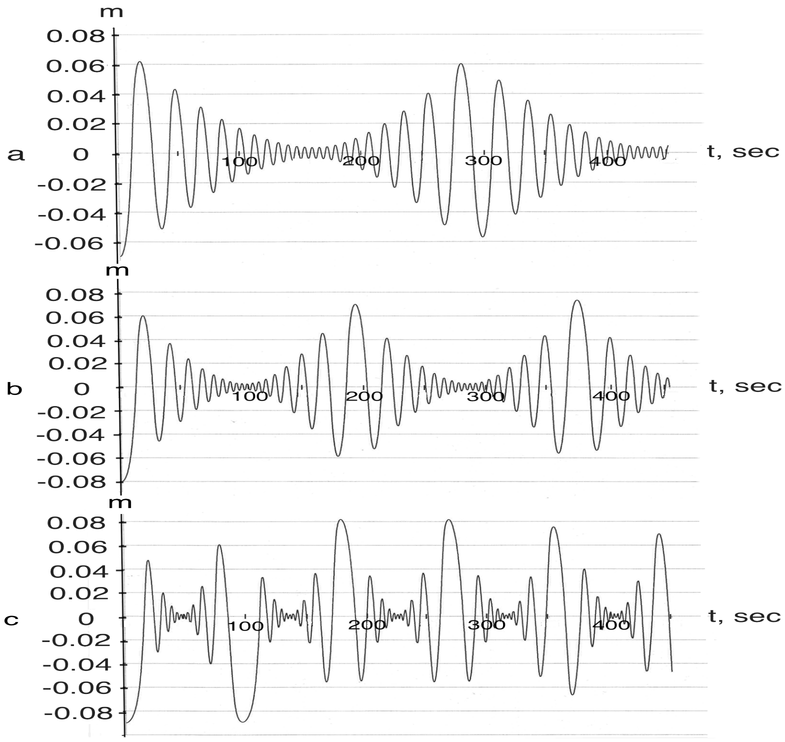

Fig.6. Graphic solutions of Eq. (19) with the effect of intermittency downstream for Re = 2300: a) *r* = 0.03m, b) *r* = 0.02m, c) *r* = 0.01m

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 11, November 2018

Based on the numerical data, presented in Table1, the critical number for laminar to turbulent transition (**LT**) in pipes with diameters in a range between 0.16m and 0.5m and vortices with r = 0.03m can be defined as:

$$\mathbf{LT} = \mathrm{Re}^2 * Ma \approx 56. \tag{20}$$

Here, Mach number $Ma = 0.5\ u_m/v_{s;}$ v_s – velocity of sound in medium. For water v_s = 1440m/sec, the suitable diameter of a pipe for of TTT is $2(y + r)$.

Examples: D = 0.16m, $0.5u_m$ = 0.0175 m/sec, Re = 2150, **LT** = 56.06;
D = 0.2m, $0.5u_m$ = 0.01505 m/sec, Re = 2315, **LT** = 56.0;
D = 0.3m, $0.5u_m$ = 0.01152 m/sec, Re = 2660, **LT** = 56.6;
D = 0.5m, $0.5u_m$ = 0.00818 m/sec, Re = 3150, **LT** = 56.36.

For comparison D = 0.14m, $0.5u_m$ = 0.01875 m/sec, Re = 2000, LT = 51.58.

The diagram of critical numbers for transition to turbulence with ratio $\lambda=0.5u_m D^{-1}$ for r=0.03m is shown in Fig. 7.

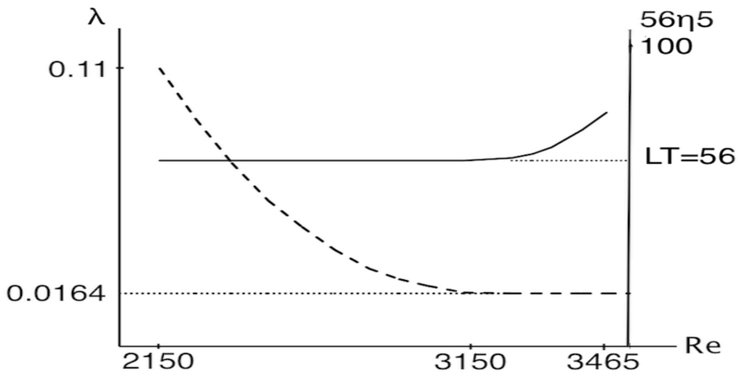

Fig 7. The diagrams of critical numbers for transition to turbulence in a pipe with ratio $\lambda=0.5u_m * D^{-1}$ (broken line), r = 0.03m

For practical applications the ratio of $0.5u_m \approx 29.5$m/ hr to a pipe with D = 0.5m and Magnus force $F\ m = \mathrm{f}\ (r^2 * u_m^2 * D^{-4})$ are assigned as minimum threshold to maintain the motion of vortices with r = 0.03m from wall to wall. The ratio λ = 0.0165 sec^{-1} characterizes the least Magnus force ability to create vortices with swing equals diameter of pipes with $D \geq$ 0.5m. So, for the pipes with diameters D greater than 0.5m, the critical Reynolds number $\mathrm{Re_c}$ for transition to turbulence can be defined as

$$\mathrm{LT} = \mathrm{Re_c} = \lambda\, D^{\,2} \nu^{\,-1} \tag{21}$$

$\lambda = 0.0165\ \mathrm{sec}^{-1}$ (Fig. 7).

For example: D = 0.8m, $\mathrm{Re_c} = 0.0165 * 0.8^{\,2} * 1.3^{-1} * 10^6 = 8123.0$

III. DISCUSSION

The distinctive features of presented vortex model and solutions of the governing equation are the following: a) The numerical analysis shows that transition to turbulence in a pipe with diameter 0.2m and vortex with radius 0.03m occurs at commonly accepted critical Re = 2300. b) The increase in intermittency and turbulence resultant due to diffusion of vortices was computed and graphically demonstrated c) The representative number of cases with different Reynolds number, piping diameters and diameters of vortices was analyzed to formulate the new numerical

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 11, November 2018

criteria for transition to turbulence d) The following is noteworthy: the intermittency factor for pipes with critical Reynolds numbers $\gamma_{2150} > \gamma_{2315} > \gamma_{660} > \gamma_{3150...}$ and accordingly distances between turbulent spikes and distance from the entrance to the pipe increase to full turbulence e) The appearance of turbulence can be detected irrespective of the vortices irregularity and sizes. Note: There is an interesting, feasible scenario. If a pipe is smooth, the turbulence occurs at large Reynolds number. However, after the turbulence is established, the velocity of flow (pressure head) can be diminished until Reynolds number becomes critical.

IV. CONCLUSION

The analysis of interaction of moving vortices with laminar flow in a pipe revealed the formation of turbulent velocity profile generated by vortices drag. The ''zero function" was introduced for elucidation of turbulent velocity profile, assuming that flow rate is constant. By introducing the eddy diffusivity model with effect of the Magnus force, new constant, critical numbers have been derived for indication of transition to turbulence in industrial pipes conveying fluid. The solutions of governing Eq.19 specify the required numerical relations for vortex circulation, Magnus force and ratio of the mean velocity to piping diameter to trigger off turbulence, Fig.7. The results graphically demonstrate the dynamics of vortices, necessity of vortices to collide with piping walls in turbulent regime, turbulent diffusion and increase of intermittency factor downstream. Albeit the propose semi – empirical model does not take into account the stochastic constituent and implies that only motion of vortices is the major factor generating turbulence, the results are in compliance with well – known, fundamental experiments. The obtained universal numbers (20) and (21) can be instrumental in engineering applications.

Nomenclature

M – mass of the vortex's core, F_m – Magnus force, F_s – resistance force, ρ - fluid density,
ν – kinematic viscosity, u_x - average speed of flow, u_m - largest speed of flow,
R, D – radius and diameter of the pipe, π = 3.14..., $A = \pi r^2$ - vortex's core cross area,
r, d - radius and diameter of vortex's core, C_r - spherical vortex's circulation,
$\mathrm{Re} = 0.5\, u_m D \nu^{-1}$, $\mathrm{Res} = y' d \nu^{-1}$, y' - vortex speed across the stream,
C_w – drag coefficient, f – shift coefficient, ω - frequency of the vortex motion,
γ - intermittency factor, w - vortex's angular velocity.

REFERENCES

[1]. Schlichting H, *Boundary Layer Theory*, Seventh Edition, McGraw-Hill Science, New York . 1979.
[2]. Davidson P. A., "Turbulence – An Introduction for Scientists and Engineers", Oxford University Press. 2004.
[3]. Rott N., "Note on the History of the Reynolds Number", Annual Rev. Fluid. Mech. 22: 1-11. www.annualreviews. org/online, 1990.
[4]. Millin T., Peixinbo J., "Recent observations of the transition to turbulence in a pipe.", Proceedings of the sixth IUTAM Simposium on Laminar-Turbulent Transition. Bangalore, India, 2004.
[5]. Willis A. P., et al.," Experimental and theoretical progress in pipe flow transition", Phil. Trans. Royal Society 366 (online), UK, 2008.
[6]. Kerswell R. R., "Recent progress in understanding the transition to turbulence in a pipe.", Nonlinearity 18, R17 - R44, 2005.
[7] Eckhardt B. E., et al., "Turbulent transition in pipe flow", Annual Rev. Fluid Mech. 39, 447, 2007.
[8]. Yokokawa M., et al., "16.4 – Tflops Direct Numerical Simulation of Turbulence by Fourier Spectral Method on the Earth Simulator. ", Proceeding of the 2002 ACM / IEEE Conference on Supercomputing, 2002.
[9]. Kaneda Y., Ishinara T., "High resolution DNS of turbulence", Journal of Turbulence, 7 (20), 1 – 17, 2006.
[10]. Davidson P. A., Kaneda Y., Sreenivasan K. R., *Ten Chapters in Turbulence*, Cambridge University Press, New York, 2013.
[11]. Aliva K., D. Moxey A., de Lozar M., Aliva D. Barkley B. Hof, " The Onset of Turbulence in Pipe Flow", Science 333 (6039), pp 192-196, 2011.
[12]. Bruno Eckhardt, "A Critical Point for Turbulence", Science, pp. 165-166, 8 July 2011.
[13]. Soumerai H, Soumerai-Bourke B. E., "Analytical method of predicting turbulence transition in pipe flow", Scientific Report 2, Article number: 214, 2012. doi: 10.1038/srep 00214.
[14]. Di Pasquale D., Roma A., Garett. S. J., "A selective review of CFD transition model", 39th AIAA Fluid Dynamics Conference, 22 - 25 June 2009.
[15] Nikuradze J. Gesetzmassigkeit der turbulent Stroming in glatten Rohren. Forschg.Arb. Ing. – Wes., 356, 1932.
[16]. Clancy L. J., *Aerodynamics*, Pitman Publishing Limited, London, 1975.
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