



# Sustainable Evaluation of the Value of Permanent Delay in Dynamic Systems

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**ABSTRACT:** The problems of constructing regularized algorithms for estimating the value of constant delay in dynamic systems are considered. When constructing regularized estimates, the regularization method and its iterated version are used. The above computational procedures make it possible to regularize the problem of estimating delays and, thus, to improve the quality indicators of the processes of control of dynamic objects.

**KEY WORDS:** dynamic system, constant lag, stable estimation algorithms, regularization, regularization parameter, iteration.

## I. INTRODUCTION

A large class of dynamic systems is known, which contain in their structure links of constant delay, as well as the application of the theory of optimal control to them [1-3]. Such systems can be attributed to numerous control systems of technological objects, the presence of delay in which is due to the properties of automation elements and digital control systems. When testing these systems, the problem of identification arises - estimating the magnitude of delays, as well as the parameters of the system as a whole. The initial data for solving the problem are measurements of the input and output signals of the system during the transition process occurring in it. The well-known and well-proven methods of identification [4,5] in the time domain, based on the use of nonlinear filtering, can be applied to this task only with sufficiently accurate a priori data. Therefore, the task of robust determination of the delay by measuring the transient processes of the system is relevant for practice.

## II. FORMULATION OF THE PROBLEM

The model of the system and measurements, taking into account fluctuation measurement errors, as well as random constant errors such as sensor zero offset and time-varying zero-drift errors, can be represented as [6]:

$$\dot{x}_k(t) = \sum_{i=1}^n f_{ki} x_i(t - \tau_{xk_i}) + \sum_{j=1}^m g_{kj} U_j(t - \tau_{uk_j}), \quad k = \overline{1, n}, \quad (1)$$

$$Z_x(t) = x(t) + v_{x0} + v_{xl}t + v_x, \quad (2)$$

$$Z_u(t) = U(t) + v_{u0} + v_{ul}t + v_u.$$

Here  $Z_x(t), Z_u(t)$  – measurement vectors of phase coordinates and control actions;  $v_{x0}, v_{u0}, v_{xl}t, v_{ul}t$  – permanent and linear measurement errors;  $v_x, v_u$  – centered high frequency measurement noise;  $x, U$  – state and control vectors.

In (1) the unknown parameters are constant coefficients.  $f_{ki}, g_{kj}, \tau_{xk_j}; i = \overline{1, n}, j = \overline{1, m}, k = \overline{1, n}$ . Here it turns out to be expedient to reduce the problem of identification to a linear one with respect to the desired parameters, which guarantees the uniqueness of the solution. For this purpose, a bilateral symmetric Laplace transform can be used.

$$\begin{aligned} \overline{L}\{x(t)\} &= X(p) + \tilde{X}(p), \quad p = \sigma + j\omega, \\ X(p) &= \int_0^{\infty} x(t)e^{-pt} dt, \quad \tilde{X}(p) = \int_{-\infty}^0 x(t)e^{pt} dt. \end{aligned} \quad (3)$$

Taking into account the integral transformation of the form (3), the transition from differential to algebraic equations is performed. In this case, the desired time lags are linearized in the parametric domain. For each of the equations in

system (1), an overdetermined system of algebraic equations is compiled, taking into account  $N$  calculated points of the complex variable  $p$ . For the  $k$ -th equation of system (1), such a system of equations can be represented as [6]:

$$p_l [Z_k(p_l) - \tilde{Z}_k(p_l)] = \sum_{i=1}^n f_{ki} \{ Z_{xi}(p_l) + \tilde{Z}_{xi}(p_l) - p_l \tau_{xki} [Z_{xi}(p_l) - \tilde{Z}_{xi}(p_l)] \} + \sum_{j=1}^m g_{kj} \{ Z_{uj}(p_l) + \tilde{Z}_{uj}(p_l) - p_l \tau_{ukj} [Z_{uj}(p_l) - \tilde{Z}_{uj}(p_l)] \} + \frac{2c_0}{p_l} + E_k(p_l), \quad l = \overline{1, N}. \tag{4}$$

Here  $c_0$  is the total time constant error, taking into account the influence of polynomial errors  $\nu_{x0}, \nu_{u0}, \nu_{xl}t, \nu_{ul}t$ , also dependent on lag values.

The system of equations (4) can be written in matrix form:

$$Y_k(p) + E_k(p) = Z_k(p) a_k. \tag{5}$$

Here  $Y_k(p) = p [Z_k(p) - \tilde{Z}_k(p)]$  - vector of images of derived measurements of phase coordinates of dimension  $N$ ;  $E_k$  - error vector due to fluctuation measurement errors  $\nu_x, \nu_u$ ;  $a_k$  - vector of required dimension parameters  $n + m + s + 1$ , where  $s$  - the number of determined constant delays in the  $k$ -th equation,

$$a_k = [f_k - f_{k1}\tau \dots f_{kn}, -f_{kn}\tau_{xkn}, g_{k1} \dots - g_{km}\tau_{ukm}, 2c_0]^T.$$

In the absence of errors  $\nu_x, \nu_u$  equation (5) is solved by holding the number of calculation points equal to the number of unknowns. When taking into account fluctuation errors, the system (5) of normal equations is over determined, and can be written as:

$$Z_k \hat{a}_k = Y_k. \tag{6}$$

In general, the solution (6) regarding  $a_k$  is complex. Since the desired parameters belong to the real value domain, (6) is replaced by a double-order system compiled for imaginary and real parts, or complex values are replaced by sums of real and imaginary parts [6]. Then the following system of equations is used:

$$R_k a_k = Q_k, \tag{7}$$

where

$$R_k = \text{Re}[Z_k] + \text{Im}[Z_k], \quad Q_k = \text{Re}[Y_k] + \text{Im}[Y_k].$$

In equation (7), the matrix can be ill-conditioned, which requires the use of regularization methods when solving equation (7) [7-11].

### III. SOLUTION OF THE TASK

In conditions where instead of the exact right side  $Q_k = \bar{Q}_k$  and matrix operator  $\bar{R}_k$  we have them approaching  $\tilde{Q}_k$  and  $\tilde{R}_k$  such that  $\|\tilde{Q}_k - \bar{Q}_k\| \leq \delta$ ,  $\|\tilde{R}_k - \bar{R}_k\| \leq h$ , by class  $G$ ,  $\gamma \equiv (\delta, h)$ , comparable in accuracy with the original data, i.e. allowable approximations to the normal solution  $a_k^0$  the equations  $\bar{R}_k a_k = \bar{Q}_k$  is a set of vectors  $a_k$ , satisfying the condition

$$\|\tilde{R}_k a_k - \tilde{Q}_k\| \leq 2(h\|a_k\| + \delta) + \tilde{\mu}, \quad \tilde{\mu} = \inf \|\tilde{R}_k a_k - \tilde{Q}_k\|,$$

where  $\inf$  taken over all vectors  $a_k$ .

If numbers  $\delta$  and  $h$  are known, then this problem can be solved using the Lagrange method of uncertain factors, i.e. find vector  $a_k^\alpha$ , minimizing smoothing functionality

$$M^\alpha [a_k, \tilde{Q}_k, \tilde{R}_k] = \|\tilde{R}_k a_k - \tilde{Q}_k\|^2 + \alpha \|a_k\|^2,$$

and the parameter  $\alpha$  determine from condition

$$\|\tilde{R}_k a_k^\alpha - \tilde{Q}_k\| = 2(h\|a_k^\alpha\| + \delta) + \tilde{\mu}.$$

If the numbers  $h$  and  $\delta$  unknown or their calculation is associated with considerable difficulties, then the regularization parameter  $\alpha$  it is advisable to determine either on the basis of the method of quasioptimality:

$$\|a_{\alpha_{i+1}} - a_{\alpha_i}\| = \min, \quad \alpha_{i+1} = \Theta \alpha_i, \quad i = 0, 1, 2, \dots, \quad 0 < \Theta < 1,$$

or relationship [7,8]:

$$r_{reg}(\alpha) = r_1(\alpha)/r(\alpha),$$

where

$$r_1(\alpha) = \|K\eta_\alpha - (K\varphi_\alpha - \tilde{f})\|_F^2, \\ \eta_\alpha = \alpha(d\varphi_\alpha/d\alpha).$$

An effective way to select a regularization parameter  $\alpha$  in conditions where quantities  $h$  and  $\delta$  unknown, is also the method of cross-significance [8], according to which the chosen value of the regularization parameter is  $\alpha_{rp}$ , delivering a minimum to the functional of the form:

$$\Phi_0(\alpha) = \frac{1}{S} \|\tilde{Q}_k - \tilde{R}_k \Delta a_k^\alpha\|^2 / [1 - Sp[\tilde{R}_k (\tilde{R}_k^T \tilde{R}_k + \alpha I)^{-1} \tilde{R}_k^T] / S]^2,$$

where  $Sp[\cdot]$  – matrix trace index;  $S$  – number of rows of the matrix  $\tilde{R}_k$ .

When constructing an approximate solution of equation (7) in the case of a reversible operator  $R_k$  a large role is also played by various iterative methods [9-11]. These methods can be either linear, to go to the next iterative approximation, you need to apply some linear operator to one or several previous approximations, or nonlinear, when the transition operator is non-linear. It is known [10] that the commonly used linear iterative methods can generate approximations and in the case of an irreversible operator  $R_k$ . Equipped with a suitable break rule  $r(\delta, h)$  these iterative processes, in turn, generate regularizing algorithms for problem (7).

From this point of view, the iterated variant of the regularization method A.N. Tikhonov [9,10]:

$$\alpha a_{k,r} + R_k^T R_k a_{k,r} = \alpha a_{k,r-1} + R_k^T Q_{k,\delta} \quad (r = 1, \dots, m). \quad (8)$$

The solution of equation (8) is given by the formula

$$a_{k,m} = (I - R_k^T R_k g_{m,\alpha} (R_k^T R_k)) a_{k,0} + g_{m,\alpha} (R_k^T R_k) R_k^T Q_{k,\delta}, \quad (9)$$

where  $a_{k,0}$  – initial approximation, a  $g_{m,\alpha}(\lambda)$  – the generating system of functions for the iterated version (8) of the method of A.N. Tikhonov.

Parameter  $r = r(\delta, h)$  in approximation (9) should be chosen in such a way [9], so that

$$r(\delta, h) \rightarrow \infty, \quad (\delta + h)^2 r(\delta, h) \rightarrow 0, \quad \text{at } \delta \rightarrow 0, \quad h \rightarrow 0.$$

Then  $a_{k,r(\delta,h)} \rightarrow a_{k,*}$  at  $\delta \rightarrow 0, \quad h \rightarrow 0$  where  $a_{k,*}$  – solution of the equation  $R_k^T R_k a_{k,*} = R_k^T Q_k$ .

Here it is advisable to use the following stopping rule of the iterative process:

Set the numbers  $b_1 > 1$  and  $b_2 \geq b_1$ . If a  $\|R_k a_{k,0} - Q_{k,\delta}\| \leq b_2(\delta + \|a_{k,0}\|h)$ , then we put  $r = 0$  and for an approximate decision we will take  $u_0$ . Otherwise, choose  $r > 0$ , at which  $b_1(\delta + \|a_{k,r}\|h) \leq \|R_{k,h} a_{k,r} - Q_{k,\delta}\| \leq b_2(\delta + \|a_{k,r}\|h)$ . If at  $r \in [0, d/(\delta + h)^2]$  the residual did not reach the level  $\|R_{k,h} a_{k,r} - Q_{k,\delta}\| \leq b_2(\delta + \|a_{k,r}\|h)$ , then search  $r$  terminated and selected  $r = d/(\delta + h)^2, \quad d = \text{const} > 0$ .

To calculate the desired vector  $a_k$  for example, other iterative schemes [10, 11] can also be used, which are adjacent to regular iterative algorithms. For the problem under consideration, a nonlinear iterative algorithm of the form [11]:

$$a_{k,r+1} = a_{k,r} - (\varepsilon_r I + R_k^T R_k)^{-1} R_k^T (R_k a_{k,r} - f_\delta), \quad a_{k,0} = 0,$$
$$\varepsilon_r = \begin{cases} \bar{\varepsilon}_r = \frac{\|R_k^T (R_k a_{k,r} - f_\delta)\|^2}{\|R_{k,h} a_{k,r} - f_\delta\|^2}, & \bar{\varepsilon}_r \geq \psi(\delta, h) > 0, \\ \psi(\delta, h), & \bar{\varepsilon}_r < \psi(\delta, h), \end{cases} \quad (10)$$

where  $\Psi(\delta, h)$  – given threshold function such that

$$\lim_{\delta \rightarrow 0, h \rightarrow 0} \Psi(\delta, h) = 0, \quad \Psi(0) = 0.$$

Then, following the theory of iterative methods [9,10], it can be shown that if

$$\lim_{\substack{\delta \rightarrow 0, h \rightarrow 0 \\ r \rightarrow \infty}} (\delta + h \|\hat{a}_k\|) \sum_{j=0}^k (\varepsilon_j)^{-1/2} = 0,$$

that

$$\lim_{\substack{\delta \rightarrow 0, h \rightarrow 0 \\ r \rightarrow \infty}} \|a_{k,r+1} - \hat{a}_{k,r}\| = 0, \quad \hat{a}_k \perp \ker R_k^T R_k.$$

The iteration process (10), while remaining non-linear at the first iterations, converges faster than a number of known iterative algorithms.

#### IV. CONCLUSION

The given stable iteration algorithms allow to increase the accuracy of calculating the magnitude of the delays in the equation of the dynamics of the object and the quality indicators of management processes.

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