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Vibration Analysis of a Flexible Beam with Interior Inlay Fluid

Yan Li, Jie Xu

Student, Nantong Polytechnic College, Nantong, China
Lecturer, Nantong Polytechnic College, Nantong, China

ABSTRACT: Under assumptions of fluid movements and flexible beam vibrations, considering the damping and axial elongation of the flexible beam, coupling nonlinear dynamic controlling equations are established for a liquid-solid system, a flexible cantilever opening beam with interior inlay fluid. In the case of free vibrations without excitations, according to the Hopf bifurcation's requirements for the characteristic polynomial's roots of the Jacobi coefficient matrix of perturbation equations, algebraic expressions, which the critical flow velocity must satisfy, are derived using relative algebraic criterions of Hopf bifurcations. In the case of forced vibrations with low fixed frequency sinusoidal excitations, the classical 4th- order Runge-Kutta method is utilized to solve state-space equations, and results show that when the flow velocity exceeds the critical velocity, the amplitude of beam's vibration response jumps.

KEYWORDS: Beam, Liquid-solid, Matrix; Hopf Bifurcation; Algebraic Criterion.

I. INTRODUCTION

In the field of practical project, there often are some typical liquid-solid coupling structures/systems with interior inlay fluid, such as pipeline structure, including oil and natural gas pipeline, and laminated panels cooling system of nuclear reactor and so on. The research on stability and vibration response characteristics is very significant to the designing of the typical structures. Paidoussis M. P.[1] studied the dynamic stability of air extraction pipe. He found the pipeline could not be vibration instability under the action of infinitely small fluid, which was also verified by corresponding experiments. Holmes[2], Bajaj[3] et al. studied the stability of fluid-conveying pipeline under different supporting conditions and proved the presence of post-instability phenomenon. Guo C.Q. et al. [4] investigated the vibration problem of laminated rectangular panels inducted by fluid. There is still have many similar investigations but details not listed here. In this paper, a flexible cantilever opening beam, with interior inlay fluid in axial channel, was investigated. Coupling dynamic controlling equations were established on certain assumptions.

In the case of free vibrations without excitations, according to the Hopf bifurcation theory and relative algebraic criterions, Hopf bifurcation critical flow velocity can be obtained. The classical 4th-order Runge-Kutta method is utilized to solve state-space equations, and results show that when the flow velocity exceeds the critical velocity, the amplitude of beam's vibration response jumps.

II. COUPLING VIBRATION CONTROLLING EQUATIONS

Consider a flexible cantilever opening beam with interior inlay fluid. The fluid flows along the axial channel in a constant flow velocity of v_F , with low fixed frequency sinusoidal excitations in the free end of the beam. Main characteristic parameters of the system are listed in Table 1. Following assumptions are taken for the system: (i) The flexible beam only can vibrate in y direction. (ii) During the vibration, at the same time, the height of flow channel (y direction) is constant along the length of the beam (x direction). (iii) The fluid is ideal fluid, inviscid and incompressible, and has constant flow velocity of v_F . Based on above assumptions, the liquid-solid coupling vibration controlling equation is following [4] .

$$EI_y \frac{\partial^4 w}{\partial x^4} + c\frac{\partial w}{\partial t} + \rho A \frac{\partial^2 w}{\partial t^2} - 2EA(\frac{\partial w}{\partial x})^2 \frac{\partial^2 w}{\partial x^2} + \frac{3}{4} EA(\frac{\partial w}{\partial x})^4 \frac{\partial^2 w}{\partial x^2}$$
$$+ M_c(\frac{\partial^2 w}{\partial t^2} + 2v_F \frac{\partial^2 w}{\partial t \partial x} + v_F^2 \frac{\partial^2 w}{\partial x^2}) = \delta(x-L)\overline{F}_y \sin(\omega t) \quad (1)$$

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where EI_y is y direction flexural rigidity of the beam. $w(x,t)$ is y direction vibration displacement. A is Cross section area of the beam. $M_c=\rho_F A_F$ (ρ_F is the fluid density and A_F is the cross section area of the flow channel) is the fluid added mass. L is length of the beam. For more conveniently analyzing formula (1), the following dimensionless quantity is introduced: $W=w/L$, $X=x/L$, $\alpha=L^2A/I$, $\beta=M_c/(M_c+\rho A)$, $T=(EI)^{0.5}t/[L^2(M_c+\rho A)]$, $C=cL^2/\sqrt{EI(Mc+\rho A)}$, $V=\sqrt{Mc/(EI)}v_F L$, $F=\bar{F}L^3/(EI)$, $\varpi=\omega L^2\sqrt{(M_c+\rho A)/(EI)}$. Substituting above dimensionless quantity into formula (1), dimensionless coupling vibration controlling equations of system can be obtained as followed.

$$\frac{\partial^2 W}{\partial T^2}+C\frac{\partial W}{\partial T}+2\sqrt{\beta}V\frac{\partial^2 W}{\partial T\partial X}+V^2\frac{\partial^2 W}{\partial X^2}-2\alpha(\frac{\partial W}{\partial X})^2\frac{\partial^2 W}{\partial X^2}+\frac{3}{4}\alpha(\frac{\partial W}{\partial X})^4\frac{\partial^2 W}{\partial X^2}=\delta(X-1)F\sin(\varpi T) \tag{2}$$

According to the constraint conditions of the beam, assuming experimental mode function $\phi_i(X)$ as main vibration mode of empty beam.

$$\begin{aligned}&\phi_i(X)=\cos(\kappa_i X)-\mathrm{ch}(\kappa_i X)+r_i[\sin(\kappa_i X)-\mathrm{sh}(\kappa_i X)]\\&r_i=\frac{\sin\kappa_i-\mathrm{sh}\,\kappa_i}{\cos\kappa_i+\mathrm{ch}\,\kappa_i}\\&\kappa_1=1.875,\kappa_2=4.694,\kappa_i=(i-0.5)\pi,(i\geq 3)\end{aligned} \tag{3}$$

Let $\eta_i(T)$ be corresponding modal coordinates, and then $W(X,T)$ can be expressed as:

$$W(X,T)=\sum_{i=1}^{\infty}\eta_i(T)\phi_i(X) \tag{4}$$

For approximate calculation, truncating higher order modes than 2, substituting formula (4) into formula (2), using modal orthogonality, multiplying $\phi_j(X)$ on the both sides of the formula and omitting higher order modes than 3, then integrating the formula in .

$$\begin{aligned}&\frac{\mathrm{d}^2\eta_1}{\mathrm{d}T^2}+a_1\frac{\mathrm{d}\eta_1}{\mathrm{d}T}+a_2\frac{\mathrm{d}\eta_2}{\mathrm{d}T}+a_3\eta_1+a_4\eta_2+a_5{\eta_1}^3+a_6{\eta_1}^2\eta_2+a_7\eta_1{\eta_2}^2+a_8{\eta_2}^3=a_9\sin(\varpi T)\\&\frac{\mathrm{d}^2\eta_2}{\mathrm{d}T^2}+b_1\frac{\mathrm{d}\eta_1}{\mathrm{d}T}+b_2\frac{\mathrm{d}\eta_2}{\mathrm{d}T}+b_3\eta_1+b_4\eta_2+b_5{\eta_1}^3+b_6{\eta_1}^2\eta_2+b_7\eta_1{\eta_2}^2+b_8{\eta_2}^3=b_9\sin(\varpi T)\end{aligned} \tag{5}$$

where

$$\begin{aligned}&a_1=C\int_0^1{\phi_1}^2\mathrm{d}X+2\sqrt{\beta}V\int_0^1\phi_1\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\mathrm{d}X,\ a_2=2\sqrt{\beta}V\int_0^1\phi_1\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\mathrm{d}X,\ a_3={\kappa_1}^4\int_0^1{\phi_1}^2\mathrm{d}X+V^2\int_0^1\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\phi_1\mathrm{d}X\\&a_4=V^2\int_0^1\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\phi_1\mathrm{d}X,\ a_5=-2\alpha\int_0^1\phi_1(\frac{\mathrm{d}\phi_1}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X,\ a_6=-2\alpha(2\int_0^1\phi_1\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X+\int_0^1\phi_1(\frac{\mathrm{d}\phi_1}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X)\\&a_7=-2\alpha(2\int_0^1\phi_1\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X+\int_0^1\phi_1(\frac{\mathrm{d}\phi_2}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X),\ a_8=-2\alpha(\int_0^1\phi_1(\frac{\mathrm{d}\phi_2}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X),\ a_9=F\phi_1|_{X=1}\end{aligned} \tag{6}$$

and

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$$b_1=2\sqrt{\beta}V\int_0^1\phi_2\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\mathrm{d}X,\ b_2=C\int_0^1\phi_2^{\ 2}\mathrm{d}X+2\sqrt{\beta}V\int_0^1\phi_2\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\mathrm{d}X,\ b_3=V^2\int_0^1\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\phi_2\mathrm{d}X$$

$$b_4=V^2\int_0^1\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\phi_2\mathrm{d}X+\kappa_2^{\ 4}\int_0^1\phi_2^{\ 2}\mathrm{d}X,\ b_5=-2\alpha\int_0^1\phi_2(\frac{\mathrm{d}\phi_1}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X,\ b_6=-2\alpha(2\int_0^1\phi_2\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X+\int_0^1\phi_2(\frac{\mathrm{d}\phi_1}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X) \tag{7}$$

$$b_7=-2\alpha(2\int_0^1\phi_2\frac{\mathrm{d}\phi_1}{\mathrm{d}X}\frac{\mathrm{d}\phi_2}{\mathrm{d}X}\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X+\int_0^1\phi_2(\frac{\mathrm{d}\phi_2}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_1}{\mathrm{d}X^2}\mathrm{d}X),\ b_8=-2\alpha(\int_0^1\phi_2(\frac{\mathrm{d}\phi_2}{\mathrm{d}X})^2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}X^2}\mathrm{d}X),\ b_9=F\phi_2|_{X=1}$$

Let $q1=\eta_1$, $q2=\dot{\eta}_1$, $q3=\eta_2, q4=\dot{\eta}_2$, and then the state equation of the system is following.

$$\begin{aligned}&\dot{q}1=q2,\quad \dot{q}2=-a_1q2-a_2q4-a_3q1-a_4q3-a_5q1^3-a_6q1^2q3-a_7q1q3^2-a_8q3^3+a_9\sin(\varpi T)\\&\dot{q}3=q4,\quad \dot{q}4=-b_1q2-b_2q4-b_3q1-b_4q3-b_5q1^3-b_6q1^2q3-b_7q1q3^2-b_8q3^3+b_9\sin(\varpi T)\end{aligned} \tag{8}$$

III. DYNAMIC HOPF BIFURCATION UNDER FREE VIBRATION OF SYSTEM

For some pipeline or interlayer structures, under the condition of free vibrations without excitations, the whole system would be vibration instability when flow velocity of the interior fluid exceeds a certain value, which indicates the occurrence of dynamic Hopf bifurcation. At this time, topological structure of system phase diagram changes, and stable limit cycles appear around the fixed point. According to bifurcation theory, an important item must be met when dynamic Hoph bifurcation taking place, that is, there must be a pair of pure characteristic roots of the Jacobi coefficient matrix of perturbation equations. For our system, truncating excitation items in the right side of formula (8), the free vibration state equation can be obtained, furthermore, linearization perturbation equation of system can be obtained as

$$\begin{bmatrix}\dot{\zeta}_1\\\dot{\zeta}_2\\\dot{\zeta}_3\\\dot{\zeta}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\-(0.8790V^2+12.3596)&-(C+4\sqrt{\beta}V)&11.7549V^2&9.5238\sqrt{\beta}V\\0&0&0&1\\-1.8736V^2&-1.5170\sqrt{\beta}V&(13.2868V^2-485.4811)&-(C+4\sqrt{\beta}V)\end{bmatrix}\begin{bmatrix}\zeta_1\\\zeta_2\\\zeta_3\\\zeta_4\end{bmatrix} \tag{9}$$

From formula (9), the characteristic polynomial of the Jacobi coefficient matrix A can be obtained as

$$|A-\lambda E|=\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0 \tag{10}$$

where

$$A_1=-2B_2,\ A_2=B_2^{\ 2}-B_6B_4-B_7-B_1,\ A_3=B_1B_2-B_4B_5-B_3B_6+B_2B_7,\ A_4=B_1B_7-B_3B_5 \tag{11}$$

and

$$\begin{aligned}&B_1=-(0.8790V^2+12.3596),\ B_2=-(C+4\sqrt{\beta}V),\ B_3=11.7549V^2\\&B_4=9.5238\sqrt{\beta}V,\ B_5=-1.8736V^2,\ B_6=-1.5170\sqrt{\beta}V,\ B_7=(13.2868V^2-485.4811)\end{aligned} \tag{12}$$

It is very difficult to validate whether formula (9) have a pair of pure characteristic roots by the method of solving characteristic roots. Reference 8 gave the following theorem to judge the real coefficient characteristic polynomial's roots condition. For the following real coefficient polynomial

$$x^n+a_1x^{n-1}+a_2x^{n-2}\cdots\cdots+a_{n-1}x+a_n=0 \tag{13}$$

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There are a pair of pure imaginary roots, and the necessary and sufficient conditions of other characteristic roots having negative real parts are: (i) $a_i > 0$ （$i = 1,2,3\cdots,n$）; (ii) $\Delta_i > 0$ （$i = n-3, n-5,\cdots\cdots$）, $\Delta_{n-1} = 0$, where Δ_i is Hurwitz determinant of polynomial. According to above theorem, it can be obtained through calculation that, when the dynamic Hopf bifurcation occurs, the critical flow velocity of interior fluid in axial channel must meet the following formulas:

$$2B_2[(B_2^2 - B_6B_4 - B_7 - B_1)(B_1B_2 - B_4B_5 - B_3B_6 + B_2B_7) - 2B_2(B_1B_7 - B_3B_5)] + (B_1B_2 - B_4B_5 - B_3B_6 + B_2B_7)^2 = 0$$
$$A_2 = B_2^2 - B_6B_4 - B_7 - B_1 > 0,\quad A_3 = B_1B_2 - B_4B_5 - B_3B_6 + B_2B_7 > 0 \quad (14)$$

Substituting parameters into the dimensionless formula, obtaining $\beta = 0.4$， $\alpha = 10^5$， $C = 0.08$, which were substituted into the formula (11), and then the critical flow velocity of dynamic Hopf bifurcation can be obtained: $V_{\text{div-dy}} = 7.3825$. The classical 4th-order Runge-Kutta method is utilized to solve state-space equations in the condition of free vibrations without without excitations.

IV. VIBRATION RESPONSE CHARACTERISTIC

In the case of forced vibrations with low fixed frequency sinusoidal excitations, the classical 4th- order Runge-Kutta method is utilized to solve state-space equations (8), and the response amplitude change law in trans-critical flow $V_{\text{div-dy}}$ is investigated. Excitation amplitude F is selected according to the principle of response amplitude is not exceed $1\times10^{-2}\text{m}$, and here, Appointed it as $F = 0.01$.

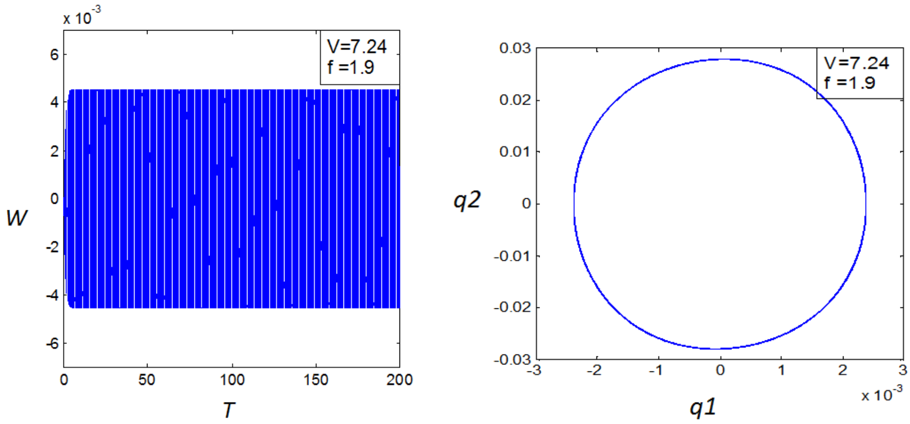

Fig.1 The time-domain curve and phase diagram of the flexible beam when $V < V_{\text{div-dy}}$ (V=7.24, f=1.9)

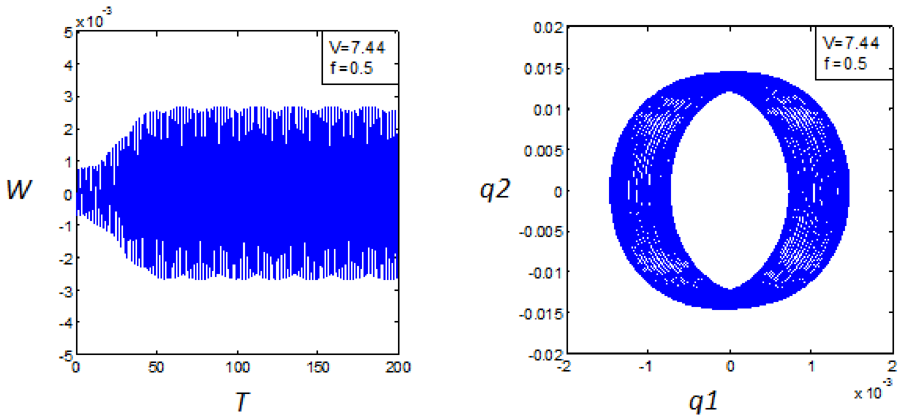

Fig.2 The time-domain curve and phase diagram of the flexible beam when $V < V_{\text{div-dy}}$ (V=7.44, f=0.5)

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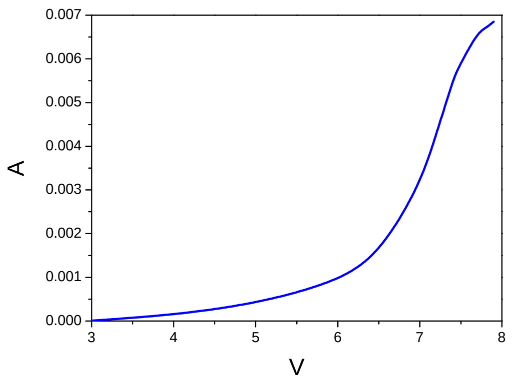

Fig.3 The effect of flow velocity on vibration response amplitude
(A is dimensionless vibration response amplitude of beam free end, V is dimensionless velocity)

When $V < V_{\text{div-dy}}$, the vibration response of flexible beam is as Fig.1 showed. When $V > V_{\text{div-dy}}$, with the change of difference between exciting frequency and vibration frequency of unperturbed vibration instability, vibration response characteristic is also different. When the difference is great, vibration response have beat frequency (Fig.2). Analysis result map (Fig.3) shows that, pre and post $V_{\text{div-dy}}$, vibration response amplitude A has a clear jump. Obviously, the jump is resulted in vibration of vibration instability, but the inherent mechanism needs a further investigation.

V. CONCLUSION

For a liquid-solid system, a flexible cantilever opening beam with interior inlay fluid, under assumptions of fluid movements and flexible beam vibrations, coupling nonlinear dynamic controlling equations are established by considering the damping and axial elongation of the flexible beam. In the case of free vibrations without excitations, the Jacobi coefficient matrix characteristic polynomial of perturbation equations are analyzed according to the Hopf bifurcation theory and relative algebraic criterions of real coefficient characteristic polynomial's roots, dynamic Hopf bifurcation critical flow velocity $V_{\text{div-dy}}$ was obtained. It was proved that the free vibration system would be vibration instability when the flow velocity of fluid exceeded critical flow velocity. The classical 4^{th}- order Runge-Kutta method is utilized to solve the vibration response when the system is under low frequency sinusoidal excitations, and results showed that when the difference between exciting frequency and vibration frequency of unperturbed vibration instability was great, vibration response had beat frequency. Whereas, when the difference is not so great, beat frequency disappeared. In the latter condition, corresponding analysis results showed that vibration response amplitude A had a clear jump when the flow velocity crossed the critical flow velocity $V_{\text{div-dy}}$.

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