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Nonlinear vibrations ribbed circular plate under influence of pulse loading

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ABSTRACT: The paper is devoted to numerical study of stressed-strain state of circular, ribbed circular plate under influence of pulse loading, taking into account of geometrical nonlinearity. Structure consists of boarding and reinforced rib which materials are identical and follow Hooke's law. Rib section is constant on radius of the plate and angular coordinate. The height of the rib and its position is set by means of individual unit function. It is considered that vibrations are excited by pulse loading enclosed on an external surface of the plate. Numerical method is applied to the problem solution. Computation scheme is based on definition of displacements and angles of rotation on mesh points, and deformations, forces, momentum and shear forces on the element center. The deflection of the central point and force depending on the rib position are computed. In particular are established that the least deflection of the central point is reached at an rib position on the middle of distance from the center to plate edge. And position of the rib near to plate edge can lead to reduction of bearing ability of the structure in comparison with not reinforced plate.

KEYWORDS: plate, geometrical nonlinearity, deformation, rib displacement, pulse loading, deflection, force.

I. INTRODUCTION

In such areas as shipbuilding, aircraft engineering and construction are widely applied thin plates, working under the influence of the high dynamical loadings [1]. For giving of necessary rigidity of a plate in such conditions it is supported with ribs which increase its durability and can transfer the efforts, close to the concentrated. Therefore, in researches of dynamic character of the plates strengthened by edges of rigidity, the special attention is given to definition strength characteristics of plates of the increased rigidity which are under the influence of compressing loads [2]. Problems of application of numerical methods, in particular, method of finite differences, for the solution of problems of dynamics of ribbed rectangular and circular plates are considered in works [3], [4]. In work [4] with application of the generalized delta-function various ways of modeling of influence of supporting ribs on dynamic behavior of plates and shells at pulse loadings are analyzed. In works [5], [6] analytical solution of the problem on stressed-strain state of flexible the ribbed plate is given at local reinforcing, recommendations about reinforcing of plates with irregularity are developed and the way and algorithm of computation of reinforced concrete plates with infringements of regularity of structure, in the form of the apertures, supporting ribs and others are given.

The problem about nonlinear vibrations of the circular plate bearing concentric rigid weight is solved in [7]. Evaluation of stressed-strain state of the ribbed spherical dome at pulse loading in [8] is given. The detailed analysis of the scientific works devoted to numerical computation of various aspects of ribbed structures, is resulted in works [9], [10]. The present paper is devoted numerical research of the stressed-strain state of ribbed circular plate under the influence of pulse loading taking into account geometrical nonlinearity.

II.FORMULATION OF THE PROBLEM

The circular plate reinforced with ring rib of rigidity and supported along the edge is considered. The rib has quadrangular cross-section and is attached on internal surface of the plate. On external surface of the plate the pulse loading P changing in time on exponential law. For the considered plate the cylindrical system of co-ordinates (r, φ, z) is used. Thus the axis z is directed perpendicularly to plate plane. It is considered that the structural system consists of a

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Vol. 5, Issue 3 , March 2018

boarding and the reinforced rib which materials are identical and submit to the Hooke's law. The rib section is constant on radius of a plate and angular co-ordinate. Height of rib $H(r)$ and its position is set by delta-function $\delta(r-r_0)$:

$$H(r)=h_0\delta(r-r_0),$$

where $\delta(r-r_0)=\begin{cases}0, & r<a,\ r>b,\\ 1, & a<r<b,\end{cases}\quad a=r_0-c/2,\quad \mathrm{b}=r_0-c/2;$

r_0 is middle point coordinate of contacting boarding and rib; $h_0,\ c-$ height and width of rib cross section.

For the description of stressed-strain state of the plate it is used the nonlinear Timoshenko type theory of plates. In this case, with the account axisymmetry of problem the motion equations of the plate become [11]:

$$(N_1r)'-N_2=r\rho[\ddot{u}(h+F)+\ddot{\psi}S];$$
$$(rQ)'+(N_1rw')'=r\rho\ddot{w}(h+F)-rP; \quad (1)$$
$$(rM_1)'-M_2-rQ=r\rho\left[\ddot{\psi}(h^3/12+J)+\ddot{u}S\right],$$

where ρ is density of material, h is thickness of boarding, $F(r),\ S(r),\ J(r)$ are area cross section area, static moment, inertia moment of rib's with unit width and height $H(r)$, respectively;
u,w are axial displacement and deflection, respectively; ψ is angle of rotation middle surface normal of plate.

Boundary conditions along edges of plate: $u=w=\psi=0$. Except boundary conditions, there are also symmetry conditions at the plate center:

$$u=\frac{\partial w}{\partial r}=\psi=0,\quad at\quad r=0.$$

Initial conditions are zero at $t=0$ i.e. $u=\dot{u}=0; w=\dot{w}=0$.

Displacements of any point on normal of middle surface of coordinate z a) $u^z=u+z\psi$; $w^z=w$ and account of axisymmetry of loading b) $v^z=v=0$, where u,v,w – displacements of middle surface along the axis coordinates; ψ is angle of rotation middle surface normal of plate.

Lengthening deformations in the middle surface [11]

$$\varepsilon_1=\frac{\partial u}{\partial r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2;\qquad \varepsilon_2=u/r, \quad (2)$$

and deformations along the thickness

$$\varepsilon_1^z=\varepsilon_1+z\frac{\partial\psi}{\partial r};\ \varepsilon_2^z=\varepsilon_2+z\frac{\psi}{r}. \quad (3)$$

Relations between stresses and deformations of boarding are [12]

$$\sigma_i^0=\frac{E}{1-\mu^2}(\varepsilon_i^z+\mu\varepsilon_j^z);\qquad \sigma_{13}^0=\frac{E}{2(1+\mu)}\varepsilon_{13}^z, \quad (4)$$

where $\varepsilon_{13}^z=f(z)\left(\frac{\partial w}{\partial r}+\psi\right);\quad j=\begin{cases}2, & at\quad i=1;\\ 1, & at\quad i=2.\end{cases}$

Here $E$and μ modulus of elasticity and Poisson's ratio of plate material; *f (z)* is the function characterizing the law of distribution of stress σ_{13}^0 on a thickness of a plate. $f(z)=f_0(z)$ for smooth part of plate and $f(z)=f_1(z)$ on ribs

$$f_o(z)=6[0{,}25-\left(\frac{z}{h}\right)^2];\ f_1(z)=\frac{3h(h+2H)}{2(h+H)^2}\left(1+2\frac{z}{h}\right)\left(1-2\frac{z}{h+2H}\right),$$

ISSN: 2350-0328
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Vol. 5, Issue 3 , March 2018

where H – height of rib.

Axial forces, shear forces and momentums on unit length, for plate and reinforced rib are

$$Q = Q^0 + Q^R;\ N_i = N_i^0 + N_i^R;\quad M_i = M_i^0 + M_i^R,\ (i = 1,2).$$

Axial forces, shear forces and momentums for boarding

$$N_i^0 = \int_{-h/2}^{h/2} \sigma_i^0 dz = \frac{Eh}{1-\mu^2}(\varepsilon_i + \mu\varepsilon_j);\ M_i^0 = \int_{-h/2}^{h/2} \sigma_i^0 z dz = DL_i;\ Q^0 = \int_{-h/2}^{h/2} \sigma_{13}^0 dz = \frac{5}{6}\frac{Eh}{2(1+\mu)}\varepsilon_{13};$$

$$L_1 = \frac{\partial\psi}{\partial r} + \mu\frac{\psi}{r};\ L_2 = \frac{\psi}{r} + \mu\frac{\partial\psi}{\partial r};\ D = \frac{Eh^3}{12(1-\mu^2)};\ (i = 1,2). \quad (5)$$

For definition of axial forces, momentums and shear forces acting on ribs, all structure is considered as a plate with step-variable thickness. Stresses acting on ribs, we will define under following formulas

$$\sigma_i^R = G(\varepsilon_i^z + \upsilon_1\varepsilon_j^z);\ \sigma_{13}^R = G_{13}\varepsilon_{13}^z; \quad (6)$$

$$G = \frac{E}{1-\mu};\qquad G_{13} = \frac{5}{12}\frac{E}{(1+\mu)},\ ,$$

where E, μ are elastic constants of material.

Axial forces, momentums and shear forces acting on sections of rib are

$$N_i^R = \int_{h/2}^{h/2+H} \sigma_i^R dz = A(\varepsilon_i + \mu\varepsilon_j) + BL_i;\ M_i^R = \int_{h/2}^{h/2+H} \sigma_i^R z dz = B(\varepsilon_i + \mu\varepsilon_j) + CL_i; \quad (7)$$

$$Q^R = \int_{h/2}^{h/2+H} \sigma_{13}^R dz = D_{13}\left(\frac{\partial w}{\partial r} + \psi\right);\ A = GF; B = GS; C = GJ; D_{13} = \mathrm{G}_{13}\mathrm{H}(\mathrm{r_i});\quad (\mathrm{i} = 1,2).$$

III. NUMERICAL METHOD OF THE PROBLEM SOLUTION

The computation scheme is based on definition of displacements and angles of rotation on mesh point, and deformations, axial forces, momentums and shear forces on the element of center. Approximation of derivatives on element has a usual appearance. For approximation of the equations of motion (1) which are aligned in central points, the central differences are used. Thus not differentiated members in the equations (1) are led to mesh by averaging of corresponding values in elements. Derivations by time approximated as

$$\left[\frac{\partial^2\varphi}{\partial t^2}\right]_i^n = \frac{1}{\tau^2}\left[\varphi_i^{n+1} - 2\varphi_i^n + \varphi_i^{n-1}\right], \quad (8)$$

where τ is step of time, n is index defining of time layer.

Derivatives in system of the differential equations (1) are replaced mentioned difference scheme which have allowed to receive following recurrent formulas

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + \tau^2\left\{U_i^n / a - b \times (\psi_i^n - bU_i^n / a)/(ac - b^2)\right\};$$

$$w_i^{n+1} = 2w_i^n - w_i^{n-1} + \tau^2 W_i^n / a; \quad (9)$$

$$\psi_i^{n+1} = 2\psi_i^n - \psi_i^{n-1} + \tau^2 \times (\psi_i^n - bU_i^n)/a/(c - b^2/a)$$

where $a = \rho(h+F)$, $b = \rho S$, $c = \rho(h^3/12 + J)$;

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 3 , March 2018

$$U_i^n = \frac{(rN_1)^n_{(i+1/2)} - (rN_1)^n_{(i-1/2)}}{\Delta r_{i+1/2} r_{i+1/2}} + \frac{(N_2)^n_{(i+1/2)} - (N_2)^n_{(i-1/2)}}{2r_{i+1/2}};$$

$$W_i^n = \frac{(rQ)^n_{(i+1/2)} - (rQ)^n_{(i+1/2)}}{\Delta r_{i+1/2} r_{i+1/2}} + \left(\frac{w_i^n - w_{i-1}^n}{\Delta r_i}\right)\frac{1}{\Delta r_{i+1/2}} \times \frac{(rN_1)^n_{(i+1/2)} - (rN_1)^n_{(i-1/2)}}{\Delta r_{i+1/2}} + \frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{(\Delta r_i)^2} \times$$

$$\frac{(N_1)^n_{(i+1/2)} + (N_1)^n_{(i-1/2)}}{2} + P^n_{i+1/2};$$

$$\psi_i^n = \frac{(rM_1)^n_{i+1/2} - (rM_1)^n_{i-1/2}}{\Delta r_{i+1/2} r_{i+1/2}} - \frac{(M_2)^n_{i+1/2} + (M_2)^n_{i-1/2}}{2r_{i+1/2}} - \frac{Q^n_{i+1/2} + Q^n_{i-1/2}}{2};$$

Expressions (9) allows from relations (2) and (3) to receive finite-difference formulas for a deformation component

$$(\varepsilon_1)^n_{(i+1/2)} = \frac{u_{i+1}^n - u_i^n}{\Delta r_{i+1/2}} + \frac{1}{2}\left(\frac{w_{i+1}^n - w_i^n}{2\Delta r_{i+1/2}}\right)^2; \ (\varepsilon_1^z)^n_{i+1/2} = (\varepsilon_1)^n_{i+1/2} + z\frac{\psi_{i+1}^n - \psi_i^n}{\Delta r_{i+1/2}};$$

$$(\varepsilon_2^z)^n_{i+1/2} = (\varepsilon_2)^n_{i+1/2} + z\frac{\psi_{i+1}^n + \psi_i^n}{2r_{i+1/2}}; \ (\varepsilon_2)^n_{(i+1/2)} = \frac{u_{i+1}^n + u_i^n}{2r_{i+1/2}}; \ (\varepsilon_{13}^z)^n_{i+1/2} = f(z)\left(\frac{w_{i+1}^n - w_i^n}{\Delta r_{i+1/2}} + \frac{\psi_{i+1}^n + \psi_i^n}{2}\right). \tag{10}$$

Values of stresses, axial forces, momentums and shear forces can evaluate by formulas (4) - (7), using expressions (10). Formulas (9) concern only internal points of mesh. For definition of values of functions on edge points boundary conditions are used a) and b). Thus, conditions of a rigid jamming are approximated accurately, and symmetry conditions is written in finite differences, using following expression

$$\left[\frac{\partial \varphi}{\partial r}\right]_1^n = \frac{3\varphi_1^n - 4\varphi_2^n + \varphi_3^n}{2\Delta r_2}. \tag{11}$$

Functions on two adjacent time layers, necessary to start computations, give initial conditions from which follow

$$w_i^1 = w_i^2 = 0. \tag{12}$$

Finite-difference expressions (9), (11) provide an approximation second order. Equalities (12) approximate initial conditions with the first order of accuracy. The general misalignment approximation of considered boundary value problem in difference relations does not exceed $O(\Delta r^2 + \tau^2)$. At approximation of steps to zero and misalignment also approximate to zero. Hence, equations in finite difference scheme approximate the initial differential equations.

Stability research finite difference schemes are complex problem. It is especially difficult to solve it for the schemes approximating multidimensional nonlinear boundary value problems. Research of stability of the scheme (9) is conducted by numerical experiments. Ranging values of steps of the mesh providing stability of computations, are defined from Courant-Friedrichs-Levy (CFL) condition [12] $\tau_1 \le \Delta r / c_1$, where $c_1 = \sqrt{E/\rho(1-\mu^2)}$.

IV.RESULTS OF NUMERICAL COMPUTATION AND THEIR ANALYSIS

By means of the equations (1) are computed deflection and axial forces of rigidly jammed ribbed circular plate, under the influence of in regular intervals distributed loading $P = P_0 e^{-t/\alpha}$. For computations accepted values $P_0 = 2{,}5 MPa$; $\alpha = 10^{-3} s$. The plate has epy ring edge with height $h_0 = 4 \cdot 10^{-2}\, m$ and width $c = 3{,}33 \cdot 10^{-2}\, m$. Geometrical and elastic parameters of plate are $R_0 = 0{,}5 m$; $h = 10^{-2}\, m$; $E = 75600 MPa$; $\mu = 0{,}3$; $\rho = 2640 kg/m^3$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 5, Issue 3 , March 2018

Displacements and forces in sections both not reinforced, and ribbed plate in cases of position of ribs on circles with radiuses $r_0 = 0{,}10;\ 0{,}25;\ 0{,}40\,m$ are computed. Results of computations are plotted in fig. 1 - 4. For the purpose of estimation of influence of the rib on deformation of the plate on time comparison of deflections ribbed plate with not reinforced boarding is made. In the fig. 1 dependences of deflection w of the central point for the rigidity of not reinforced and ribbed plate, in points $r_0 = 0{,}10;\ 0{,}25;\ 0{,}40\,m$ are presented. From diagrams follow that the greatest value of displacement is reached in case of not reinforced plate at $t = 12{,}6 \cdot 10^{-3} s$ ($w = 56{,}28 \cdot 10^{-3} m$); for the reinforced plate in points $r_0 = 0{,}10\,m$ at $t = 14{,}7 \cdot 10^{-3} s$ ($w = 43{,}54 \cdot 10^{-3} m$); $r_0 = 0{,}25\,m$ at $t = 14{,}7 \cdot 10^{-3} s$ ($w = 36{,}51 \cdot 10^{-3} m$); $r_0 = 0{,}40\,m$ at $t = 12{,}6 \cdot 10^{-3} s$ ($w = 48{,}85 \cdot 10^{-3} m$). The least deflection is reached at the rib position on the middle of distance from the center to plate edge.

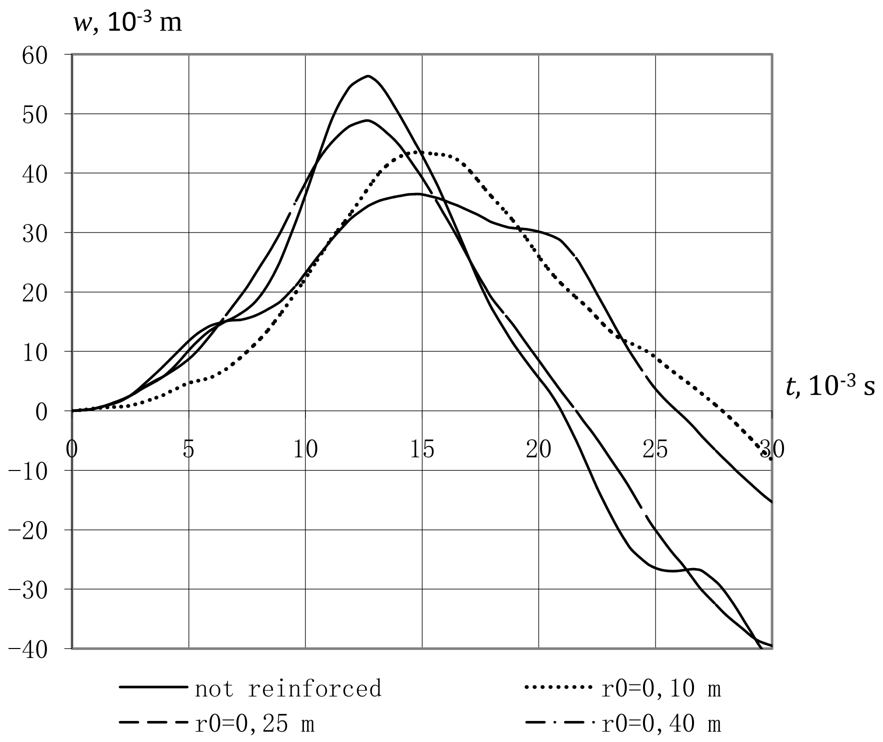

Fig.1.Deflections of central point of not reinforced and reinforced plate in cases of positions rib on points $r_0 = 0{,}10;\ 0{,}25;\ 0{,}40\,m$

In fig. 2 diagrams of dependences of deflections by radial co-ordinate are plotted. The presented curves show that the central point will be has the least deflections at position of the rib more close to it ($r_0 = 0{,}10\,m$) and at rib position on the middle of radius of the plate ($r_0 = 0{,}25\,m$). Without dependence from the position of the rib the greatest deflection on the central point in comparison with other points of the plate always has. Thus on radius fading character on enough smooth curve has deflection changes. The central point at position of the rib near to plate edge has the deflection close to a deflection of not reinforced plate.

In Figures 3 and 4 for time moment distributions of longitudinal force N_2 and shear forces Q are plotted. Diagrams show its jumping change. Jump takes place in points of the rib position and in their small neighborhood. Value of jump is less at $r_0 = 0{,}10\ m$ ($7{,}63 \cdot 10^{-6} Pa$), it is more at $r_0 = 0{,}40\,m$ ($13{,}72 \cdot 10^{-6}\ Pa$), and an average at $r_0 = 0{,}25\,m$ (

ISSN: 2350-0328
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Vol. 5, Issue 3 , March 2018

$11{,}34 \cdot 10^{-6} Pa$). The axial force N_2 for not reinforced plate has no jumps, changes smoothly, aspiring to zero and having the greatest value at $r = 0$ ($8{,}09 \cdot 10^{-6} Pa$). Values N_2 in cases of presence of the rib also aspire to zero with increases of radial coordinate.

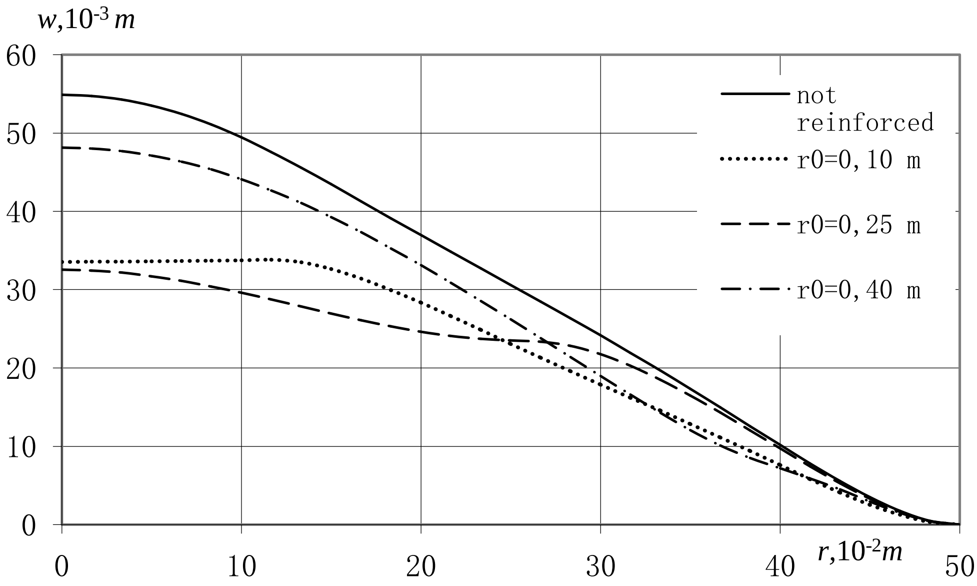

Fig.2. Influence of rib position on deflection w at the moment $t = 1{,}2 \cdot 10^{-3} s$ for different values of $r_0 = 0{,}10;\ 0{,}25;\ 0{,}40 m$

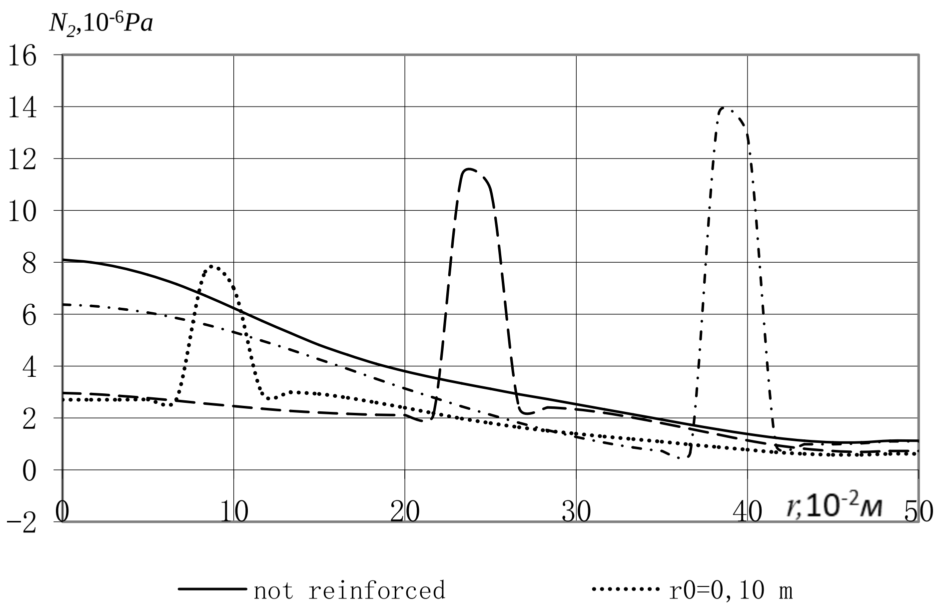

Fig.3. Dependence of force N_2 in cases of rib position on various distances for time moment $t = 1{,}2 \cdot 10^{-3} s$.
$r_0 = 0{,}10;\ 0{,}25;\ 0{,}40 m$

ISSN: 2350-0328
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Vol. 5, Issue 3 , March 2018

From fig. 4 follows that shear force also changes smoothly, having maximum points. For example, the greatest value is reached in a point $13{,}33 \cdot 10^{-2} m$, equal $30{,}33 \cdot 10^{-6} Pa$. Thus, unlike other characteristics (deflection, force, momentum) values of force Q for the fixed moment of the time for not reinforced plate can, does not surpass corresponding values for the reinforced plate at $r_0 = 0{,}10 m$ but at values $r_0 = 0{,}25\, m$ and $0{,}40 m$ they surpass them. The shear force at the rib position on the middle radius of plate, i.e. at $r_0 = 0{,}25\, m$ has the least values in comparison with other cases of the rib position.

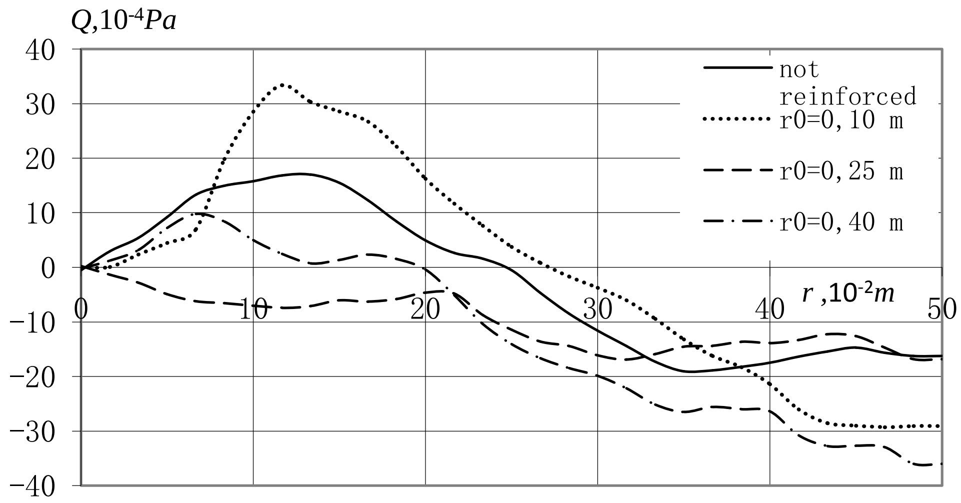

Fig.4. Dependence between force Q and r in cases of rib position on various distances for time moment $t = 1{,}2 \cdot 10^{-3} s$.
$r_0 = 0{,}10;\ 0{,}25;\ 0{,}40 m$

V. CONCLUSION

The rib located near to a pole of the circular plate, reduce the maximum values of amplitude, the period and frequency of vibrations of the central point, and also value of bending deformations in the central part of the plate. Not reinforced plate has the greatest deflection of the central point. Its least deflection is reached at rib position on the middle of distance from the center to plate edge.

Position of the rib near to plate edge can lead to reduction of bearing ability of the structure in comparison with not reinforced plate.

Rigidity of epy plate can be increased at the expense of increase in a thickness of reinforced ribs at constant value of their cross-section area. Axial force has jump in the point of rib arranged and its small neighborhood. With removal of a point of an arrangement of an edge from the center of a plate value of jump increases. Shear force decreases with increase of values of radial coordinate. Thus, for some moments of time, unlike other mechanical characteristics (deflection, axial force), values shear forces for not reinforced plate can be less, than corresponding values of this force for the reinforced plate.

REFERENCES

[1.] Jigalko Yu. P., Dimitrieva L. M., Dynamics of ribbed plates and shellsк// Research on the theory of plates and shells, 1978, issue 13, PP..3–30(in Russian).
[2.] KayukYa.F. Geometrically Nonlinear Problems in the Theory of Plates and Shells, NaukovaDumka, Kyiv, 1987(in Russian).

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 5, Issue 3 , March 2018

[3.] GaliyevSh.U., Karshiyev A.B. Numerical analysis of models accuracy of ribbed spherical dome, subjected to pulse loading// Problems of Strength.- 1990. - No5. - PP. 83-86(in Russian).
[4.] Khalmuradov R.I., KarshiyevA.B.Computationofstructureelementswithinfringementsofregularityofstructure. - Tashkent, 2002.-285 p(in Russian).
[5.] Khalmuradov R.I.,MikhaylovB.K. Analytical solution of the problem on stressed-strain state flexible plate at local reinforcing// Researches on the building mechanics and reliability of building strutures. - M, 1992. - PP.153-160(in Russian).
[6.] Khalmuradov R.I.Computationmethodofribbedplates//Uzbekjournalofproblemsofmechanics,1995-6-PP.23-32(in Russian).
[7.] Chiang D.C, Chen S.S. Large amplitude vibration of a circular plate with concentric rigid mass. – "Pap. ASME", 1971, NAPMW-11, 7p.
[8.] Karshiyev A.B.Research of the stressed-strain state of ribbed spherical dome at pulse loading// Uzbek journal of problems of mechanics,2000-6-PP.51-54(in Russian).
[9.] Benerjee S., Kundu T. Symmetric and anti-symmetric rayleigh-lamb modes in sinusoidally corrugated waveguides - International Journal of Solids and Structures, 43 (21) (2006), pp. 6551-6567
[10.] Benerjee S., Kundu T. Elastic wave propagation in sinusoidally corrugated waveguides -Journal of Acoustical Society of America, 119 (4) (2006), pp. 2006-2017.
[11.] Volmir A.C. Nonlinear dynamics of plates and shells.-M.: Nauka,1972-432p (in Russian).
[12.] Galiyev Sh. U., Babich Yu. N., Juraxovskiy C.B., Nechitaylo N. V,Romashenko V.A. Numerical modeling of wave processes in the limited environments.-Kiev.: Naukova Dumka,1989.-199p(in Russian).